Introduction

- question 1:
  Give an example of a equation and and a differential equation.
  answer 1:
  For instance:
  1. An equation: $x^2 + 3x + 2 = 0$.
  2. A differential equation $y'(x) + y(x) = \sin(x)$
- Question 2:
  What is the difference between them?
  answer 2:
  The main difference is that solutions of equations are numbers and solutions of differential equations are functions. These are maps from sets (parts) of $\mathbb{R}$ (real numbers) to sets (parts) of $\mathbb{R}$

Introduction

For you there are three kinds of differential equations which you can solve by hand (analytically):

- the separable first order differential equation,

  example1: $y'(x) = y^2(x)x$,

- the first order linear differential equation,

  example2: $xy'(x) + y(x) = x$ and

- the second order linear differential equation with constant coefficients

  example3: $y''(x) + 2y'(x) + y(x) = \sin(x)$

During the lectures we shall pay attention to them and explain their importance

(you will find theory about it in the book of Stewart chapter 9 and 17).

Are you well suited for modeling?

- Question1: Give the derivatives of $x^2, \sin(x), \sin(x^2)$

  Answer1:

  $2x, \cos(x), \cos(x^2)2x$

- Question2: Given the differential equation $xy'(x) + y(x) = 0$ (a first order differential equation). Which of the given functions is a solution: $\sin(x)$, 1 , 0, $\frac{1}{x}$?

  Answer2:

  Only 0 and $\frac{1}{x}$

- Question3: Solve $x^2 + x - 2 = 0$, $xy(x) = x + 1$, $xy'(x) = x + 1$, $y'(x) = y(x)$, $y''(x) = -y(x)$

  Answer3:

  x is 1 or -2 (numbers!), $y(x) = \frac{x+1}{x}$ (one function!) , $y(x) = x - \frac{1}{x^2} + C$ with C a constant (a lot of functions!), $y(x) = Ke^x$ with K a constant, $y(x) = C_1 \cos(x) + C_2 \sin(x)$ with C_1 and C_2 constants

Modeling with MAPLE, a start:

- Always start a Maple-sheet in **worksheetmode**.
- Further we expect knowledge of the following maple commands: restart, diff, solve,unapply, op, and plot.
- A Maple Demo 1 and comments Examine!! the Maple sheet of demo 1 for getting the meaning of the commands.
- Special attention for unapply:

The Maple-command "unapply"

The command unapply is strongly connected with meaning or definition of a function f

Question:

What is de difference between f and $f(x)$?

Answer:

Function (Map) f stands for a action on elements of a set A to elements of a set (another) B.

To define a function three things are needed:

- two sets A and B,
- and how the action is.

The Maple-command "unapply"

Some examples:

- Function f defined by: $A = \mathbb{R}$,$B = \mathbb{R}$ (the sets) with $f(x) = x^2$ (defines the action).The action on 2 by f gives 4.
- Function g defined by: $A = \mathbb{R}^2$,$B = \mathbb{R}$ (the sets) with $g(x, y) = \sqrt{x^2 + y^2}$ (defines the action).The action on $(4, 3)$ by g gives 10.

Remark: In "modeling course" it is common that the sets of the action are not defined. In this case one mostly takes for first set A the "largest" set for which the action is defined, for set B mostly $\mathbb{R}$. So
$f(x) = \sqrt{x}$ means the action between the sets $\mathbb{R}_0^+$ and $\mathbb{R}$.

The Maple-command "unapply"

Suppose that in Maple H is declared as the expression $x^2 + \sqrt{x}$.
The Maple action F:=unapply(H,x); means that F is defined as a function (action) between the sets $\mathbb{R}_0^+$ and $\mathbb{R}$. The result of action of F on (number) x is the (number) $x^2 + \sqrt{x}$.
The latter is noted as $F(x)$.

An application:

Question:

Why are differential equations important?

Answer:

Because of the second Law of Newton:

$\mathbf{F}^* = -m\mathbf{a}$ and

$\sum_i \mathbf{F}_i = \mathbf{0}$

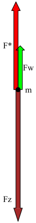
These vector equations result (after defining coordinates) in differential equation(s).

An application:

Question: Describe the motion of a point mass falling down from rest under influence of gravity with air friction.

Answer:

- What is the FBD:



- Given from the FBD: the vectorequation :
  $\mathbf{F}^* + \mathbf{F}_w + \mathbf{F}_z = \mathbf{0}$.
- We need one axis, we choose the x-axis downwards. What is de corresponding differential equation?
  $m\ddot{x} + c_w\dot{x} = mg$. (linear friction)
  if v is defined as $\dot{x}$ then we get:
  $m\dot{v} + c_w v = mg$
  with initial condition $v(0) = 0$.
- After the thinking give a Maple solution.