Introduction

➤ question 1:
Give an example of a equation and and a differential equation.

answer 1:
For instance:

1. An equation: \(x^2 + 3x + 2 = 0\).
2. A differential equation \(y'(x) + y(x) = \sin(x)\)

➤ Question 2:
What is the difference between them?

answer 2:
The main difference is that solutions of equations are numbers and solutions of differential equations are functions. These are maps from sets (parts) of \(\mathbb{R}\) (real numbers) to sets (parts) of \(\mathbb{R}\)
Introduction

For you there are three kinds of differential equations which you can solve by hand (analytically):

- **the separable first order differential equation**,  
  example 1: \( y'(x) = y^2(x)x \),

- **the first order linear differential equation**,  
  example 2: \( xy'(x) + y(x) = x \) and

- **the second order linear differential equation with constant coefficients**,  
  example 3: \( y''(x) + 2y'(x) + y(x) = \sin(x) \)

During the lectures we shall pay attention to them and explain their importance

( you will find theory about it in the book of Stewart chapter 9 and 17).
Are you well suited for modeling?

▶ **Question 1:** Give the derivatives of $x^2$, $\sin(x)$, $\sin(x^2)$

**Answer 1:**
$2x$, $\cos(x)$, $\cos(x^2)2x$

▶ **Question 2:** Given the differential equation $xy'(x) + y(x) = 0$ (a first order differential equation). Which of the given functions is a solution: $\sin(x)$, $1$, $0$, $\frac{1}{x}$?

**Answer 2:**
Only $0$ and $\frac{1}{x}$

▶ **Question 3:** Solve $x^2 + x - 2 = 0$, $xy(x) = x + 1$, $xy'(x) = x + 1$, $y'(x) = y(x)$, $y''(x) = -y(x)$

**Answer 3:**
$x$ is $1$ or $-2$ (numbers!), $y(x) = \frac{x+1}{x}$ (one function!), $y(x) = x - \frac{1}{x^2} + C$ with $C$ a constant (a lot of functions!), $y(x) = Ke^x$ with $K$ a constant, $y(x) = C_1 \cos(x) + C_2 \sin(x)$ with $C_1$ and $C_2$ constants
Modeling with MAPLE, a start:

- Always start a Maple-sheet in **worksheetmode**.
- Further we expect knowledge of the following maple commands: `restart`, `diff`, `solve`, `unapply`, `op`, and `plot`.
- A Maple Demo 1 and comments **Examine!!** the Maple sheet of demo 1 for getting the meaning of the commands.
- Special attention for `unapply`:
The Maple-command "unapply"

The command *unapply* is strongly connected with meaning or definition of a function $f$

**Question:** What is the difference between $f$ and $f(x)$?

**Answer:** Function (Map) $f$ stands for a *action* on elements of a set $A$ to elements of a set (another) $B$.

To define a function three things are needed:

- two sets $A$ and $B$,
- and how the action is.
The Maple-command "unapply"

Some examples:

- Function $f$ defined by: $A = \mathbb{R}, B = \mathbb{R}$ (the sets) with $f(x) = x^2$ (defines the action). The action on 2 by $f$ gives 4.

- Function $g$ defined by: $A = \mathbb{R}^2, B = \mathbb{R}$ (the sets) with $g(x, y) = \sqrt{x^2 + y^2}$ (defines the action). The action on $(4, 3)$ by $g$ gives 10.

Remark: In "modeling course" it is common that the sets of the action are not defined. In this case one mostly takes for first set $A$ the "largest" set for which the action is defined, for set $B$ mostly $\mathbb{R}$. So $f(x) = \sqrt{x}$ means the action between the sets $\mathbb{R}_0^+$ and $\mathbb{R}$.
The Maple-command "unapply"

Suppose that in Maple $H$ is declared as the expression $x^2 + \sqrt{x}$.
The Maple action $F := \text{unapply}(H, x)$; means that $F$ is defined as a function (action) between the sets $\mathbb{R}^+_0$ and $\mathbb{R}$. The result of action of $F$ on (number) $x$ is the (number) $x^2 + \sqrt{x}$. The latter is noted as $F(x)$. 
An application:

Question: Why are differential equations important?

Answer: Because of the second Law of Newton:

\[ F^* = -ma \]

and

\[ \sum_i F_i = 0 \]

These vector equations result (after defining coordinates) in differential equation(s).
An application:

Question: Describe the motion of a point mass falling down from rest under influence of gravity with air friction.

Answer:

▸ What is the FBD:

◮ Given from the FBD: the vector equation:
  \[ F^* + F_w + F_z = 0. \]

◮ We need one axis, we choose the x-axis downwards. What is the corresponding differential equation?
  \[ m\ddot{x} + c_w \dot{x} = mg. \] (linear friction)
  if \( v \) is defined as \( \dot{x} \) then we get:
  \[ m\dot{v} + c_w v = mg \]
  with initial condition \( v(0) = 0. \)

◮ After the thinking give a Maple solution.