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- Maple mostly (depending on F) gives this solution:

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In the homogeneous part you see the expression $\sqrt{b^2-4\,ca}$, in the case of a negative argument of the square root, Maple still gives an answer which is expressed in sin- and cos-functions.

An application part 1

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We investigate in Demo 2, part 1 with Maple some Mass, Spring and Damper systems without external forces

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Examine the sheet especially the proper use of unapply
(We define a function of four variables.)

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We investigate in Demo 2, part 2 with Maple some Mass, Spring and Damper systems *with* external forces

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  $Heaviside(t) = 0$ if $t < 0$ and $Heaviside(t) = 1$ if $t \geq 0$

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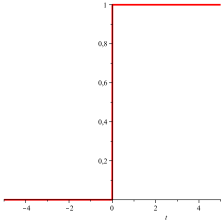

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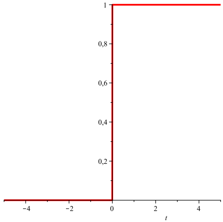

- The function has a derivative which is named as $Dirac - \delta$-function. (In mathematical sense it is not normal function is an example of so called Distributions or Verallgemeinte Functionen (German))
  For application of this see the Maple Demo part 2.

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Given on $\mathbb{R}$ the function f with $f(x) = x$ if $x \leq 0$ and $f(x) = \sin(x)$ if $x \geq 0$

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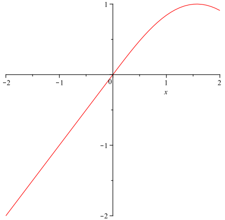

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Example 2:

Given on $[-2\pi, 2\pi]$ the function $g(x) = \sin(x)$ and glue function $f(x) = -1$ if $g'(x) > 0$ and $f(x) = -1$ if $g'(x) \leq 0$

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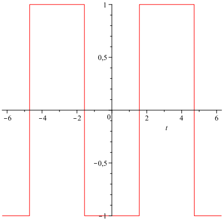

Some special functions: mechanical friction

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Example 3:

You can describe the Dynamic Mechanical friction force $MF(t)$ with magnitude 1 [N] for instance as $MF(t) = -1$ if $x'(t) > 0$ and $MF(t) = 1$ if $x'(t) < 0$ where $x(t)$ is displacement function of a mass.

An application part 2

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Examine this!

The cliff hanger:

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Given:

$$\ddot{x}(t) + 0.1\dot{x}(t) + x(t) = 2(Heaviside(t-1) - Heaviside(t-5) + 0.1MF(t)$$

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Answer:
The DE is a mathematical representation of a point mass ($m = 1$[kg]), attached on a spring (with constant 1[N/m]),with linear air drag, (constant 0.1[Ns/m]), exserted by a pulse force of magnitude of 2 [N] from second 1 to second 5, and influenced by a mechanical friction force of magnitude 0.1 [N].

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See last part of the Maple sheet for a numerical solution of the problem.