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are different notations for the same DE.

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A first order DE with general form $y' = F(x, y)$, is called separable if we can write $F(x, y)$ as a product $g(x)h(y)$ where g is a function of the only variable x , and h a function of the only variable y .

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Answer: Only the first and the last one.

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- ▶ Solving for y gives (in steps):
 - ▶ $|y| = e^{x+C}$,
 - ▶ $|y| = e^x e^C = Ke^x$, with K a positive constant

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- ▶ The final answer is $T(t) = 20 + 80e^{-Kt}$, for getting the value of K an extra data is needed.

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- ▶ The Maple answer:

$$y(x) = \left(\int Q(x) e^{\int P(x) dx} dx + _C1 \right) e^{-\int P(x) dx}$$

According to the Maple answer is calculation of the solution of this type of DE nothing more than calculation of two integrals.

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If not: try a numerical solver. (see Maple Demo):

(Explanation of numerical methods is topic of the next Lecture.)

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