# A little Exam:

Classify the following DE-'s:

- DE1: $y' = y^2 + xy$. Type1: First order not sep. not lin.
- DE2: $y' = y + xy$. Type2: First order sep. and lin.
- DE3: $y' = xy^2$. Type3: First order sep. and not lin.
- DE4: $y' = y + x$. Type4: First order not sep. and lin.
- DE5: $y'' = y + xy'$. Type5: Second order not lineair with constant coefficients, still linear
- DE6: $\frac{y''+2y+y'}{x^2+\sin(x)} = 1$. Type6: Second order lineair with constant coefficients
- DE7: $\frac{v''}{(\sqrt{1+v'^2})^3} = \frac{M(x)}{E(x)I(x)}$. Type7: First order Sep.

There is Mars in offing(!!) fore the first one who recognize the last DE(It determines the the shape of the inflection line)!

# Programming; the for loop:

- Question What is the result of the following Maple-command lines:

  value:= 0; n := 5; for i from 1 to n do value := value+i end do

- Answer: 15
- Question Calculate the product $n \cdot (n-1) \cdot (n-2)...32 \cdot 1$ using a "for-loop" for $n = 10$
- Maple-command lines:

  Answer: n := 10; produkt := 1; for k from 2 to n do produkt := produkt*k: od: produkt;

Remark: If you look to the command lines you will see two types of closures of the "for-loop".

# Creating a own Maple-command: The procedure

In the next maple command lines is the *new command, procedure* productnaturalnumbers(n) defined:

```
> productnaturalnumbers := proc (n)
local produkt, k;
produkt := 1;
for k from 2 to n
do
produkt := produkt*k
end do;
return produkt end proc;
```

From now we we can use the command productnaturalnumbers, for instance:

```
> productnaturalnumbers(10);
```

Result: 628800

# Creating a own Maple-command: The procedure

Remarks:

- In this procedure is are defined two local variables, ’produkt’ and ’k’, this means that "outside" the procedure they have no meaning.
- The procedure ’productnaturalnumbers’ is a function on the natural numbers its action on the number n is: the product of of all numbers from 1 up to n.
- For other example see the maple sheet.

# A numerical DE-solver: Euler-method:

Given the general first order DE: $y'(x) = F(x, y(x))$.
with initial condition $y(0) = 1$.
For this DE we can sketch a so called field plot which is a collection of line segments in the $xy$-plane. pause
This field plot is calculated as follows:

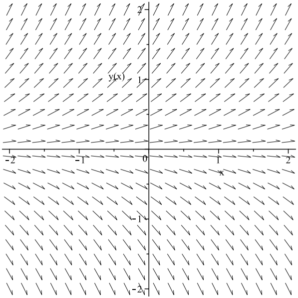
- Take a point $(a, b)$ of the $xy$-plane.
- Calculate $F(a, b)$.
- Draw a tiny line segment in the $xy$-plane at place $(a, b)$ with slope $F(a, b)$.
- Do the same with a couple of other points. In this way a field plot is coming up.
- The Maple command for this is dfieldplot, see also the Maple-sheet.

# A numerical DE-solver: Euler-method:

## A Example:

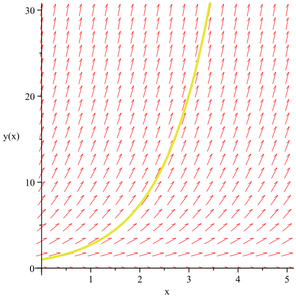
Given the DE $y'(x) = y(x)$.

The belonging field is pictured as:



# A numerical DE-solver: Euler-method:

From this plot it is easy to recognize the general solutions, for instance the solution which satisfy the initial condition: $y(0) = 1$, in a picture we get:




We see in the last picture the yellow line following the line segments. The Maple-command for the last plot is Deplot.

# A numerical DE-solver: Euler-method:

Question: How does Maple do this?
Answer: With the method of Euler!
Question: What is this?

# A numerical DE-solver: Euler-method:

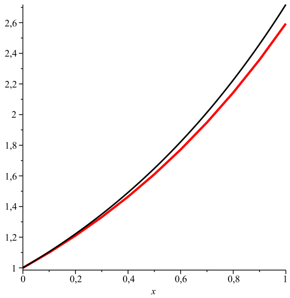
- For explaining this method we take as starting point the DE $y'(x) = y(x)$, with initial condition $y(0) = 1$.
- Aim: We want to approximate the value of $y(1)$.
- To do this we divide the interval $[0, 1]$ into 10 subintervals: [0,0.1]; [0.1,0.2];...;[0.9,1].
- We start at point $(0, 1)$ and we follow the line segment with slope 1 (why?) up to $x = 0.1$.
- We reach the point $(0.1, 1.1)$. From this point we repeat the same process as before:

  We start at point $(0.1, 1.1)$ follow the line segment with slope 1.1 (why?) up to $x$=0.2.
- We reach the point $(0.2, 1.21)$. From this we repeat again and again until we reach $x = 1$.
- With the aid of Maple we get the following list:

  [0, 1], [1/10, 11/10], [1/5, 121/100], [3/10, 1331/1000], [2/5, 14641/10000], [1/2, 161051/100000], [3/5, 1771561/1000000], [7/10, 19487171/10000000], [4/5, 214358881/100000000], [9/10, 2357947691/1000000000], [1, 25937424601/10000000000]

# A numerical DE-solver: Euler-method:

So the approximation of $y(1)$ is $\frac{25937424601}{10000000000} = 2.593...$
We know of course the exact solution of the DE : $y(x) = e^x$.
So the exact answer is $e$, this equals to 2.718281828.
A picture of exact solution and intermediate approximations is:



# A numerical DE-solver: Euler-method:

If we formalize above for the general case then we have with the computer a powerful tool for "solving" DE-'s
De formalization:

- Given $y'(x) = F(x, y(x))$, with initial condition $y(a) = c$ with $a$ and $c$ are given numbers.
- Aim: Approximate $y(b)$ where $b$ is number greater than $a$.
- We get the two recursion relations:

$$x_n = x_0 + h \cdot n; y_n = y_{n-1} + h \cdot F(x_{n-1}, y_{n-1})$$

  wherein $h = \frac{b-a}{n}$ (called step size), $n$ is number of steps,
  $F(x_{n-1}, y_{n-1})$ the slope of the line segment at point $(x_{n-1}, y_{n-1})$.
- The starting points are $x_0 = a$ and $y_0 = c$.
- The approximation for $y(b)$ is $y_n$.
- See the Maple-demo and feel yourself happy: There connection between theory and practise.