A self made DE-solver:

Gegeven de volgende procedure EULERMETHODELIJST:

```
EULERMETHODELIJST := proc (n, a, b, F, x0, y0)
local stapgrootte, X, Y, i, LIJST;
stapgrootte := (b-a)/n; X := x0; Y := y0; LIJST := [X, Y];
for i to n
do
    Y := Y+stapgrootte*F(X, Y);
    X := X+stapgrootte;
    LIJST := LIJST, [X, Y]
end do;
return LIJST
end proc
```

Solving rules for DE-'s:

Solving rules:

- For the DE-'s we try to solve them by hand if they are separable.
- If not separable judge if maple can find a exact solution, if so, find it.
- If not exact solvable by Maple try to sketch a solution with aid of Euler or a numerical solver of Maple.

Solving DE-'s:

The given DE-'s again:

- DE1: $y' = y^2 + xy$. Type1: First order not sep. not lin.
- DE2: $y' = y + xy$. Type2: First order sep. and lin.
- DE3: $y' = xy^2$. Type3: First order sep. and not lin.
- DE4: $y' = y + x$. Type4: First order not sep. and lin.
- DE5: $y'' = y + xy'$. Type5: Second order not lineair with constant coefficients, still linear
- DE6: $\frac{y''+2y+y'}{x^2+\sin(x)} = 1$. Type6: Second order lineair with constant coefficients
- DE7: $\frac{v''}{(\sqrt{1+v'^2})^3} = \frac{M(x)}{E(x)I(x)}$. Type7: First order Sep.

Solving DE1:

Question:

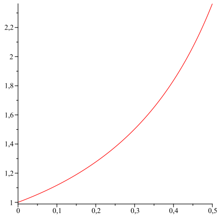
Given $y' = y^2 + xy$ with initial condition $y(0) = 1$.

Solve it!

Answer:

DE1 is First order not sep. not lin.

So we start a numerical approximation with Euler. (See Maple sheet) and get the picture:



Solving DE2:

Question:
Given $y' = y + xy$ with initial condition $y(0) = 1$.
Solve it!

Answer:
DE2 is sep.
So we solve it by hand:

- We rewrite DE2 as: $dy = y(1 + x)dx$
- Separation: $\frac{1}{y}dy = (1 + x)dx$
- Integration: $\ln|y| = x + \frac{1}{2}x^2 + K$ (K a constant)
- Simplify: $y(x) = Ce^{x+\frac{1}{2}x^2}$ (C a constant)
- Initial condition: $y(x) = e^{x+\frac{1}{2}x^2}$
- Check with Maple: right!

Solving DE3:

Question:

Given $y' = xy^2$ with initial condition $y(0) = 1$.

Solve it!

Answer:

DE3 is sep.

So we solve it by hand:

- We rewrite DE3 as: $dy = xy^2dx$
- Separation: $\frac{1}{y^2}dy = xdx$
- Integration: $-\frac{1}{y} = \frac{1}{2}x^2 + K$ (K a constant)
- Simplify: $y(x) = -\frac{2}{x^2+C}$ (C a constant)
- Initial condition: $y(x) = -\frac{2}{x^2-2}$
- Check with Maple: right!

Solving DE4:

Question:
Given $y' = y + x$ with initial condition $y(0) = 1$.
Solve it!
Answer:
DE3 is not sep. but linear
So Maple probably gives an exact solution.
From this we get:

$$y(x) = -x - 1 + 2e^x$$

Solving DE5:

Question:

Given $y'' = y + xy'$ with initial conditions $y(0) = 0$ and $y'(0) = 1$.

Why two conditions?

Solve it!

Answer:

DE5 is a second order linear DE (with nonconstant coefficients)

We try Maple to get an exact solution.

From this we get:

$$y\left(x\right) = 1/2\,\mathrm{erf}\left(1/2\,\sqrt{2}x\right)\mathrm{e}^{1/2\,x^2}\sqrt{\pi}\sqrt{2}.$$

An exact (!!) solution with a unfamiliar, but known by Maple, function *erf*.

Footnote:

The definition of *erf* is

$$erf(x) = \frac{2\int_0^x e^{-t^2}dt}{\sqrt{\pi}}$$

Solving DE6:

Question:
Given $\frac{y''+2y+y'}{x^2+\sin(x)} = 1$ with initial conditions $y(0) = 0$ and $y'(0) = 1$.
Why two conditions?
Solve it!
Answer:
DE6 is a second order linear DE (with constant coefficients)
So Maple probably gives an exact solution.
Running Maple we get as solution:

$$y(x) = \frac{11}{28} e^{-1/2\,x} \sin\left(1/2\sqrt{7}x\right)\sqrt{7} + 3/4\, e^{-1/2\,x} \cos\left(1/2\sqrt{7}x\right)$$
$$-1/2\,x - 1/4 + 1/2\,x^2 + 1/2\sin(x) - 1/2\cos(x)$$

Solving DE7:

Question:
Given $\frac{v''}{(\sqrt{1+v'^2})^3} = \frac{M(x)}{E(x)I(x)}$ (which is one of the core DE-'s of mechanics of materials!!).
Solve it (general solution, no initial conditions) for
$M(x) = 1, I(x) = 1, E(x) = 1$!

Answer:
DE7 is sep. after the substitution $u = v'$.So we solve it by hand:

- We rewrite DE7 after substitution $u = v'$ as: $\frac{u'}{\left(\sqrt{1+u^2}\right)^3} = 1$
- Separation: $\frac{du}{\left(\sqrt{1+u^2}\right)^3} = dx$
- Integration: $\frac{u}{\sqrt{1+u^2}} = (x + K)$ (K a constant)
- Solving for u we get : $u(x) = \pm\sqrt{\frac{(x+K)^2}{1-(x+K)^2}}$
- With $u = v'$ we get $v'(x) = \pm\sqrt{\frac{(x+K)^2}{1-(x+K)^2}}$
- Again integration: $v(x) = \pm\sqrt{1 - (x + K)^2} + L$ (L a constant)
- Squaring gives: $(v(x) - L)^2 + (x + K)^2 = 1$, so solutions of DE7 (under given conditions) are parts of circles
- Check with Maple: right!