

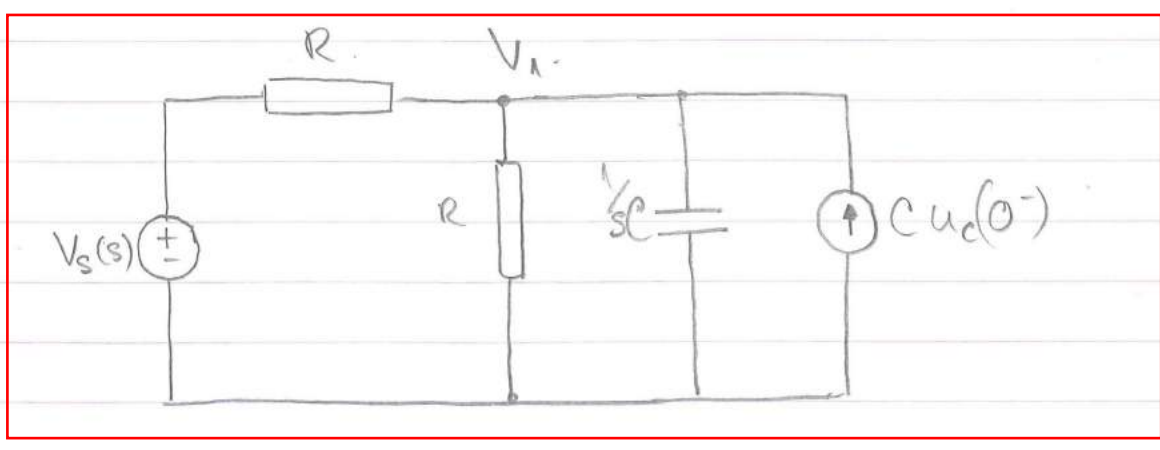
a) Determine $V_S(s) = \mathcal{L}\{v_S(t)\}$.

b) Determine $u_c(t)$.

a) $v_S(t) = t H(t) - t H(t-1)$
 $= t H(t) - (t-1) H(t-1) - H(t-1)$ 0.5 points

$V_S(s) = \frac{1}{s^2} - \frac{1}{s^2} e^{-s} - \frac{1}{s} e^{-s} = \frac{1 - e^{-s} - s e^{-s}}{s^2}$ (v.s) 0.5 points

b) Initial conditions.
 $u_c(0^-) = 1V$ 0.05 points



0.15 points

$$\frac{V_1 - V_S}{R} + \frac{V_1}{R} + sCV_1 = C u_c(0^-)$$

$$V_1(2 + sRC) = RC u_c(0^-) + V_S \Rightarrow V_1(2 + s) = V_S + 1$$

$$V_1 = \frac{1}{2+s} \left[\frac{1 - e^{-s} - s e^{-s}}{s^2} + 1 \right] = \frac{1 + s^2 - (1+s)e^{-s}}{s^2(2+s)} = u_c$$

$$\frac{1 + s^2}{s^2(2+s)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{2+s} \quad \text{0.3 points}$$

$$A(2s + s^2) + B(2 + s) + Cs^2 = (A+C)s^2 + (2A+B)s + 2B = 1 + s^2$$

↓

$$A + C = 1 \Rightarrow C = 1 - A = 5/4$$

$$2A + B = 0 \Rightarrow A = -B/2 = -1/4$$

$$2B = 1 \Rightarrow B = 1/2$$

0.3 points

$$(A+C)s^2 + (2A+B)s + 2B = 1 + s$$

↓

$$A + C = 0 \Rightarrow C = -1/4$$

$$2A + B = 1 \Rightarrow A = 1/2(1 - B) = 1/4$$

$$2B = 1 \Rightarrow B = 1/2$$

0.2 points

$$u_c(t) = \left[-1/4 + 1/2t + 5/4 e^{-2t} \right] H(t) - \left[1/4 + 1/2(t-1) - 1/4 e^{-2(t-1)} \right] H(t-1)$$

$$= \left[-1/4 + 1/2t + 5/4 e^{-2t} \right] H(t) - \left[-1/4 + 1/2t - 1/4 e^{-2(t-1)} \right] H(t-1)$$

$$H(s) = \frac{20 \cdot (s+1) \cdot (s^2 + 230s + 6000)}{(s+100) (s^2 + 150s + 5000)}$$

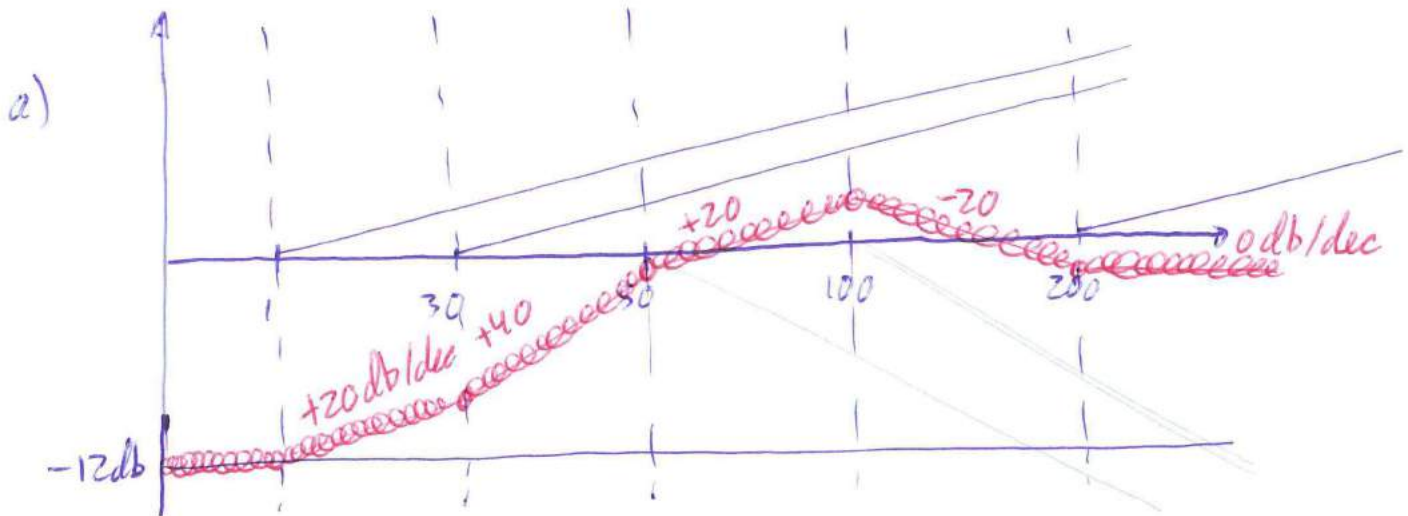
$$\textcircled{*} s^2 + 230s + 6000 = (s+200) \cdot (s+30)$$

$$s = \frac{-230 \pm \sqrt{230^2 - 4 \cdot 6000}}{2} = \frac{-230 \pm 170}{2} = \begin{cases} -200 \\ -30 \end{cases}$$

$$\textcircled{*} s^2 + 150s + 5000 = (s+50) (s+100)$$

$$s = \frac{-150 \pm \sqrt{150^2 - 4 \cdot 5000}}{2} = \frac{-150 \pm 50}{2} = \begin{cases} -100 \\ -50 \end{cases}$$

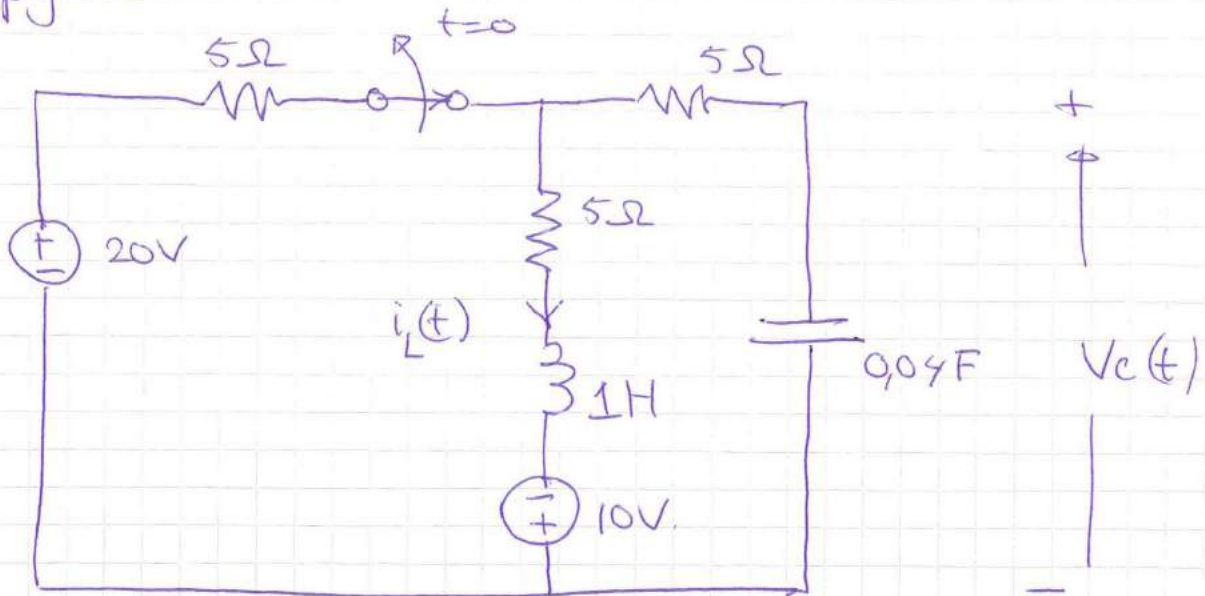
$$H(s) = \frac{20 (s+1) (s+30) (s+200)}{(s+50) (s+100)^2} = \frac{0.24 (s+1) \cdot \left(\frac{s}{30} + 1\right) \left(\frac{s}{200} + 1\right)}{\left(\frac{s}{50} + 1\right) \cdot \left(\frac{s}{100} + 1\right)^2}$$



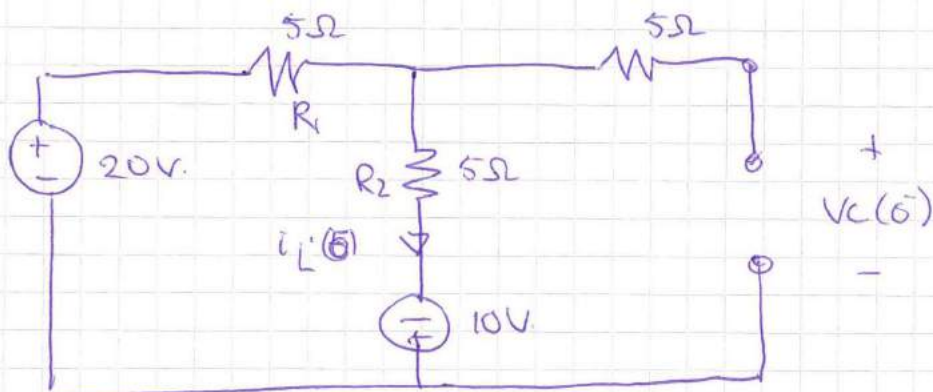
b) $|H(s)| \approx 20 \cdot \log 0.24 \approx -12 \text{ dB} \quad \forall \omega < 1 \text{ rad/s}$

c) $|H(s)| \approx \lim_{\omega \rightarrow \infty} 20 \cdot \log \left| \frac{20 (j\omega + 1) (j\omega + 30) (j\omega + 200)}{(j\omega + 50) (j\omega + 100)^2} \right| \approx 26 \text{ dB}$

Opgave 3



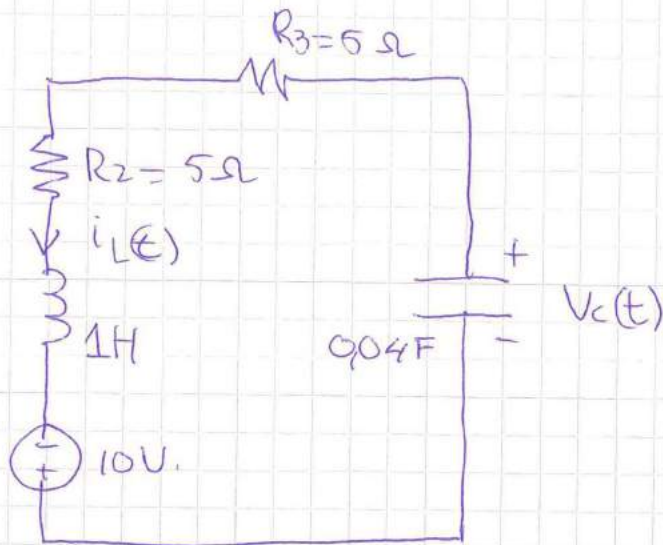
a) Beginvoorwaarden $V_C(t)$ & $i_L(t)$ bij $t=0^-$
 voor $t=0^-$



$$i_L(0^-) = \frac{20 + 10}{R_1 + R_2} = \frac{30}{10} = \underline{\underline{3A}}$$

$$V_C(0^-) = -10 + 3 \cdot 5 = \underline{\underline{5V}}$$

b) Voor $t > 0$ s



De maasvergelijking voor $i_L(t)$ voor $t > 0$ s wordt:

$$-10 - V_C(t) + \frac{1}{C} \int i_L(t) dt + i_L(t) \cdot (R_2 + R_3) + L \frac{di_L(t)}{dt} = 0.$$

c)

$$L \frac{d^2 i_L(t)}{dt^2} + (R_2 + R_3) \frac{di_L(t)}{dt} + \frac{1}{C} i_L(t) = 0.$$

$$\frac{d^2 i_L(t)}{dt^2} + \frac{R_2 + R_3}{L} \frac{di_L(t)}{dt} + \frac{1}{LC} i_L(t) = 0.$$

$$s^2 + 10s + 25 = 0.$$

$$(s+5)(s+5) = 0.$$

$$\left. \begin{array}{l} s_1 = -5 \\ s_2 = -5 \end{array} \right\} \text{ twee gelijk oplossingen.}$$

d) Twee gelijke oplossing dus kritisch gedempt.

$$\text{Algemene oplossing voor } i_L(t) = A e^{-5t} + B \cdot t e^{-5t} + C.$$

Met $C = 0$ want $i_L(t) = 0$ voor $t \rightarrow \infty$.

$$e) \quad i_L(t) = A e^{-5t} + B \cdot t \cdot e^{-5t} \Rightarrow i_L(0) = A + B \cdot 0 = A \quad \left. \vphantom{i_L(t)} \right\} \Rightarrow$$

$$i_L(0^+) = i(0) = 3A \quad \leftarrow \text{uit vraag a)}$$

$$A = 3.$$

Maken we nu gebruik van de maasvergelijking uit b) dan vinden we.

$$15 = i_L(0^+) (R_2 + R_3) + L \frac{di_L(0^+)}{dt}$$

met $\frac{di_L(0^+)}{dt} = -5A + B$.

en $i_L(0^+) = 3$

$$15 = 3 \cdot (5 + 5) + 1 \cdot [-5A + B] \Rightarrow$$

$$15 = 30 + [-15 + B] \quad \left. \vphantom{15} \right\} \Rightarrow$$

(met $A = 3$.)

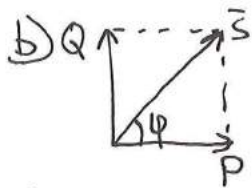
$$15 = 15 + B \Rightarrow$$

$$B = 0$$

Dus: $i_L(t) = 3 e^{-5t} \quad A$

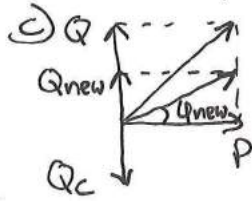
$$i_L(0,2196) = 3 e^{-5 \cdot 0,2196} = \underline{\underline{1 \text{ A}}}$$

a) De stroom loopt achter op de spanning (lagging/inductief).



$$\cos \varphi = 0,4 \rightarrow \varphi = 66,42^\circ$$

$$\tan \varphi = \frac{Q}{P} \rightarrow \boxed{Q = P \tan \varphi = 916,52 \text{ VAR}}$$



$$\varphi_{\text{new}} = \arccos(0,85) = 31,79^\circ$$

$$Q_{\text{new}} = P \tan \varphi_{\text{new}} = 247,9 \text{ VAR}$$

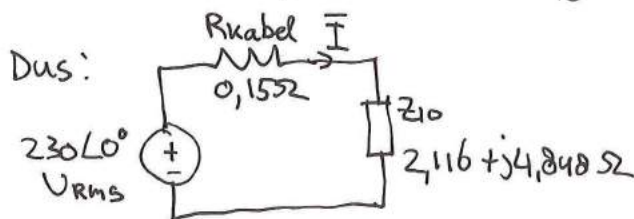
$$Q_c = Q_{\text{new}} - Q = -668,62 \text{ VAR}$$

$$\text{en ook: } Q_c = -\omega C U_{\text{RMS}}^2 \rightarrow \boxed{C = \frac{-Q_c}{\omega U_{\text{RMS}}^2} = 40,23 \mu\text{F}}$$

d) Eerst de impedantie van 1 belasting bepalen:

$$\bar{S} = 400 + j916,52 \text{ VA} \quad \text{en} \quad \bar{S} = \frac{U_{\text{RMS}}^2}{Z^*} \rightarrow Z = \frac{U_{\text{RMS}}^2}{\bar{S}^*} = 2,116 + j4,848 \Omega$$

$$10 \text{ belastingen parallel: } Z_{10} = \frac{Z}{10} = 2,116 + j4,848 \Omega$$



$$\bar{I} = \frac{\bar{V}}{Z_{\text{tot}}} = \frac{230 \angle 0^\circ}{2,116 + j4,848 + 0,15}$$

$$\bar{I} = 18,197 - j38,934 \text{ A}_{\text{RMS}}$$

$$\text{Dus de kabel verliezen zijn: } \boxed{P_{\text{kabel}} = |\bar{I}|^2 R_{\text{kabel}} = 277,05 \text{ W}}$$

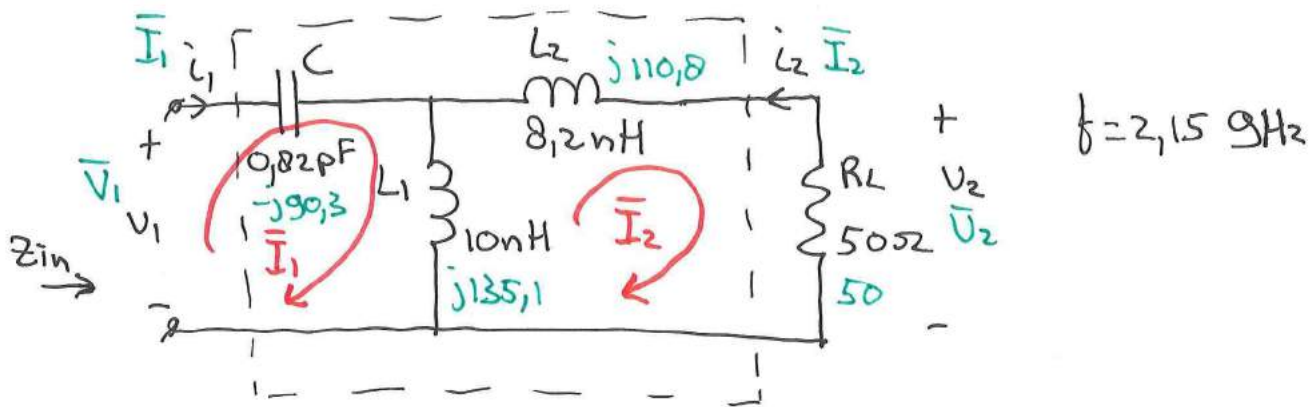
e) $\cos \varphi = 0,85$ dus $\bar{S} = 400 + j247,9 \text{ VA}$

$$\text{Impedantie 1 belasting: } Z = \frac{U_{\text{RMS}}^2}{\bar{S}^*} = 9,551 + j5,9217 \Omega$$

$$\text{Impedantie 10 belastingen: } Z_{10} = \frac{Z}{10} = 9,555 + j5,922 \Omega$$

$$\text{De stroom } \bar{I}: \bar{I} = \frac{230 \angle 0^\circ}{9,555 + j5,922 + 0,15} = 17,269 - j10,537 \text{ A}_{\text{RMS}}$$

$$\text{Dus de verliezen zijn nu: } \boxed{P_{\text{kabel}} = |\bar{I}|^2 R_{\text{kabel}} = 61,39 \text{ W}}$$



$$a) Z_C = \frac{1}{j\omega C} = -j90,3 \Omega$$

$$Z_{L1} = j\omega L_1 = j135,1 \Omega$$

$$Z_{L2} = j\omega L_2 = j110,8 \Omega$$

$$b) \bar{I}_1: \bar{V}_1 = -j90,3 \bar{I}_1 + j135,1 (\bar{I}_1 - \bar{I}_2) \rightarrow \boxed{j44,8 \bar{I}_1 - j135,1 \bar{I}_2 = \bar{V}_1} \quad (1)$$

$$\bar{I}_2: 0 = j110,8 \bar{I}_2 + 50 \bar{I}_2 + j135,1 (\bar{I}_2 - \bar{I}_1) \rightarrow \boxed{-j135,1 \bar{I}_1 + (50 + j245,9) \bar{I}_2 = 0} \quad (2)$$

$$c) \text{ uit (2) volgt } \bar{I}_2: \bar{I}_2 = \frac{j135,1 \bar{I}_1}{50 + j245,9}$$

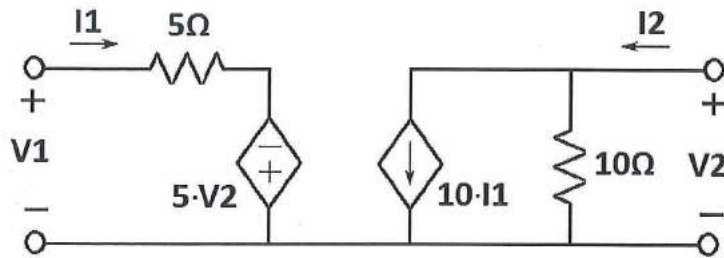
$$\text{invullen in (1): } \bar{V}_1 = j44,8 \bar{I}_1 + \frac{135,1^2}{50 + j245,9} \bar{I}_1 \quad \bar{I}_1 = \bar{I}_1$$

$$\text{Dus: } Z_{in} = \frac{\bar{V}_1}{\bar{I}_1} = j44,8 + \frac{135,1^2}{50 + j245,9} = \boxed{14,49 - j26,48 \Omega}$$

$$d) Z_{11} = \left. \frac{\bar{V}_1}{\bar{I}_1} \right|_{\bar{I}_2=0} = -j90,3 + j135,1 = \boxed{j44,8 \Omega}$$

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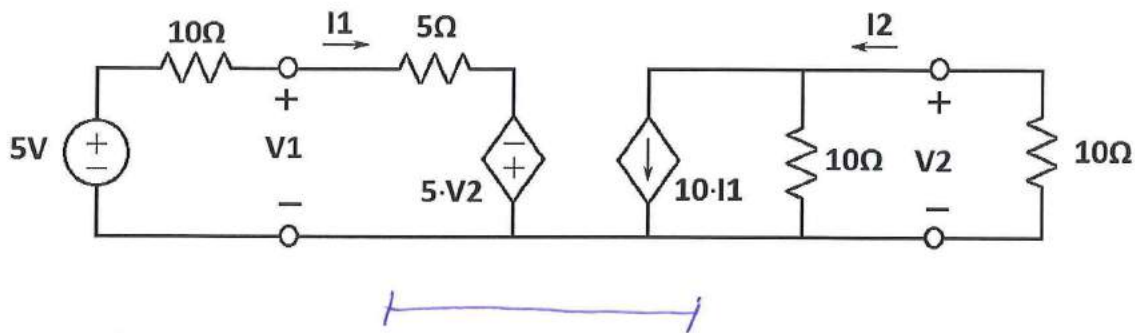
Qa) Calculate the admittance parameters y for the two-port network circuit in the following figure.



NOTE that: $I_1 = y_{11} \cdot V_1 + y_{12} \cdot V_2$

$I_2 = y_{21} \cdot V_1 + y_{22} \cdot V_2$

Qb) Using the calculated admittance parameters y , calculate the currents I_1 and I_2 for the following circuit.



a) Y_{11} $V_2 \equiv 0 \rightarrow Y_{11} \equiv \frac{I_1}{V_1} \equiv \frac{1}{5} \equiv 0.2 \text{ S}$

Y_{12} $V_1 \equiv 0 \rightarrow Y_{12} \equiv \frac{I_1}{V_2} \equiv \frac{5V_2/5}{V_2} \equiv 1 \text{ S}$

Y_{21} $V_2 \equiv 0 \rightarrow Y_{21} \equiv \frac{I_2}{V_1} \equiv \frac{2 \cdot V_1}{V_1} \equiv 2 \text{ S}$

$I_1 \equiv V_1/5 \rightarrow I_2 \equiv 10 \cdot I_1 \equiv 2 \cdot V_1$

Y_{22} $V_1 \equiv 0 \rightarrow Y_{22} \equiv \frac{I_2}{V_2} \equiv \frac{V_2/10 + 10V_2}{V_2} \equiv 10 + 1/10 \equiv 10.1 \text{ S}$

$I_1 \equiv \frac{5V_2}{5} \equiv V_2$

$I_2 \equiv \frac{V_2}{10} + 10V_2$

$$Y \equiv \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \equiv \begin{bmatrix} 0'2 & 1 \\ 2 & 10'1 \end{bmatrix}$$

$$\boxed{b)} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \equiv \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \rightarrow \begin{aligned} V_1 &\equiv 5 - 10 \cdot I_1 \\ V_2 &\equiv -10 I_2 \end{aligned}$$

$$I_1 \equiv 0'2 \cdot V_1 + V_2 \equiv 0'2(5 - 10 I_1) - 10 I_2 \equiv 1 - 2 I_1 - 10 I_2 \rightarrow$$

$$I_2 \equiv 2 \cdot V_1 + 10'1 \cdot V_2 \equiv 2(5 - 10 I_1) - 10'1 I_2 \equiv 10 - 20 I_1 - 10'1 I_2 \rightarrow$$

$$\left. \begin{aligned} 3 I_1 + 10 I_2 &\equiv 1 \\ 20 I_1 + 10'2 I_2 &\equiv 10 \end{aligned} \right\} \Rightarrow \begin{bmatrix} 3 & 10 \\ 20 & 10'2 \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \equiv \begin{bmatrix} 1 \\ 10 \end{bmatrix}$$

$$I_1 \equiv \frac{\begin{vmatrix} 1 & 10 \\ 10 & 10'2 \end{vmatrix}}{\begin{vmatrix} 3 & 10 \\ 20 & 10'2 \end{vmatrix}} \equiv \frac{10'2 - 100}{306 - 200} \equiv \frac{2}{106} \equiv 18'8 \text{ mA}$$

$$I_2 \equiv \frac{\begin{vmatrix} 3 & 1 \\ 20 & 10 \end{vmatrix}}{\begin{vmatrix} 3 & 10 \\ 20 & 10'2 \end{vmatrix}} \equiv \frac{30 - 20}{106} \equiv \frac{10}{106} \equiv 94'3 \text{ mA}$$