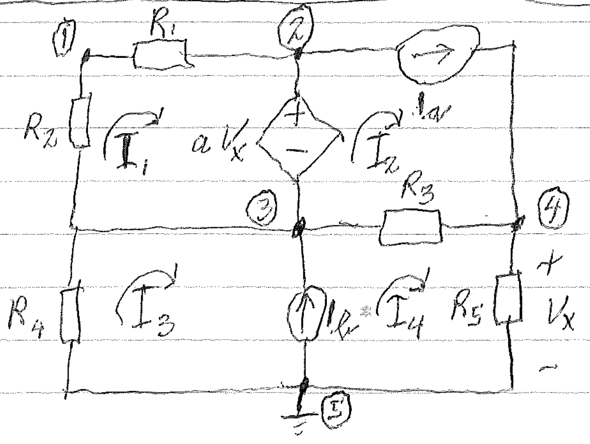
Opgave 1

Bart Tuinema

a)



b) $N = 5$ knooppunten

$B = 8$ takken

$N-1 = 4$ vergelijkingen

$B-N+1 = 4$ vergelijkingen

c)

① $\frac{V_1 - V_2}{R_1} + \frac{V_1 - V_3}{R_2} = 0$

② + ③ $\frac{V_2 - V_1}{R_1} + I_a + \frac{V_3 - V_1}{R_2} + \frac{V_3}{R_4} - I_b + \frac{V_3 - V_4}{R_3} = 0$

$V_2 = V_3 + aV_x = V_3 + aV_4$ (randvoorwaarde bron)

④ $-I_a + \frac{V_4 - V_3}{R_3} + \frac{V_4}{R_5} = 0$

d) $\boxed{1}$ $R_2 I_1 + R_1 I_1 + a V_x = 0$ $\qquad V_x = R_5 I_4$

$R_2 I_1 + R_1 I_1 + a R_5 I_4 = 0$

$\boxed{2}$ $I_2 = I_v$

$\boxed{3} + \boxed{4}$ $R_4 I_3 + R_3 (I_4 - I_2) + R_5 I_4 = 0$

$I_4 - I_3 = I_g$ (randvoorwaarde bron)

e) Maschmethode:

$\boxed{1}$ $200 I_1 + 100 I_1 + 6 \cdot 600 \cdot I_4 = 0$

$300 I_1 + 3600 I_4 = 0 \rightarrow 3I_1 + 36 I_4 = 0$ (iv)

$\boxed{2}$ $I_2 = 12 \text{ mA}$

$\boxed{3} + \boxed{4}$ $100 \cdot I_3 + 300(I_4 - 12m) + 600 I_4 = 0$

$100 I_3 + 900 I_4 = 3600m$

$I_3 + 9 I_4 = 36m$ (i)

$I_4 - I_3 = 14m \rightarrow I_4 = 14m + I_3$ (ii)

(ii) in (i) $I_3 + 9(14m + I_3) = 36m$

$10 I_3 = 36m - 126m = -90m$

$I_3 = -9 \text{ mA}$ (iii)

(iii) in (ii) $I_4 = 14m \cancel{+} - 9m = 5 \text{ mA}$ (v)

(v) in (iv) $3I_1 + 36 \cdot 5m = 0$

$3I_1 = -180m$

$I_1 = -60 \text{ mA}$

$V_x = R_5 I_4 = 600 \cdot 5m = 3V$ $\quad I_{R_1} = I_1 = -60 \text{ mA}$

e) knooppuntsmethode:

① $\frac{V_1 - V_2}{100} + \frac{V_1 - V_3}{200} = 0 \qquad (\times 200)$

$2V_1 - 2V_2 + V_1 - V_3 = 0$

$\underline{3V_1 - 2V_2 - V_3 = 0}$

②+③ $\frac{V_2 - V_1}{100} + 12m + \frac{V_3 - V_1}{200} + \frac{V_3}{100} - 14m + \frac{V_3 - V_4}{300} = 0 \quad (\times 600)$

$6V_2 - 6V_1 + 7200m + 3V_3 - 3V_1 + 6V_3 - 8400m + 2V_3 - 2V_4 = 0$

$\underline{-9V_1 + 6V_2 + 11V_3 - 2V_4 = 1,2}$

(bron) $V_2 = V_3 + 6 \cdot V_4$

$\underline{V_2 - V_3 - 6V_4 = 0}$

④ $-12m + \frac{V_4 - V_3}{300} + \frac{V_4}{600} = 0 \qquad (\times 600)$

$-7200m + 2V_4 - 2V_3 + V_4 = 0$

$\underline{-2V_3 + 3V_4 = 7,2}$

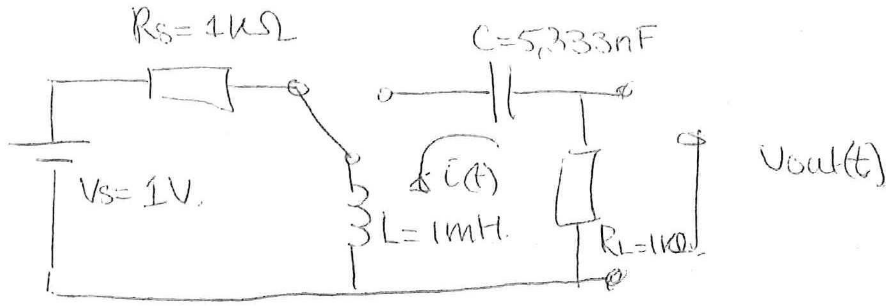
$$\left[\begin{array}{cccc|c} 3 & -2 & -1 & 0 & 0 \\ -9 & 6 & 11 & -2 & 1.2 \\ 0 & 1 & -1 & -6 & 0 \\ -9 & 0 & -2 & 3 & 7.2 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -\frac{2}{3} & -\frac{1}{3} & 0 & 0 \\ 0 & 0 & 8 & -2 & 1.2 \\ 0 & 1 & -1 & -6 & 0 \\ 0 & 0 & -2 & 3 & 7.2 \end{array}\right] \sim$$

$$\left[\begin{array}{cccc|c} 1 & -\frac{2}{3} & -\frac{1}{3} & 0 & 0 \\ 0 & 1 & -1 & -6 & 0 \\ 0 & 0 & 1 & -\frac{3}{2} & -3.6 \\ 0 & 0 & 8 & -2 & 1.2 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -\frac{2}{3} & -\frac{1}{3} & 0 & 0 \\ 0 & 1 & -1 & -6 & 0 \\ 0 & 0 & 1 & -\frac{3}{2} & -3.6 \\ 0 & 0 & 0 & 10 & 30 \end{array}\right] \sim$$

$$\left[\begin{array}{cccc|c} 1 & -\frac{2}{3} & -\frac{1}{3} & 0 & 0 \\ 0 & 1 & -1 & 0 & 18 \\ 0 & 0 & 1 & 0 & 0.9 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -\frac{2}{3} & 0 & 0 & 0.3 \\ 0 & 1 & 0 & 0 & 18.9 \\ 0 & 0 & 1 & 0 & 0.9 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right] \sim$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 12.9 \\ 0 & 1 & 0 & 0 & 18.9 \\ 0 & 0 & 1 & 0 & 0.9 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right]$$

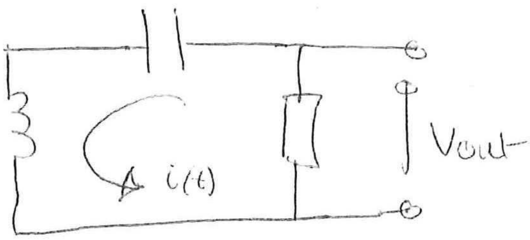
$V_x = V_4 = 3V$ $\qquad I_{R_1} = \dfrac{V_1 - V_2}{R_1} = \dfrac{12.9 - 18.9}{100} = -60\ mA$



a) De beginvoorwaarden op $t = 0^-$

$$\boxed{\begin{aligned} i_L(0^-) &= 1\,mA \\ V_C(0^-) &= 0\,V \end{aligned}}$$

b) op $t > 0$



Maasvergelijking in differentiaalvorm

$$L\frac{di}{d(t)} + V_C(t_0) + \frac{1}{C}\int_{t_0}^{t} i(x)\,dx + i(t)\,R_L = 0$$

$$\boxed{\frac{d^2 i(t)}{d^2 t} + \frac{R_L}{L}\frac{d\,i(t)}{dt} + \frac{1}{LC}\,i(t) = 0}$$

c) karakteristieke vergelijking

$$\boxed{s^2 + \frac{R_L}{L}s + \frac{1}{LC} = 0} \Rightarrow s^2 + 1e6\,s + 1{,}875e11 = 0$$

oplossingen. $S_{1,2} = -\dfrac{R_L}{2L} \pm \dfrac{\sqrt{\left(\frac{R_L}{L}\right)^2 - \frac{4}{LC}}}{2} = -0{,}5e6 \pm 0{,}25e6$

$$\boxed{S_1 = -0{,}25e6\ ,\ S_2 = -0{,}75e6}$$

dit is een overgedempt systeem.

d)

algemene oplossing.

$$i(t) = K_1 e^{-0{,}25e6\,t} + K_2\, e^{-0{,}75e6\,t} + K_3$$

$$\boxed{K_3 = 0 \quad \text{omdat} \quad i(\infty) = 0.}$$

(1). $i(0) = 1mA = K_1 + K_2$

verder is

$$\frac{d\,i(t)}{dt} = -0{,}25e6\,K_1 e^{-0{,}25e6t} - 0{,}75e6\,K_2\, e^{-0{,}75e6t}$$

dus. op $t=0$.

$$\left.\frac{d\,i(t)}{dt}\right|_{t=0} = -0{,}25e6\,K_1 - 0{,}75e6\,K_2$$

Invullen in de Maasvergelijking.

$$L \left.\frac{d\,i(t)}{dt}\right|_{t=0} + V_C(0) + i(0)\,R_L(t) = 0.$$

$$L(-0{,}25e6\,K_1 - 0{,}75e6\,K_2) + 0{,}001 \cdot R_L = 0.$$

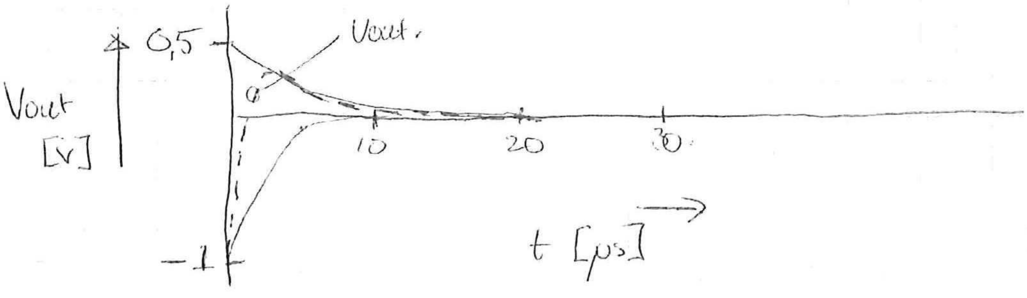
$$0{,}25e6\,K_1 + 0{,}75e6\,K_2 = \frac{1}{L} \Rightarrow$$

$$\begin{array}{lrcl} (2) & K_1 + 3K_2 & = & 0{,}004 \\ -(1) & -(K_1 + K_2) & = & -0{,}001 \\ \hline & 2K_2 & = & 0{,}003 \end{array}$$

$$\boxed{K_2 = 0{,}0015 \quad \& \quad K_1 = -0{,}0005.}$$

$V_{out} = -\,i(t) \cdot R_L$

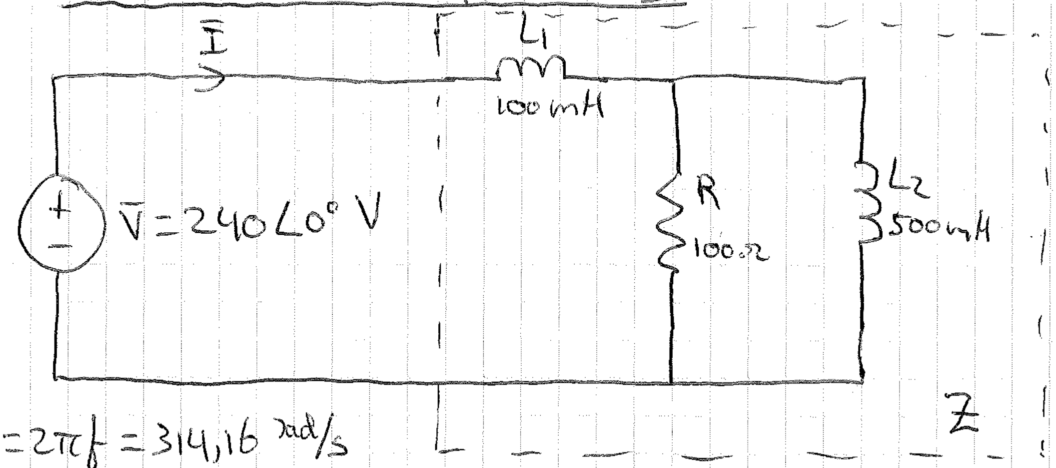
$$\boxed{V_{out} = 0{,}5\,e^{-0{,}25e6\,t} - 1{,}5 \cdot e^{-0{,}75e6\,t} \quad V}$$



Hertentamen 12 April 2013

M. Pelk 2013

(3)



$\omega = 2\pi f = 314{,}16$ rad/s

a) $Z = j\omega L_1 + \dfrac{R \cdot j\omega L_2}{R + j\omega L_2} = j31{,}42 + \dfrac{100 \cdot j157{,}08}{100 + j157{,}08} = 71{,}16 + j76{,}72\,\Omega$

polair: $Z = 104{,}64\angle 47{,}15°\ \Omega$

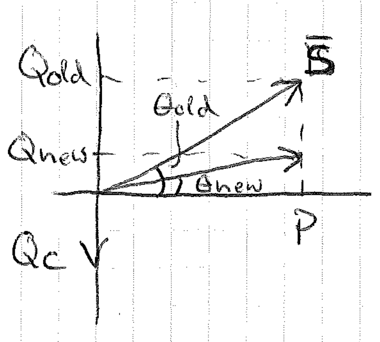
Dus: $\bar{I} = \dfrac{\bar{V}}{Z} = \dfrac{240\angle 0°}{104{,}64\angle 47{,}15°} = \boxed{2{,}294\angle -47{,}15°}$ A

b) Vermogensfactor: $pf = \cos(\theta_v - \theta_I) = \cos(47{,}15°) = \boxed{0{,}68 \text{ lagging}}$

Inductief → stroom loopt achter op spanning

c) $P = \frac{1}{2}|\bar{V}||\bar{I}| \cdot \cos(\theta_v - \theta_I) = \boxed{187{,}2\ W}$

d) Capaciteit/Condensator



$\theta_{old} = 47{,}15°$, $\theta_{new} = \arccos(0{,}98) = 11{,}48°$

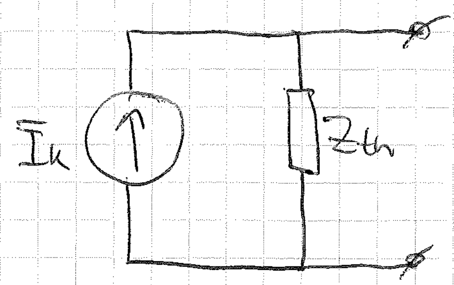
$Q_{old} = \frac{1}{2}|\bar{V}||\bar{I}|\sin(\theta_{old}) = 201{,}8$ VAR

$Q_{new} = P \cdot \tan(\theta_{new}) = 38{,}02$ VAR

$Q_c = Q_{new} - Q_{old} = -163{,}8$ VAR

$Q_c = -\omega C \frac{1}{2}|\bar{V}|^2 \rightarrow \boxed{C = 18{,}1\ \mu F}$

e)



$$\bar{I}_k = \frac{240\angle 0^\circ}{j\omega L_1} = \boxed{7{,}639\angle -90^\circ \text{ A}}$$

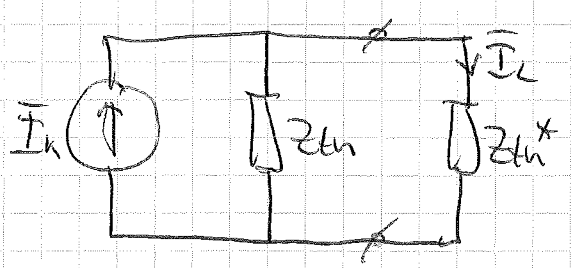
$$Z_{th} = j\omega L_1 // R // j\omega L_2 \quad , \quad L_v = \frac{L_1 L_2}{L_1 + L_2} = 83{,}3\text{mH}$$

$$Z_{th} = \frac{R \cdot j\omega L_v}{R + j\omega L_v} = \frac{100 \cdot j26{,}18}{100 + j26{,}18}$$

$$\boxed{Z_{th} = 6{,}41 + j24{,}5 \ \Omega}$$

Belasting voor max. vermogensoverdracht: $\boxed{Z_L = Z_{th}^* = 6{,}41 - j24{,}5 \ \Omega}$

f)



$$\bar{I}_L = \bar{I}_k \cdot \frac{Z_{th}}{Z_{th} + Z_{th}^*} = \bar{I}_k \cdot \frac{Z_{th}}{2\,\mathrm{Re}[Z_{th}]}$$

$$P_L = \frac{\bar{I}_L \bar{I}_L^*}{2} \cdot \mathrm{Re}[Z_{th}^*] = \frac{\bar{I}_k \cdot \bar{I}_k^*}{2} \cdot \frac{Z_{th} Z_{th}^*}{4\,\mathrm{Re}[Z_{th}]^2} \cdot \mathrm{Re}[Z_{th}^*]$$

$$P_L = \frac{|I_k|^2 \cdot |Z_{th}|^2}{8\,\mathrm{Re}[Z_{th}]} = \frac{7{,}639^2 \cdot 25{,}32^2}{8 \cdot 6{,}41}$$

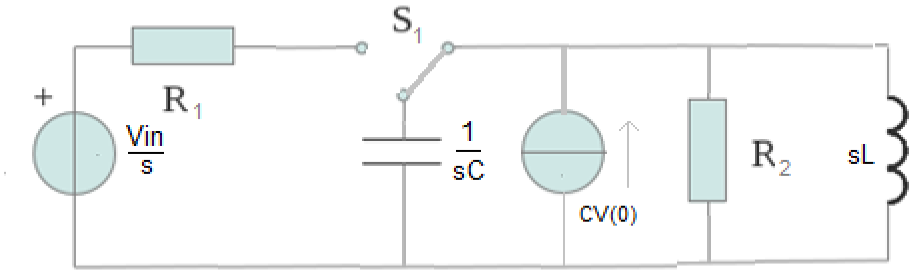
$$\boxed{P_{L,max} = 729{,}5 \text{ W}}$$

Opgave 4

a) Vc = Vin = 5V

b) Het fasor-domein is gedefinieerd voor het harmonisch regime. Dit betekent dat alle signalen periodiek (combinaties van sinusvormige signalen) moeten zijn. Door de schakelaar heeft het circuit in dit voorbeeld ondergaat dit circuit een stapresponsie, hetgeen niet in het harmonisch regime te beschrijven is.

c)



d) Parallel RLC-circuit. KCL toepassen:

$$V_c\left(\frac{1}{sL}+\frac{1}{R_2}+sC\right)-CV_c(0)=0$$
$$V_c=\frac{CV_c(0)}{\frac{1}{sL}+\frac{1}{R_2}+sC}$$
$$I_l=\frac{V_c}{sL}=\frac{CV_c(0)}{1+\frac{sL}{R_2}+s^2LC}=\frac{1}{1+\frac{5}{6}s+\frac{1}{6}s^2}=\frac{6}{6+5s+s^2}$$

e) Ontbinden in factoren:

$$I_l=\frac{6}{6+5s+s^2}=\frac{6}{(s+2)(s+3)}$$

Breuksplitsen:

$$I_l=\frac{A}{s+2}+\frac{B}{s+3}=\frac{6}{(s+2)(s+3)}$$
$$\frac{A(s+3)+B(s+2)}{(s+2)(s+3)}=\frac{6}{(s+2)(s+3)}$$
$$\begin{cases} A+B=0 \\ 3A+2B=6 \end{cases} \quad \rightarrow \quad B=-6, A=6$$
$$I_l=\frac{6}{s+2}-\frac{6}{s+3}$$

Terug transformeren middels de tabellen:

$$I_l=6\exp(-2t)-6\exp(-3t)$$

5a) $V_2 = V_1 \cdot \frac{-R_3}{R_1+R_2} = -1V \cdot \frac{40k\Omega}{30k\Omega} = -\frac{4}{3}V$

b) $H(j\omega) = \frac{V_2(j\omega)}{V_1(j\omega)} = \frac{V_2(j\omega)}{V_C(j\omega)} \cdot \frac{V_C(j\omega)}{V_1(j\omega)}$

$$= -\frac{R_3}{R_2} \cdot \frac{R_2 /\!/ \frac{1}{j\omega C}}{\left(R_2 /\!/ \frac{1}{j\omega C}\right) + R_1}$$

$$= -\frac{R_3}{R_2} \cdot \frac{\frac{R_2}{1+j\omega R_2 C}}{\frac{R_2}{1+j\omega R_2 C} + R_1}$$

$$= -\frac{R_3}{\cancel{R_2}} \cdot \frac{\cancel{R_2}}{R_2 + (1+j\omega R_2 C)R_1}$$

$$= \frac{-R_3}{R_1 + R_2 + j\omega R_1 R_2 C}$$

c) $\omega_c = \omega \Big|_{\omega R_1 R_2 C = R_1 + R_2} \implies \omega_c = \frac{R_1+R_2}{R_1 R_2 C}$

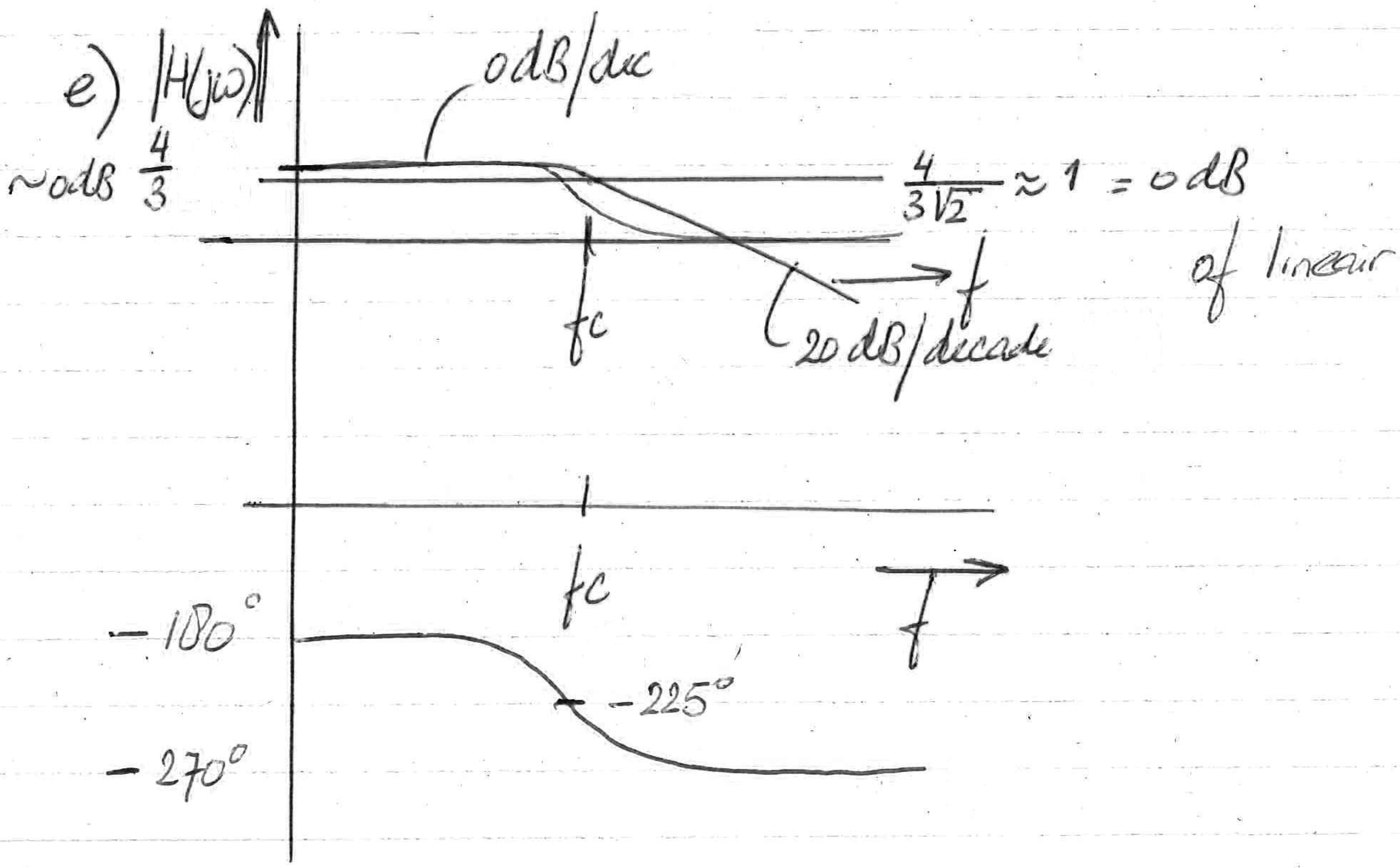
$$= \frac{1}{(R_1 /\!/ R_2)C}$$

$\omega_c = 150\ rad/s$

$f_c = \frac{\omega_c}{2\pi}$

d) laagdoorlaat

e)

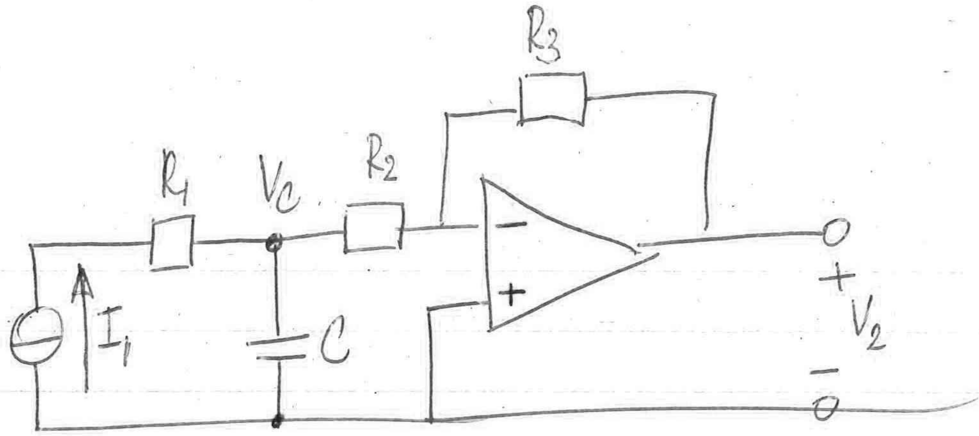


f) $$A = \frac{1}{\mu} = \frac{V_1}{V_2}\Bigg|_{I_2=0} = \frac{1}{H(j\omega)} = \frac{R_1 + R_2 + j\omega R_1 R_2 C}{-R_3}$$

$$B = \frac{1}{\gamma} = \frac{V_1}{I_2}\Bigg|_{V_2=0} = 0.$$

$$C = \frac{1}{\zeta} = \frac{I_1}{V_2}\Bigg|_{I_2=0} \Rightarrow \text{Z.O.Z.}$$

$$D = \frac{1}{\alpha} = \frac{I_1}{I_2}\Bigg|_{V_2=0} = 0.$$



$$\frac{V_2}{I_1} = \frac{V_2}{V_C} \cdot \frac{V_C}{I_1} = -\frac{R_3}{R_2} \cdot \left(R_2 // \frac{1}{j\omega C}\right)$$

$$= -\frac{R_3}{\cancel{R_2}} \cdot \frac{\cancel{R_2}}{1 + j\omega R_2 C}$$

$$= \frac{-R_3}{1 + j\omega R_2 C}$$

$$\Rightarrow C = \frac{1 + j\omega R_2 C}{-R_3}$$