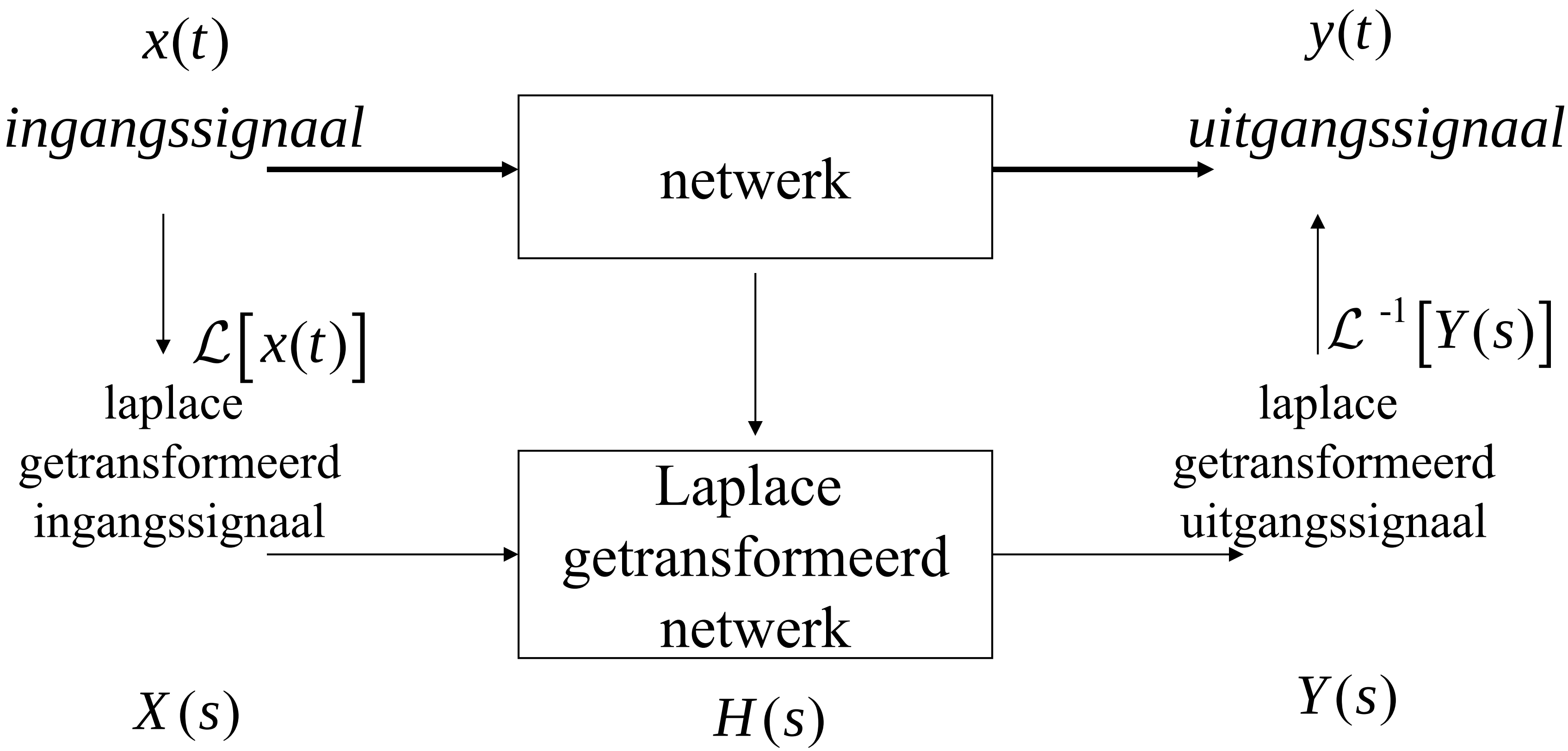
Laplace vs. tijd



De Laplace-transformatie

$$f(t) = 0 \quad \text{voor} \quad t < 0$$

$$s = \text{complexe frequentie} = \sigma + j\omega$$

$$F(s) = \mathcal{L}\left[f(t) \right] = \int_{0^-}^{\infty} f(t) e^{-st} dt$$

Inverse Laplace-transformatie:

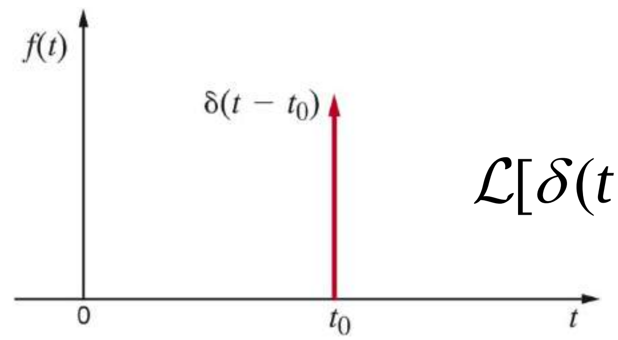
$$f(t) = \mathcal{L}^{-1}\left[F(s) \right] = \frac{1}{2\pi j} \int F(s) e^{st} \, ds$$

TABLE 13.2 Some useful properties of the Laplace transform

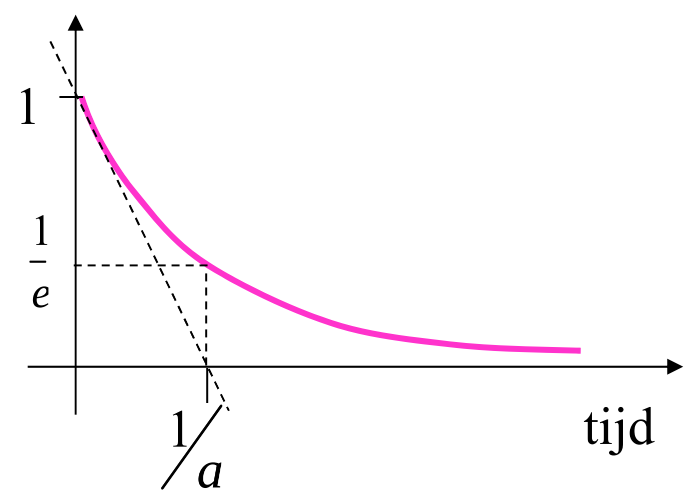
PROPERTY NUMBER	f(t)	F(s)
1. Magnitude scaling	$Af(t)$	$A\mathbf{F}(s)$
2. Addition/subtraction	$f_1(t) \pm f_2(t)$	$\mathbf{F}_1(s) \pm \mathbf{F}_2(s)$
3. Time scaling	$f(at)$	$\frac{1}{a}\mathbf{F}\left(\frac{s}{a}\right), a > 0$
4. Time shifting	$f(t - t_0)u(t - t_0), t_0 \geq 0$	$e^{-t_0 s}\mathbf{F}(s)$
	$f(t)u(t - t_0)$	$e^{-t_0 s}\mathcal{L}[f(t + t_0)]$
5. Frequency shifting	$e^{-at}f(t)$	$\mathbf{F}(s + a)$
6. Differentiation	$\frac{d^n f(t)}{dt^n}$	$s^n\mathbf{F}(s) - s^{n-1}f(0) - s^{n-2}f^1(0) \cdots - s^0 f^{n-1}(0)$
7. Multiplication by t	$tf(t)$	$-\frac{d\mathbf{F}(s)}{ds}$
	$t^n f(t)$	$(-1)^n \frac{d^n\mathbf{F}(s)}{ds^n}$
8. Division by t	$\frac{f(t)}{t}$	$\int_s^\infty \mathbf{F}(\lambda)\, d\lambda$
9. Integration	$\int_0^t f(\lambda)\, d\lambda$	$\frac{1}{s}\mathbf{F}(s)$
10. Convolution	$\int_0^t f_1(\lambda)f_2(t - \lambda)\, d\lambda$	$\mathbf{F}_1(s)\mathbf{F}_2(s)$

TABLE 13.1 Short table of Laplace transform pairs

f(t)	F(s)
$\delta(t)$	1
$u(t)$	$\frac{1}{s}$
e^{-at}	$\frac{1}{s+a}$
t	$\frac{1}{s^2}$
$\frac{t^n}{n!}$	$\frac{1}{s^{n+1}}$
te^{-at}	$\frac{1}{(s+a)^2}$
$\frac{t^n e^{-at}}{n!}$	$\frac{1}{(s+a)^{n+1}}$
$\sin bt$	$\frac{b}{s^2+b^2}$
$\cos bt$	$\frac{s}{s^2+b^2}$
$e^{-at}\sin bt$	$\frac{b}{(s+a)^2+b^2}$
$e^{-at}\cos bt$	$\frac{s+a}{(s+a)^2+b^2}$



$$\mathcal{L}[\delta(t-t_0)] = e^{-st_0}$$



Begin- en eindwaarde-theorema

BEGINWAARDE THEOREMA

Indien Laplace transformaties

van $f(t)$ en $\dfrac{df(t)}{dt}$ bestaan:

$$\lim_{t\to 0} f(t) = \lim_{s\to\infty} sF(s)$$

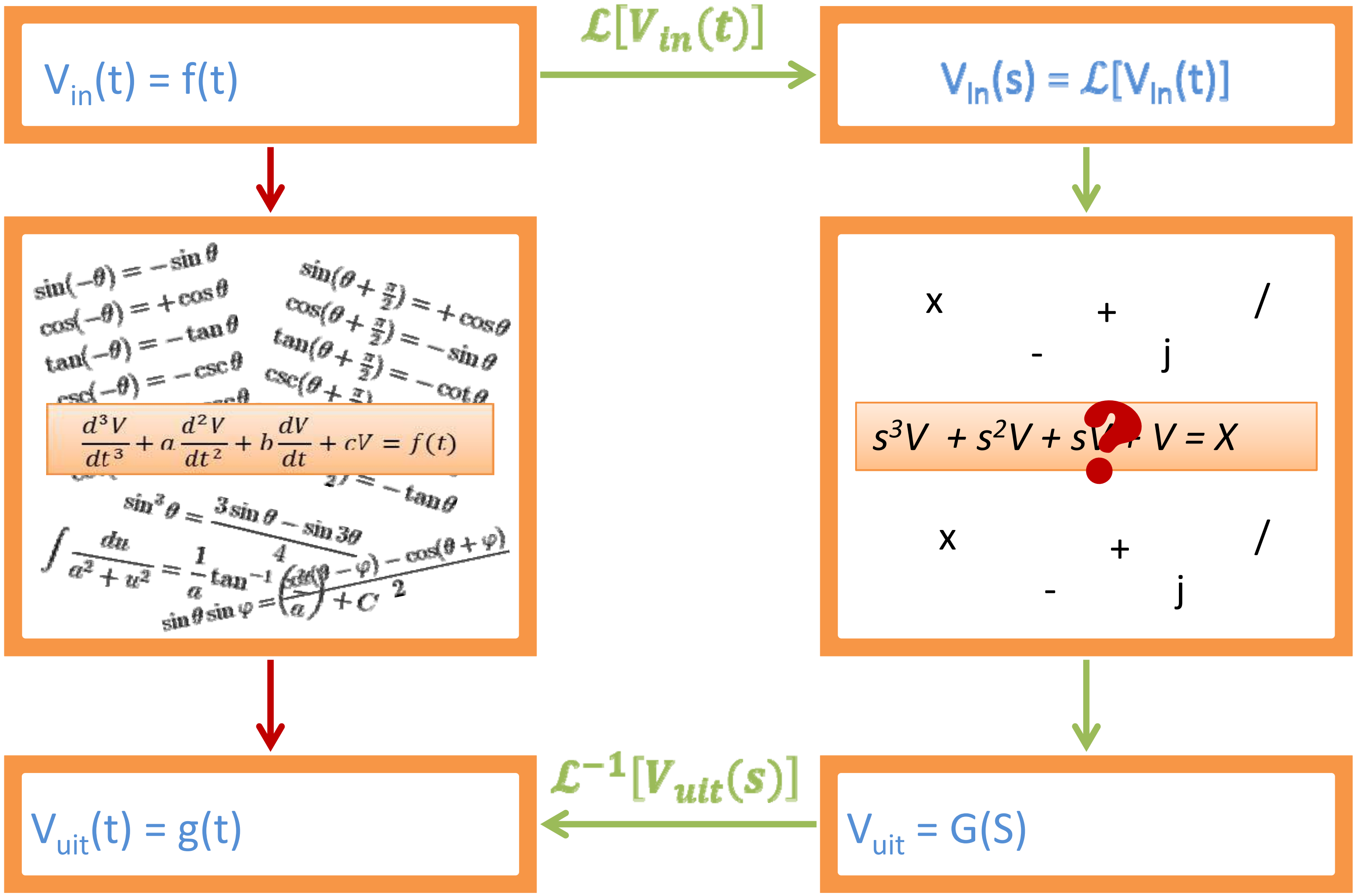
EINDWAARDE THEOREMA

Indien Laplace transformaties van

$f(t)$, $\dfrac{df(t)}{dt}$ en $\lim_{t\to\infty} f(t)$ bestaan:

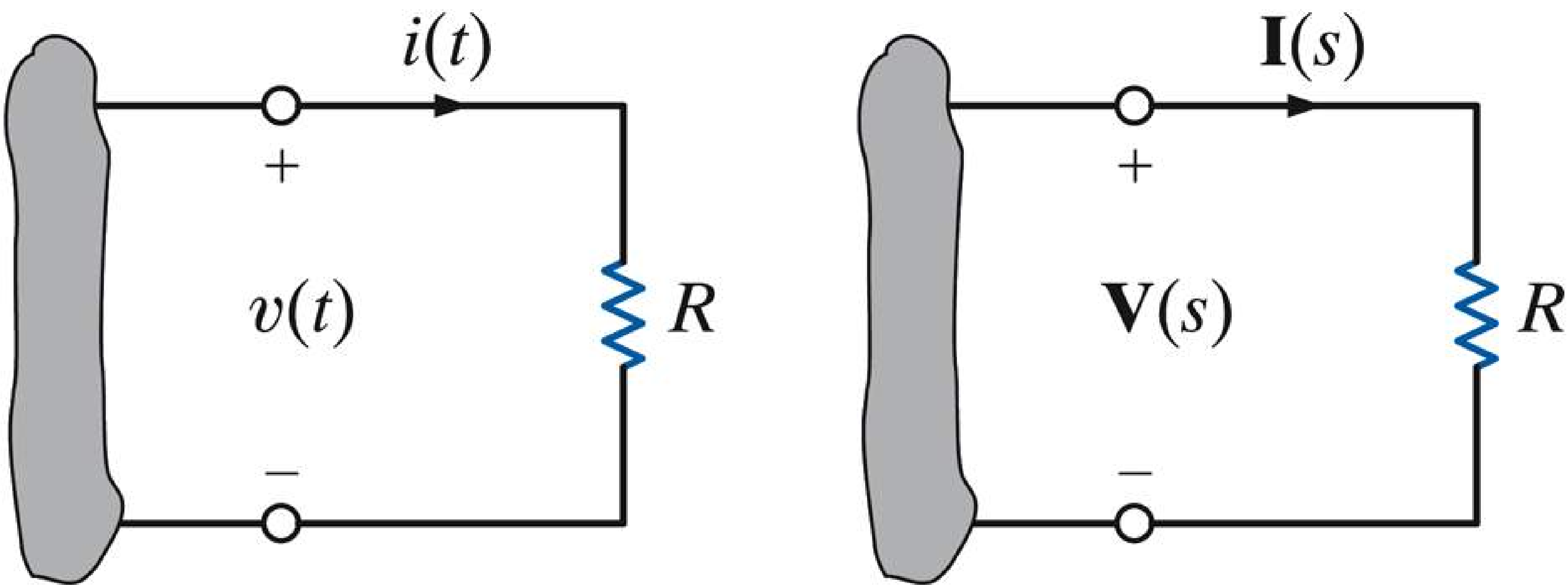
$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$$

Deze week


V_in(t) = f(t)
ℒ[V_in(t)]
V_In(s) = ℒ[V_In(t)]
d³V/dt³ + a d²V/dt² + b dV/dt + cV = f(t)
x + / - j
s³V + s²V + sV + V = X
x + / - j
ℒ⁻¹[V_uit(s)]
V_uit(t) = g(t)
V_uit = G(S)

Laplace-transformatie en circuit-elementen: 1, de resistantie

$v(t) = Ri(t)$ $\mathbf{V}(s) = R\mathbf{I}(s)$



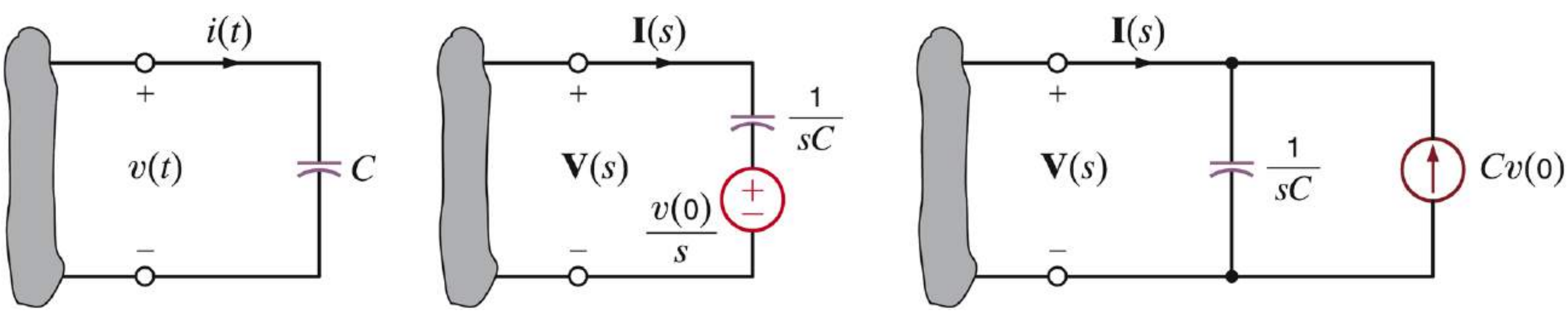
Laplace-transformatie en circuit-elementen: 2, de capaciteit

$$v(t) = \frac{1}{C}\int_0^t i(x)dx + v(0)$$

$$i(t) = C\frac{dv(t)}{dt}$$

$$\mathbf{V}(s) = \frac{\mathbf{I}(s)}{sC} + \frac{v(0)}{s}$$

$$\mathbf{I}(s) = sC\mathbf{V}(s) - Cv(0)$$



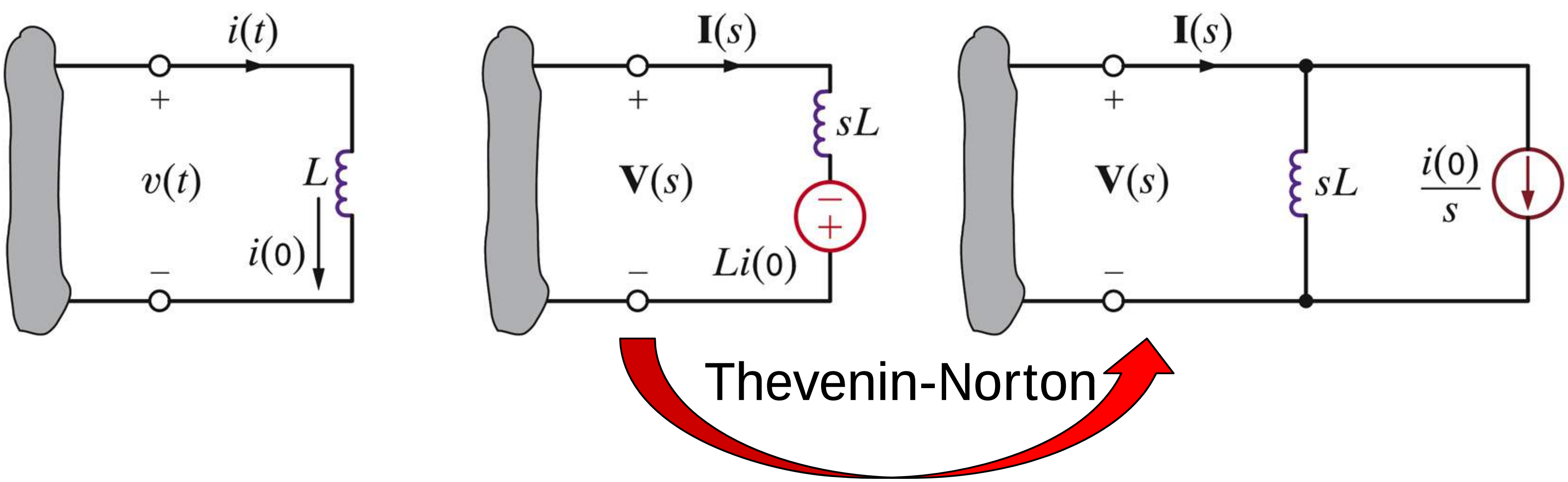
Laplace-transformatie en circuit-elementen: 3, de inductantie

$$v(t) = L\frac{di(t)}{dt}$$

$$i(t) = \frac{1}{L}\int_0^t v(x)dx + i(0)$$

$$\mathbf{V}(s) = sL\mathbf{I}(s) - Li(0)$$

$$\mathbf{I}(s) = \frac{\mathbf{V}(s)}{sL} + \frac{i(0)}{s}$$



PROBLEM-SOLVING STRATEGY

The Laplace Transform and Transient Circuits

Step 1. Assume that the circuit has reached steady state before a switch is moved. Draw the circuit valid for $t = 0^-$ replacing capacitors with open circuits and inductors with short circuits. Solve for the initial conditions: voltages across capacitors and currents flowing through inductors. Remember $v_C(0-) = v_C(0+) = v_C(0)$ and $i_L(0-) = i_L(0+) = i_L(0)$.

Step 2. Draw the circuit valid for $t > 0$. Use circuit analysis techniques to determine the differential or integrodifferential equation that describes the behavior of the circuit.

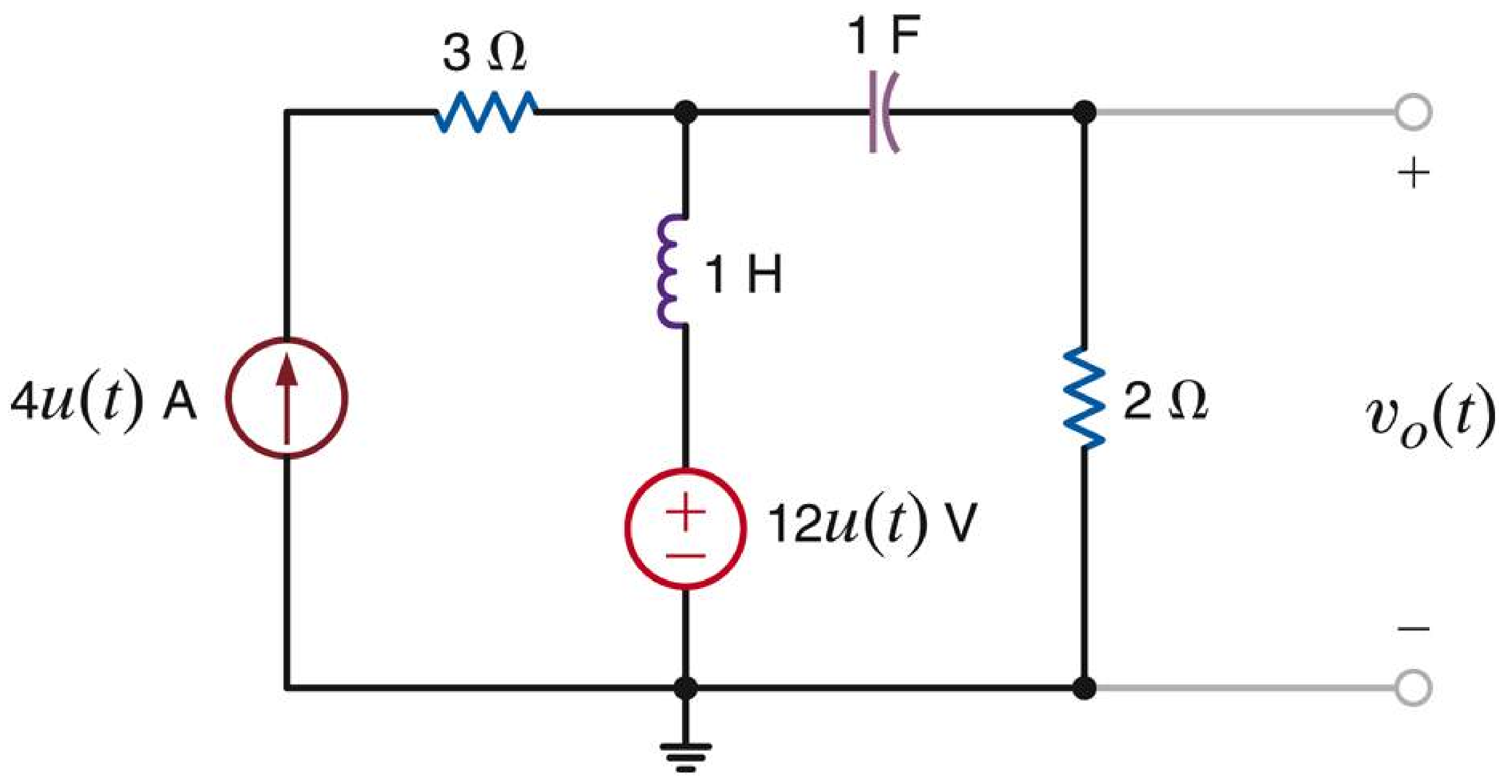
Step 3. Convert this differential/integrodifferential equation to an algebraic equation using the Laplace transform.

Step 4. Solve this algebraic equation for the variable of interest. Your result will be a ratio of polynomials in the complex variable s.

Step 5. Perform an inverse Laplace transform to solve for the circuit response in the time domain.

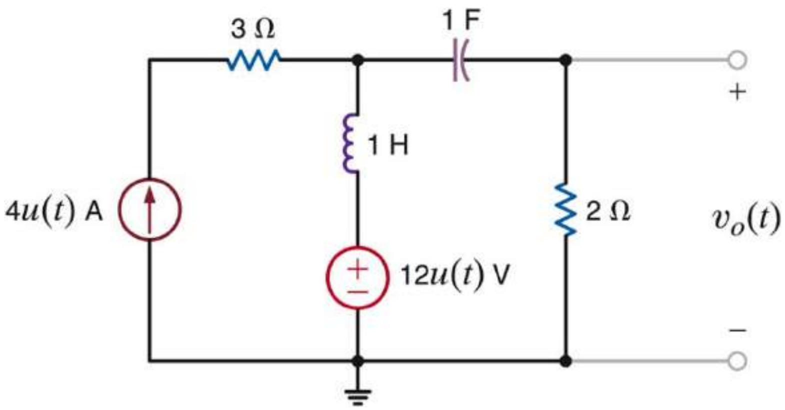
Voorbeeld (Example 14.3), 1

Gegeven:

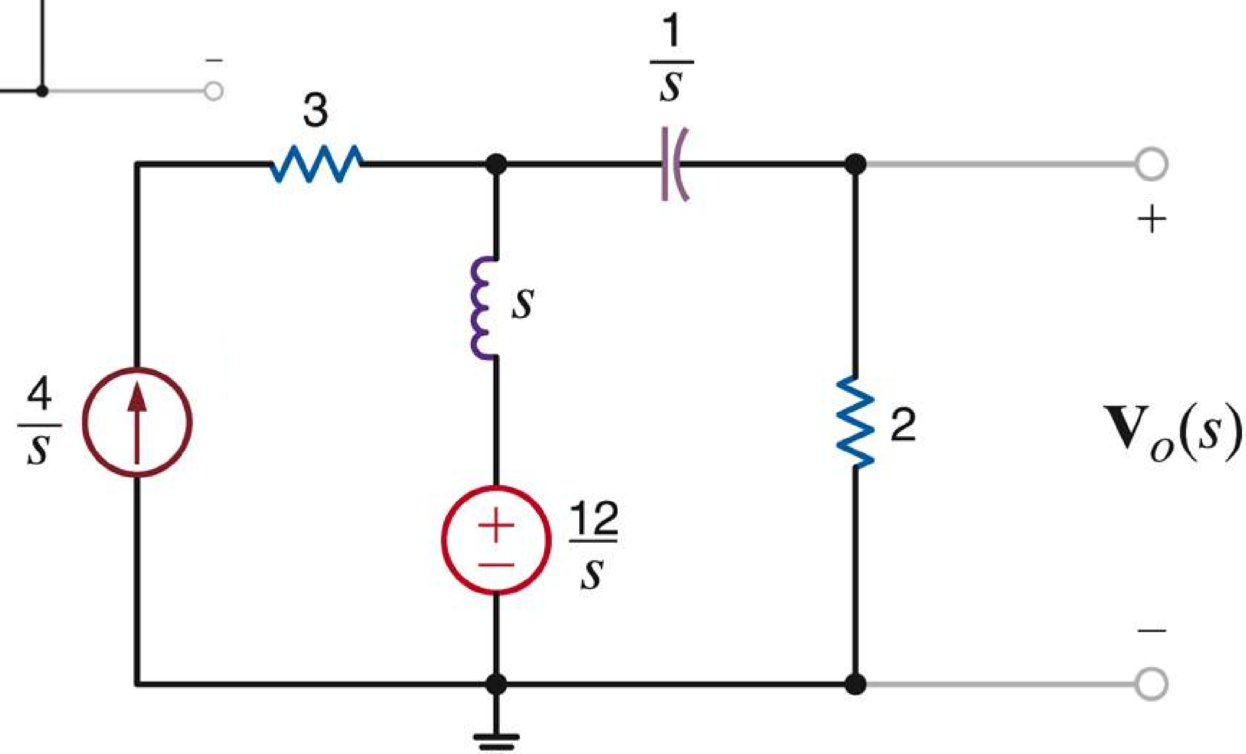


Gevraagd: $v_o(t)$

Voorbeeld (Example 14.3), 2

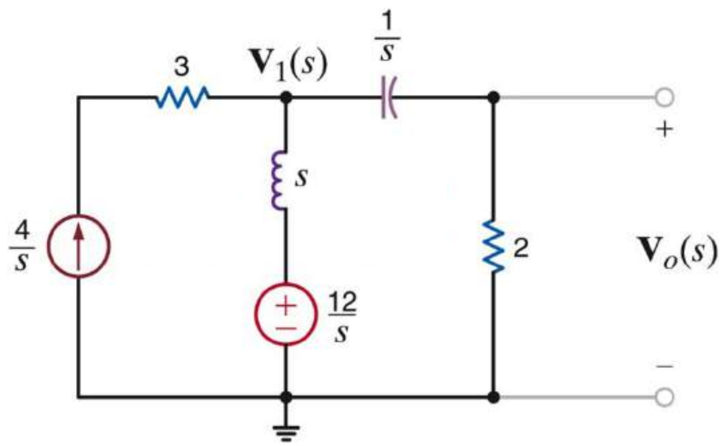


Getransformeerd netwerk



Voorbeeld (Example 14.3), 3

M.b.v. knooppuntsmethode



KCL op knooppunt $\mathbf{V}_1(s)$

$$-\frac{4}{s}+\frac{\mathbf{V}_1(s)-\frac{12}{s}}{s}+\frac{\mathbf{V}_1(s)}{\frac{1}{s}+2}=0$$

$$\mathbf{V}_1(s)=\frac{4(s+3)(2s+1)}{s(s^2+2s+1)}$$

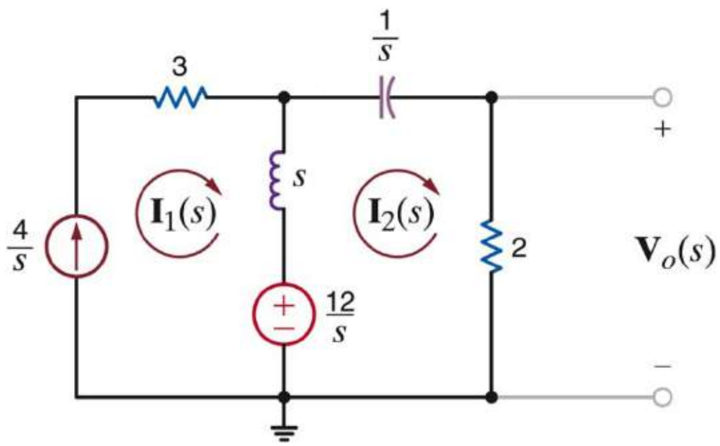
Toepassen spanningsdeling:

$$\mathbf{V}_o(s)=\mathbf{V}_1(s)\left(\frac{2}{\frac{1}{s}+2}\right)=\mathbf{V}_1(s)\left(\frac{2s}{2s+1}\right)$$

$$=\frac{8(s+3)}{(s+1)^2}$$

Voorbeeld (Example 14.3), 4

M.b.v. maasmethode



KVL op maas $I_1(s)$

$$\frac{12}{s} - [\mathbf{I}_2(s) - \mathbf{I}_1(s)]s - \frac{\mathbf{I}_2(s)}{s} - 2\mathbf{I}_2(s) = 0$$

$$\mathbf{I}_1(s) = \frac{4}{s}$$

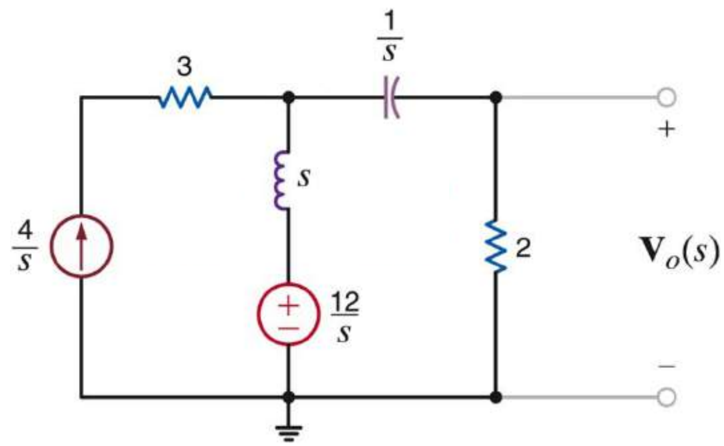
$$\mathbf{I}_2(s) = \frac{4(s+3)}{(s+1)^2}$$

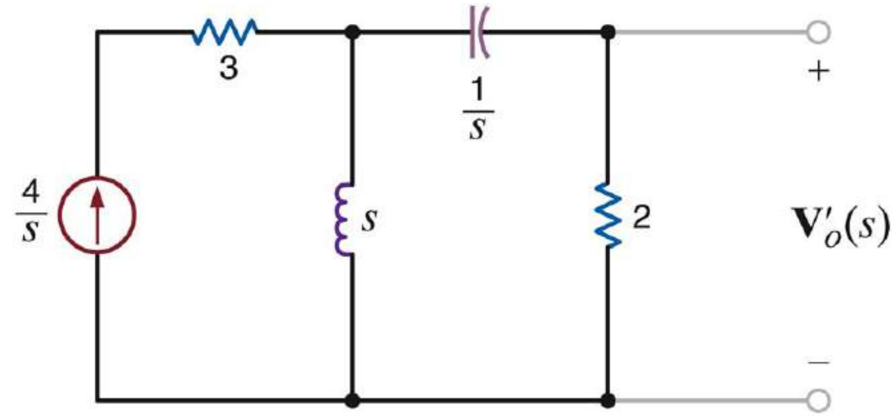
$$\mathbf{V}_o(s) = \frac{8(s+3)}{(s+1)^2}$$

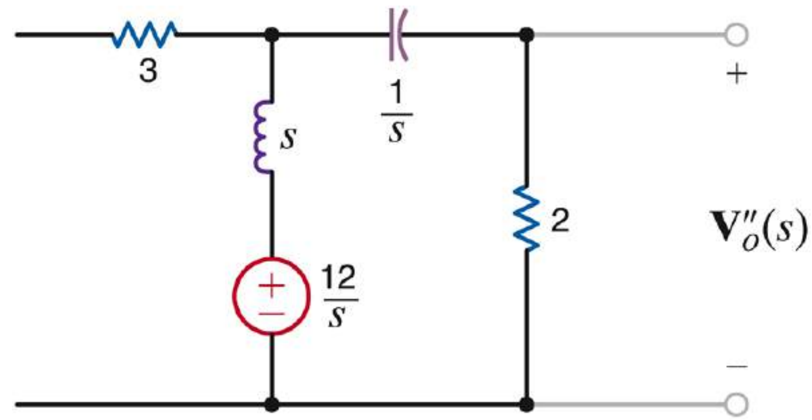
De resistantie van 3 ohm komt niet in deze uitdrukking voor. Waarom niet?

Voorbeeld (Example 14.3), 5

M.b.v. superpositie (1)

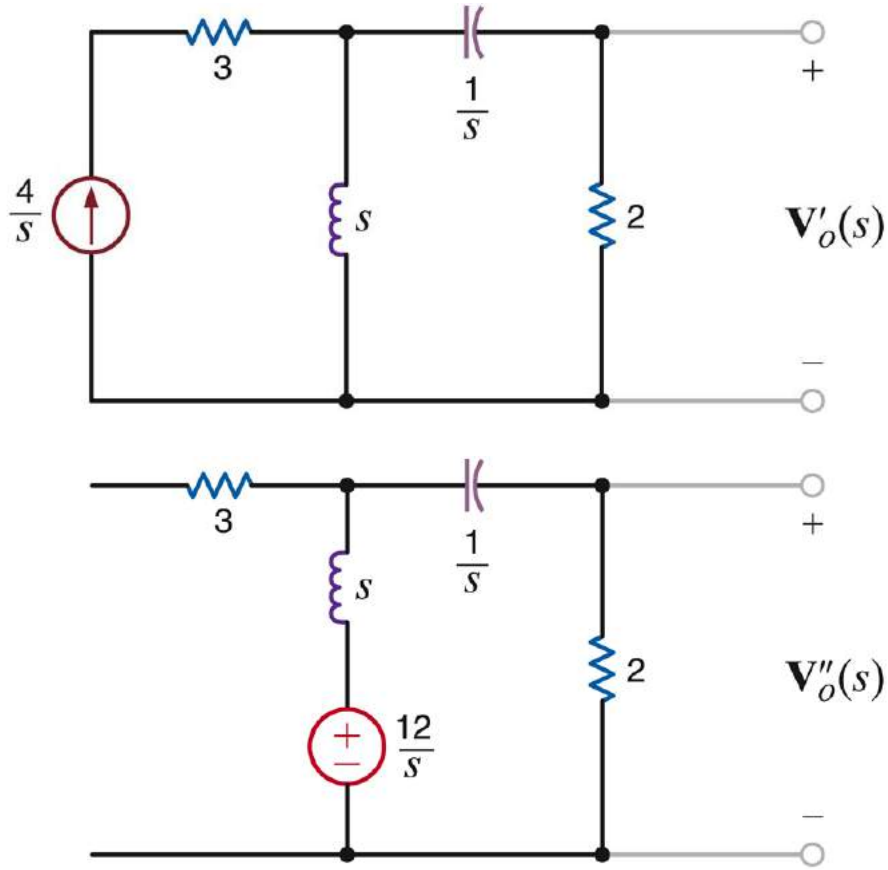






Voorbeeld (Example 14.3), 6

M.b.v. superpositie (2)



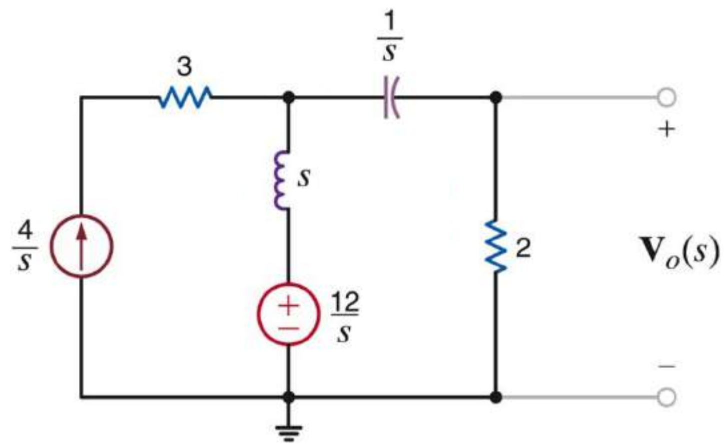
$$\mathbf{V}'_o(s) = \left[\frac{\frac{4}{s}s}{s+\frac{1}{s}+2}\right] = \frac{8s}{s^2+2s+1}$$

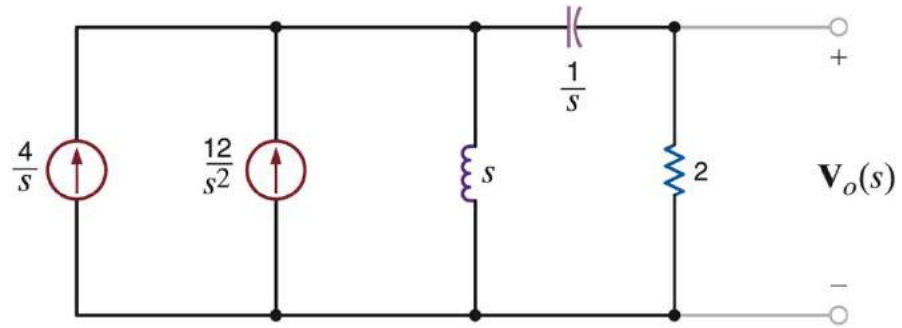
$$\mathbf{V}''_o(s) = \left[\frac{\frac{12}{s}2}{s+\frac{1}{s}+2}\right] = \frac{24}{s^2+2s+1}$$

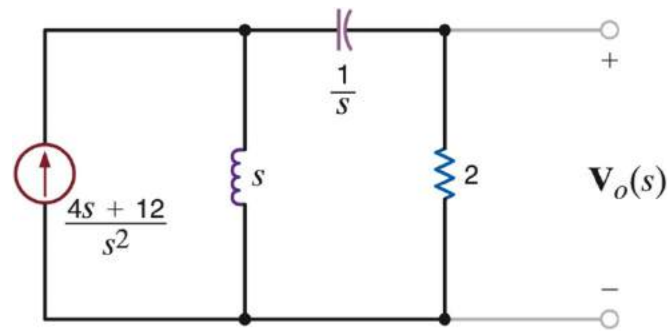
$$\mathbf{V}_o(s) = \mathbf{V}'_o(s) + \mathbf{V}''_o(s) = \frac{8(s+3)}{(s+1)^2}$$

Voorbeeld (Example 14.3), 7

M.b.v. bron-transformatie (Thevenin-Norton)

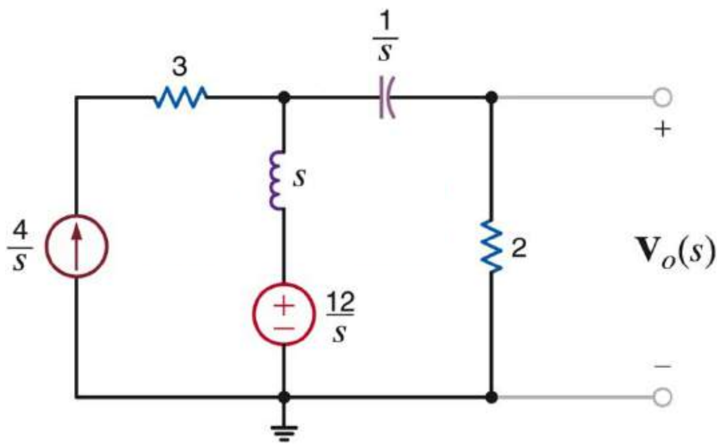


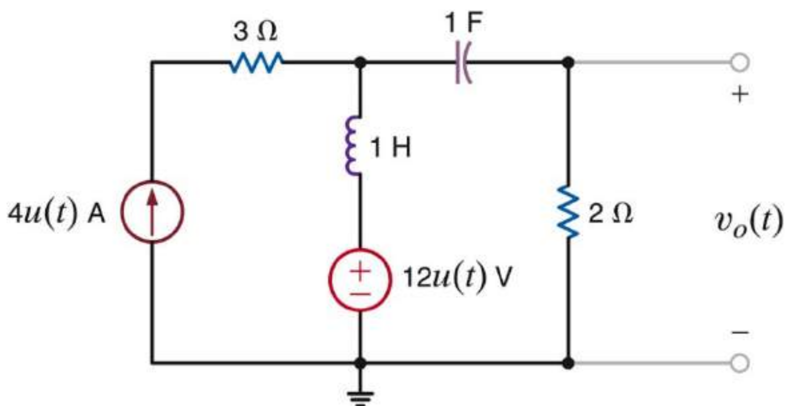




Voorbeeld (Example 14.3), 8

Terug-transformatie





$$\mathbf{V}_o(s) = \frac{8(s+3)}{(s+1)^2}$$

$$= \frac{K_{11}}{(s+1)^2} + \frac{K_{12}}{s+1}$$

$$= \frac{16}{(s+1)^2} + \frac{8}{s+1}$$

$$v_o(t) = \left(16te^{-t} + 8e^{-t}\right)u(t)\ \text{V}$$

Overdrachtsfunctie

Engels: transfer or network function

$$\mathbf{H}(s) = \frac{\mathbf{Y}_o(s)}{\mathbf{X}_i(s)}$$

en dus:

$$\mathbf{Y}_o(s) = \mathbf{H}(s) \cdot \mathbf{X}_i(s)$$

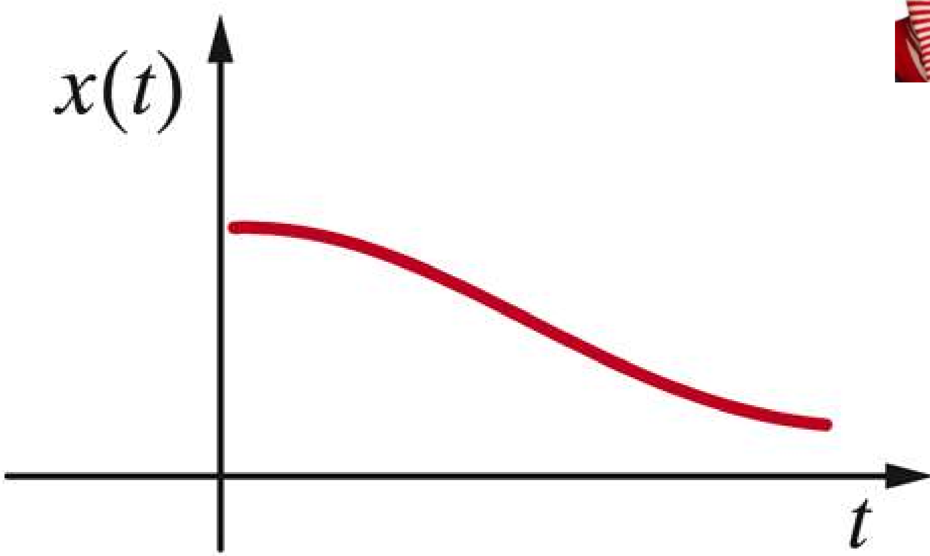
De overdrachtsfunctie geeft belangrijke informatie over de werking en stabiliteit van het systeem

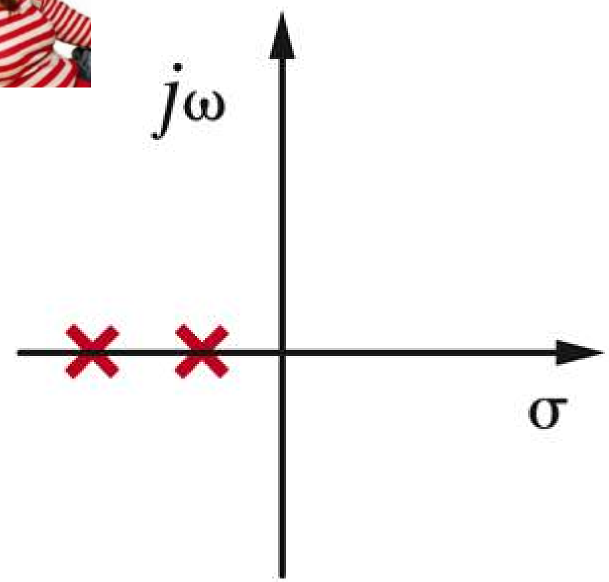
Opfrissen: karakteristieke vergelijking (1)

Geval 1: overgedempt circuit

$s_1 \neq s_2$ en s_1 en s_2 zijn reëel; discriminant: $b^2-4c > 0$

$$x(t) = k_1 e^{s_1 t} + k_2 e^{s_2 t} + k_3$$





Opfrissen: karakteristieke vergelijking (2)

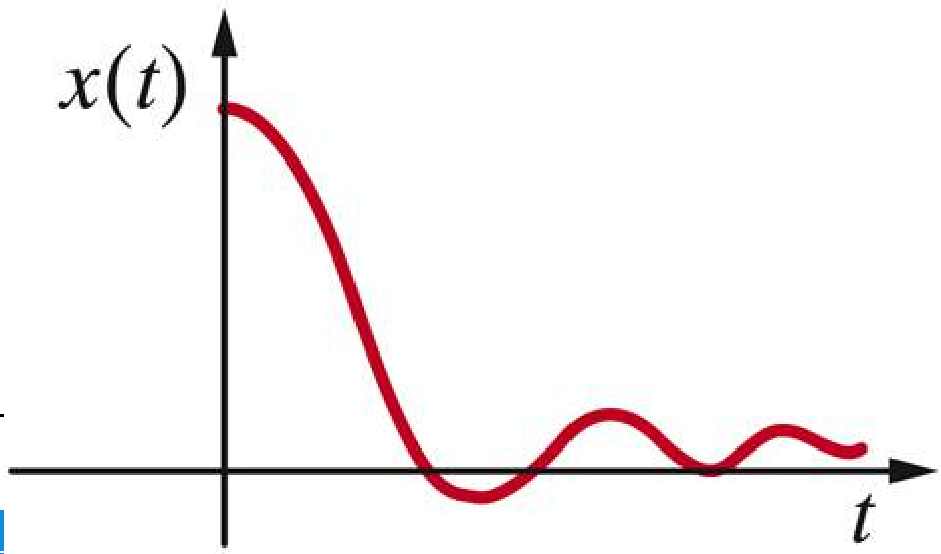
Geval 2: ondergedempt circuit

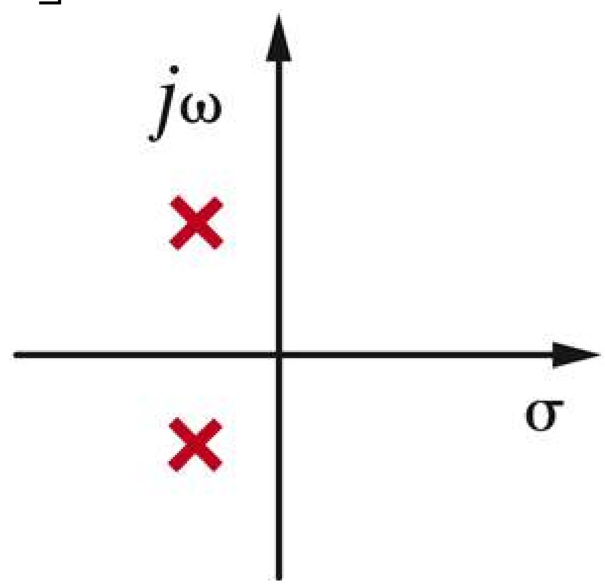
$s_1 \neq s_2$ en s_1 en s_2 zijn complex; discriminant: b^2-4c < 0

$$x(t) = k_1 e^{s_1 t} + k_2 e^{s_2 t} + k_3$$

$$s_1, s_2 = -\frac{b}{2} \pm j\frac{\sqrt{\left(4c - b^2\right)}}{2} = -\sigma \pm j\omega_d$$

$$x(t) = e^{-\sigma t}\left[A\cos\left(\omega_d t\right) + B\sin\left(\omega_d t\right)\right] + k_3$$

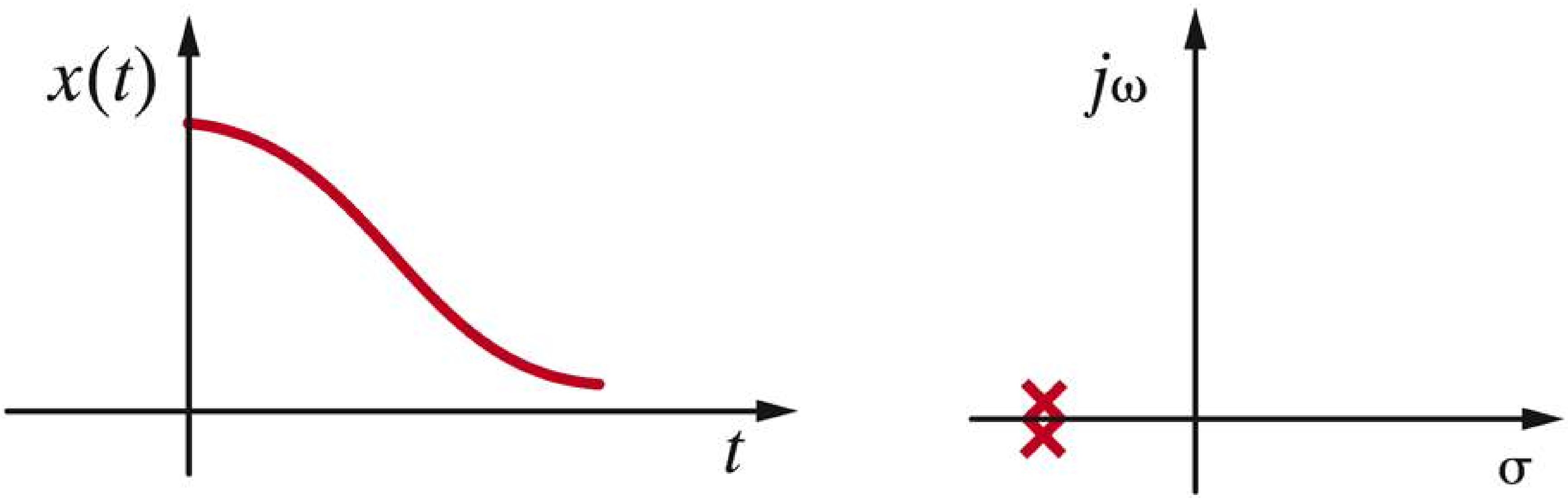




Opfrissen: karakteristieke vergelijking (3)

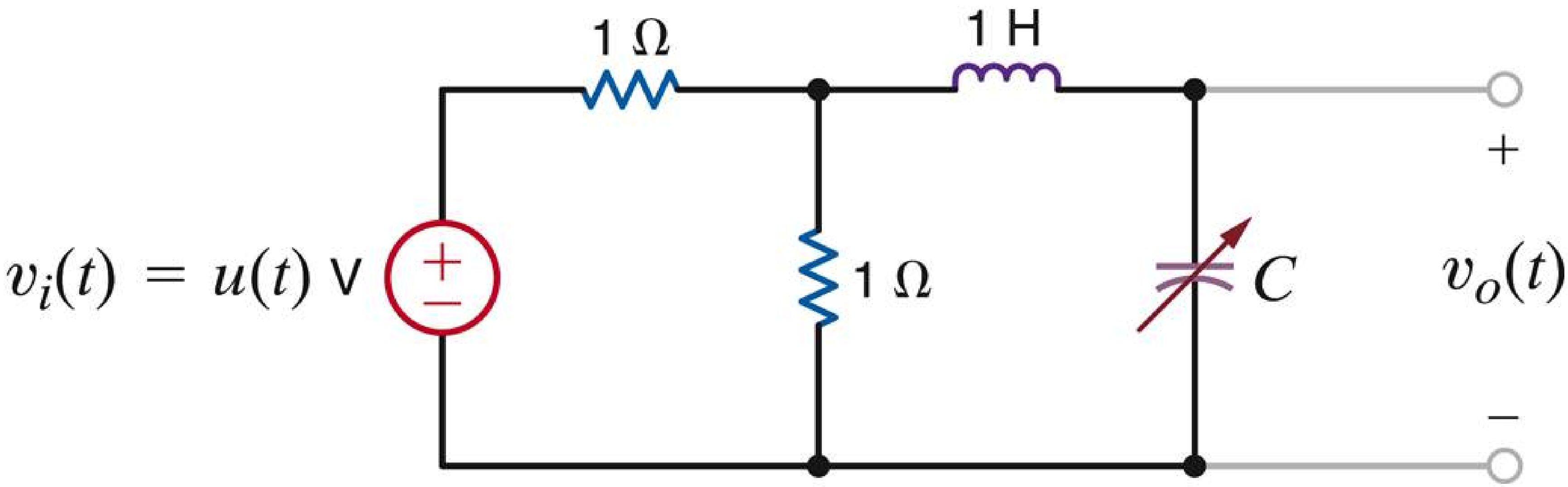
Geval 3: kritisch gedempt circuit
$s_1 = s_2$; discriminant: $b^2\text{-}4c = 0$

$$x(t) = (k_1 + k_2 t)e^{s_1 t} + k_3$$



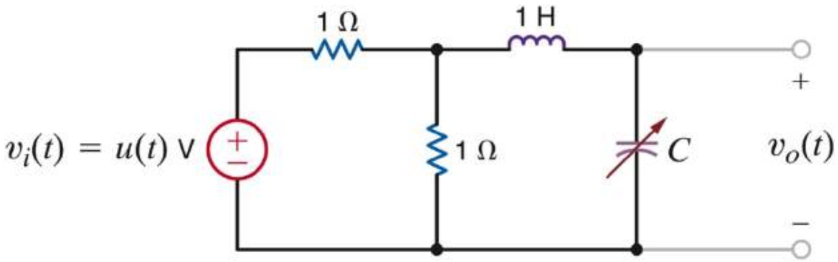
Voorbeeld (Example 14.7), 1

Gegeven:

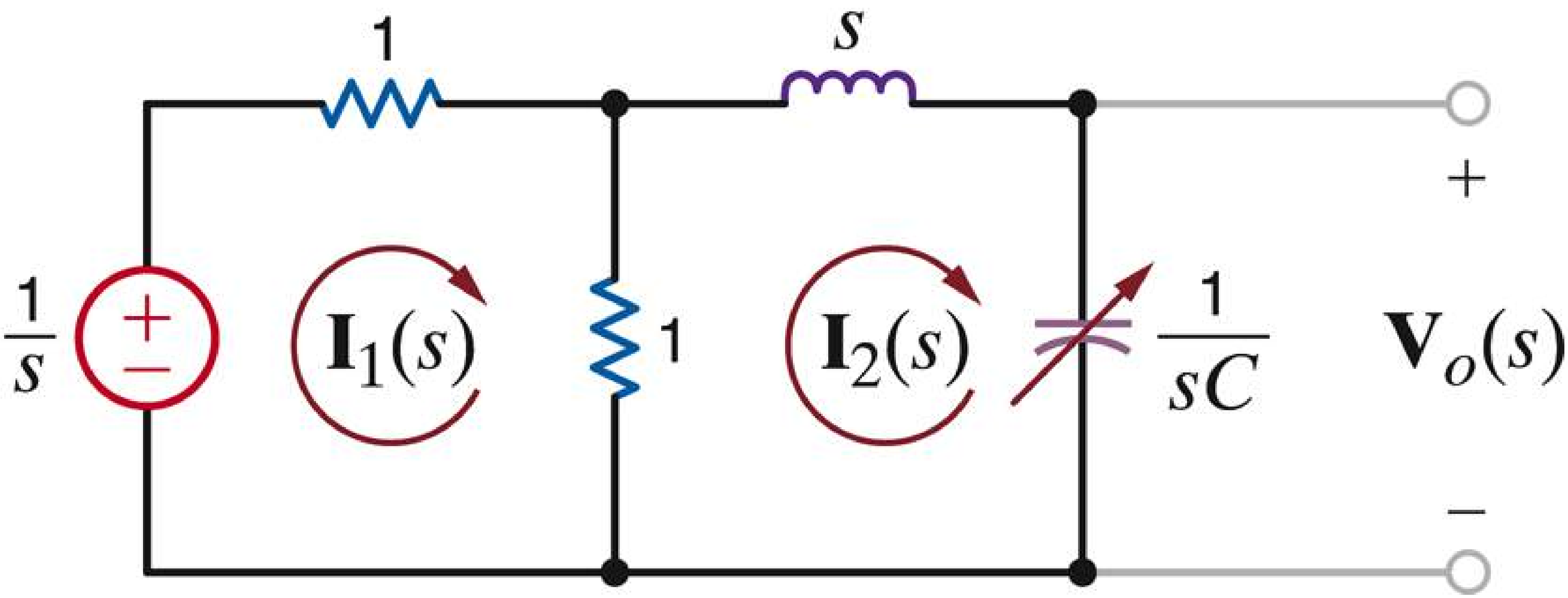


Gevraagd: $\mathbf{V}_o(s)/\mathbf{V}_i(s)$ en de stapresponsie

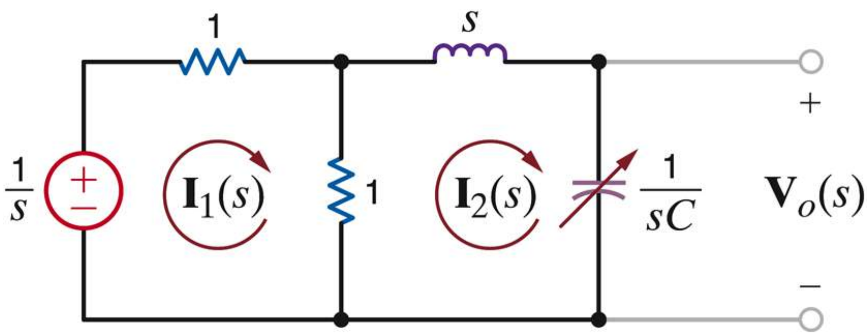
Voorbeeld (Example 14.7), 2



Getransformeerd netwerk



Voorbeeld (Example 14.7), 3



$$2\mathbf{I}_1(s) - \mathbf{I}_2(s) = \mathbf{V}_i(s)$$

$$-\mathbf{I}_1(s) + \left(s + \frac{1}{sC} + 1\right)\mathbf{I}_2(s) = 0$$

$$\mathbf{V}_o(s) = \frac{1}{sC}\mathbf{I}_2(s)$$

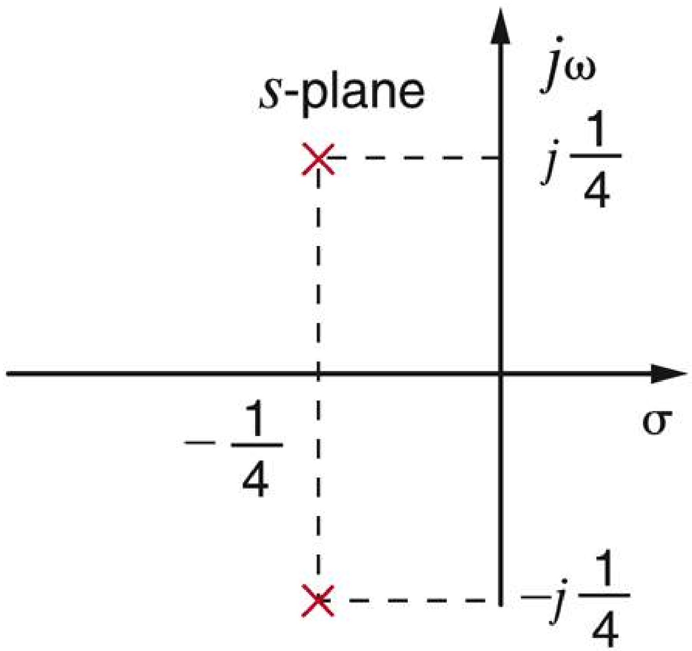
$$\mathbf{H}(s) = \frac{\mathbf{V}_o(s)}{\mathbf{V}_i(s)} = \frac{\frac{1}{2C}}{s^2 + \frac{1}{2}s + \frac{1}{C}}$$

$$\mathbf{V}_o(s) = \frac{\frac{1}{2C}}{s\left(s^2 + \frac{1}{2}s + \frac{1}{C}\right)}$$

3 gevallen:
- *a.* $C = 8\text{F}$
- *b.* $C = 16\text{F}$
- *c.* $C = 32\text{F}$

Voorbeeld (Example 14.7), 4

$C = 8\text{F}$



$$\mathbf{H}(s) = \frac{\mathbf{V}_o(s)}{\mathbf{V}_i(s)} = \frac{\frac{1}{2C}}{s^2 + \frac{1}{2}s + \frac{1}{C}}$$

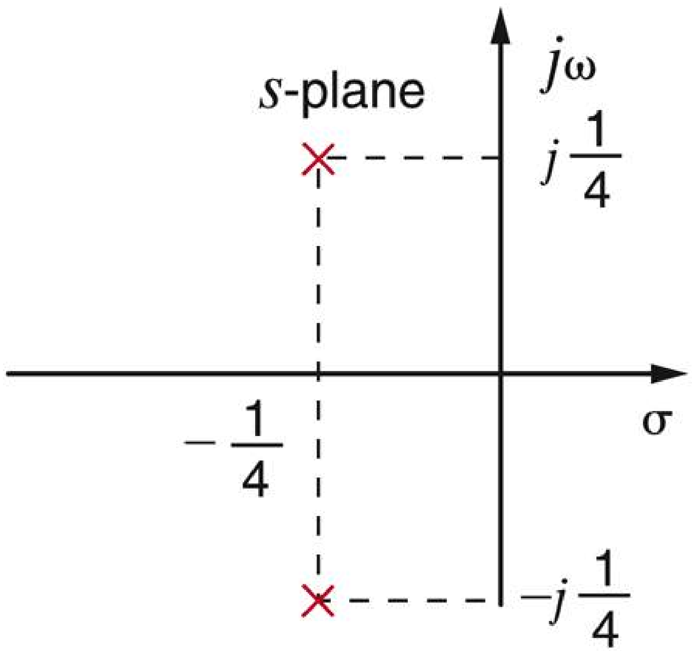
$$= \frac{\frac{1}{16}}{s^2 + \frac{1}{2}s + \frac{1}{8}}$$

$$= \frac{\frac{1}{16}}{\left(s + \frac{1}{4} - \frac{j}{4}\right)\left(s + \frac{1}{4} + \frac{j}{4}\right)}$$

$$\mathbf{V}_o(s) = \frac{\frac{1}{16}}{s\left(s + \frac{1}{4} - \frac{j}{4}\right)\left(s + \frac{1}{4} + \frac{j}{4}\right)}$$

Voorbeeld (Example 14.7), 4

$C = 8\text{F}$



$$\mathbf{V}_o(s) = \frac{\frac{1}{16}}{s\left(s+\frac{1}{4}-\frac{j}{4}\right)\left(s+\frac{1}{4}+\frac{j}{4}\right)}$$

$$= \frac{\frac{1}{2}}{s} + \frac{-\frac{1}{2}(s+\frac{1}{4})}{(s+\frac{1}{4})^2+(\frac{1}{4})^2} + \frac{-\frac{1}{2}(\frac{1}{4})}{(s+\frac{1}{4})^2+(\frac{1}{4})^2}$$

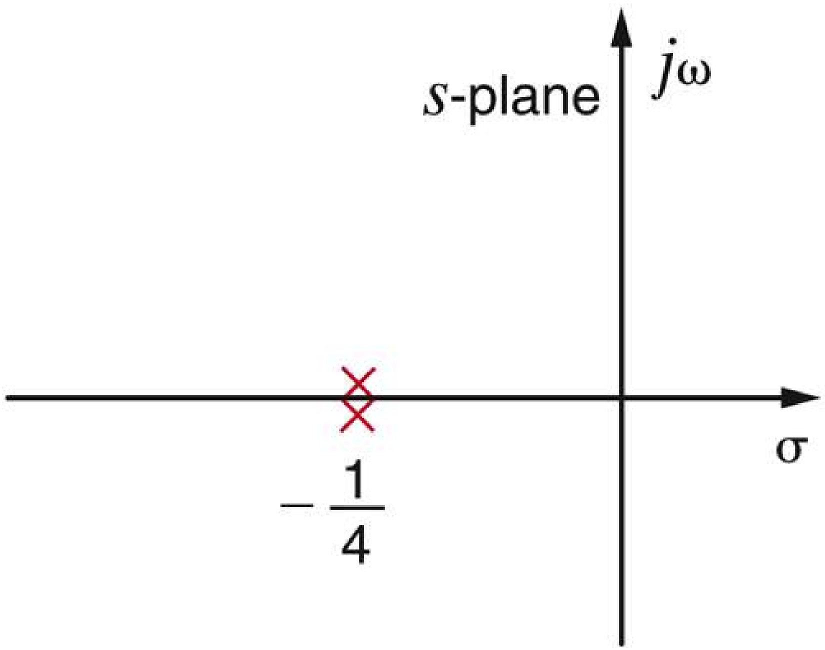
$$v_o(t) = \left[\frac{1}{2} + \frac{1}{\sqrt{2}}e^{-t/4}\cos\left(\frac{t}{4}+135°\right)\right]u(t)\ \text{V}$$

$e^{-at}\sin bt$	$\frac{b}{(s+a)^2+b^2}$
$e^{-at}\cos bt$	$\frac{s+a}{(s+a)^2+b^2}$

Volgt ook uit eindwaardestelling

Voorbeeld (Example 14.7), 5

$C = 16\text{F}$



$$\mathbf{H}(s) = \frac{\mathbf{V}_o(s)}{\mathbf{V}_i(s)} = \frac{\frac{1}{2C}}{s^2 + \frac{1}{2}s + \frac{1}{C}}$$

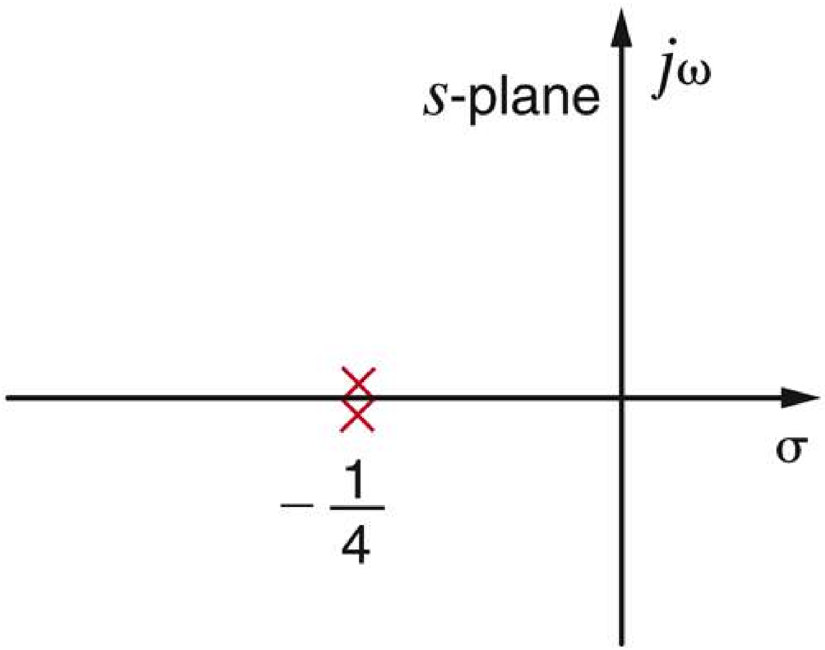
$$= \frac{\frac{1}{32}}{s^2 + \frac{1}{2}s + \frac{1}{16}}$$

$$= \frac{\frac{1}{32}}{\left(s + \frac{1}{4}\right)^2}$$

$$\mathbf{V}_o(s) = \frac{\frac{1}{32}}{s\left(s + \frac{1}{4}\right)^2}$$

Voorbeeld (Example 14.7), 5

$C = 16\text{F}$



$$\mathbf{V}_o(s) = \frac{\frac{1}{32}}{s\left(s+\frac{1}{4}\right)^2} = \frac{\frac{1}{2}}{s} + \frac{-\frac{1}{2}}{s+\frac{1}{4}} + \frac{-\frac{1}{8}}{\left(s+\frac{1}{4}\right)^2}$$

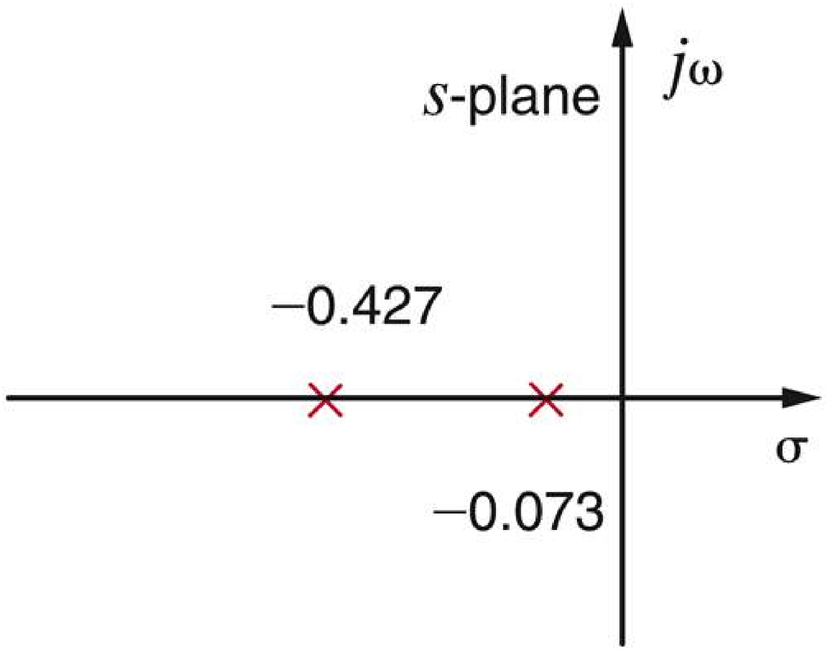
$$v_o(t) = \left[\frac{1}{2} - \left(\frac{t}{8} + \frac{1}{2}\right)e^{-t/4}\right]u(t)\ \text{V}$$

e^{-at}	$\frac{1}{s+a}$
te^{-at}	$\frac{1}{(s+a)^2}$

Volgt ook uit eindwaardestelling

Voorbeeld (Example 14.7), 6

$C = 32\text{F}$



$$\mathbf{H}(s) = \frac{\mathbf{V}_o(s)}{\mathbf{V}_i(s)} = \frac{\frac{1}{2C}}{s^2 + \frac{1}{2}s + \frac{1}{C}}$$

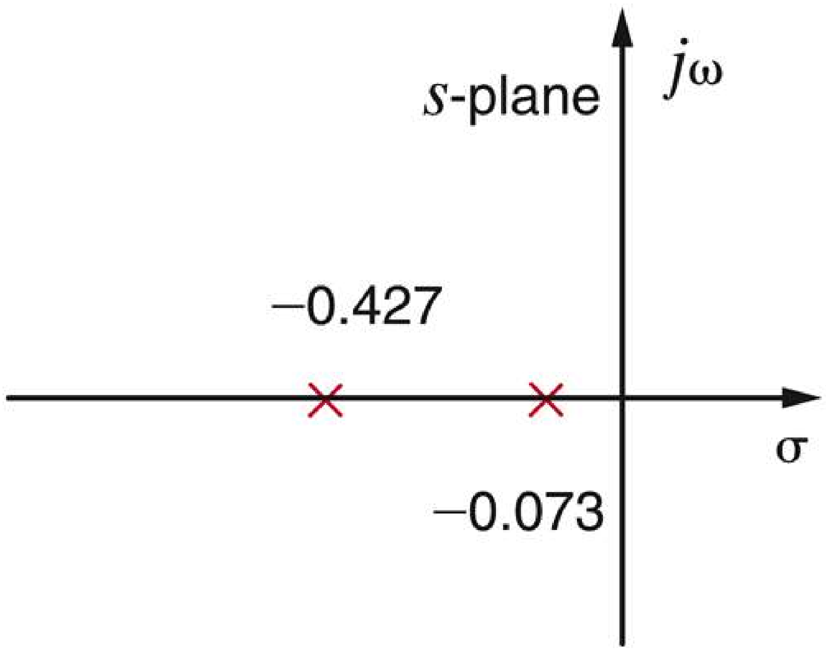
$$= \frac{\frac{1}{64}}{s^2 + \frac{1}{2}s + \frac{1}{32}}$$

$$= \frac{\frac{1}{64}}{(s+0,427)(s+0,073)}$$

$$\mathbf{V}_o(s) = \frac{\frac{1}{64}}{s(s+0,427)(s+0,073)}$$

Voorbeeld (Example 14.7), 6

$C = 32\text{F}$



$$\mathbf{V}_o(s) = \frac{\frac{1}{64}}{s(s+0{,}427)(s+0{,}073)}$$

$$= \frac{\frac{1}{2}}{s} + \frac{0{,}103}{s+0{,}427} + \frac{-0{,}603}{s+0{,}073}$$

$$v_o(t) = \left[\frac{1}{2} + 0{,}103e^{-0{,}427t} - 0{,}603e^{-0{,}073t}\right]u(t)\ \text{V}$$

e^{-at}	$\frac{1}{s+a}$

Volgt ook uit eindwaardestelling

Steady-state responsie

$$x(t) = X_M \cos\left(\omega_0 t + \theta\right)$$

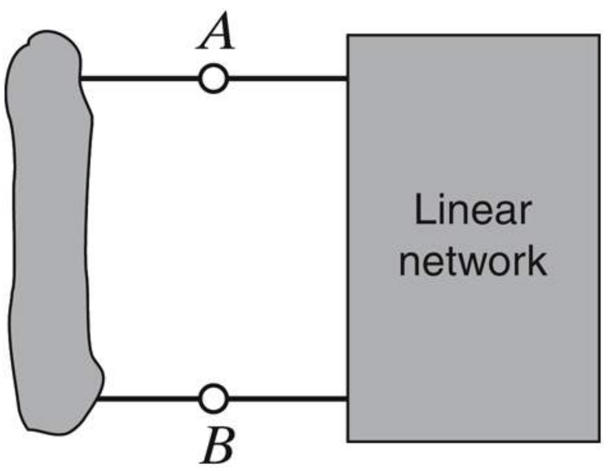
$$y_{ss}(t) = X_M \left|\mathbf{H}(j\omega_0)\right| \cos\left[\omega_0 t + \mathbf{\Phi}(j\omega_0) + \theta\right]$$

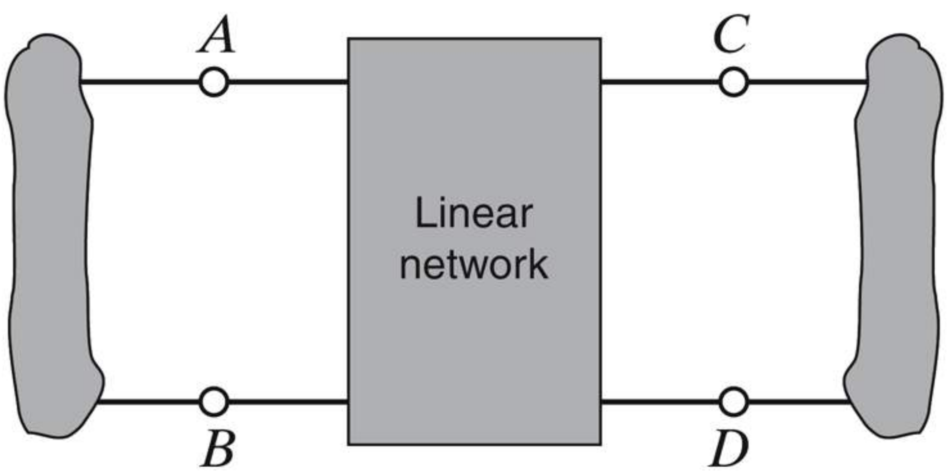
Geeft aan hoe het Fourier-domain gerelateerd is aan het Laplace-domein:

$$s = j\omega$$

$$H(j\omega)$$

Tweepoorten (1)



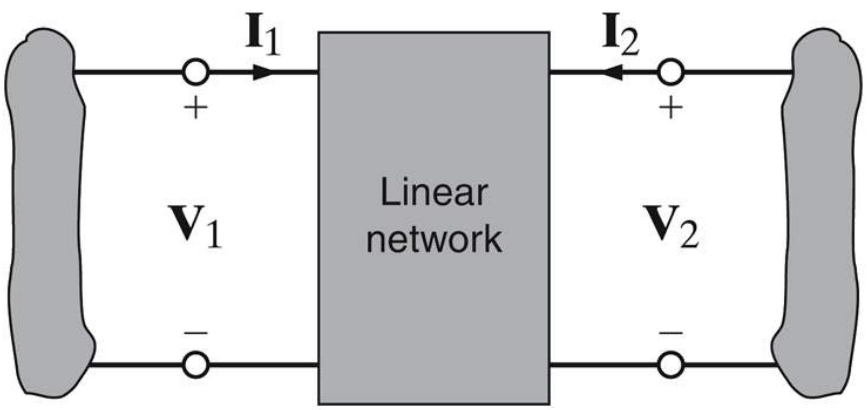


- Een-poort
  - Twee klemmen
    - Resistantie
    - Inductantie
    - Capaciteit
  - 1 relatie tussen spanning en stroom
- Tweepoort
  - Twee poorten
  - Vier klemmen
    - Magnetisch gekoppeld paar inductanties
    - Transformator
    - Elektronische schakeling
  - 4 relaties tussen spanningen en stromen

Tweepoorten (2)

- 4 (belangrijkste) soorten parameters
  - Admittantie-parameters
    - Drukken de 2 stromen uit als functie van de 2 spanningen
  - Impedantie-parameters
    - Drukken de 2 spanningen uit als functie van de 2 stromen
  - Hybride parameters
    - V_1 en I_2 als functie van V_2 en I_1
  - Ketting- of transmissie-parameters
    - V_1 en I_1 als functie van V_2 en I_2
    - Handig voor cascade-schakelingen
- Vaak uitgedrukt en geschreven in matrix-vorm
- Kan alleen wanneer relaties linear en dus superpositie geldt; dus geen onafhankelijke bronnen en alle beginvoorwaarden zijn nul

Admittantie-parameters



$$I_1 = y_{11}V_1 + y_{12}V_2$$
$$I_2 = y_{21}V_1 + y_{22}V_2$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Kortsluit-ingangsadmittantie

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$$

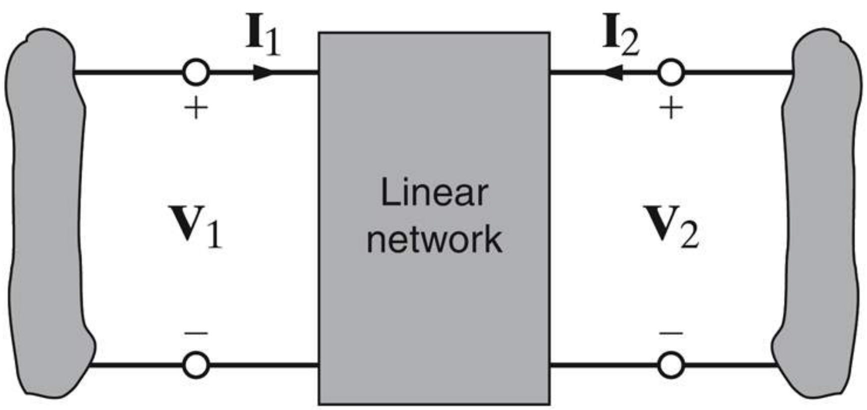
$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

$$y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

Kortsluit-uitgangsadmittantie

Impedantie-parameters



Open-klem ingangsimpedantie

$$\mathbf{z}_{11} = \left.\frac{\mathbf{V}_1}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0}$$

$$\mathbf{z}_{12} = \left.\frac{\mathbf{V}_1}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0}$$

$$\mathbf{z}_{21} = \left.\frac{\mathbf{V}_2}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0}$$

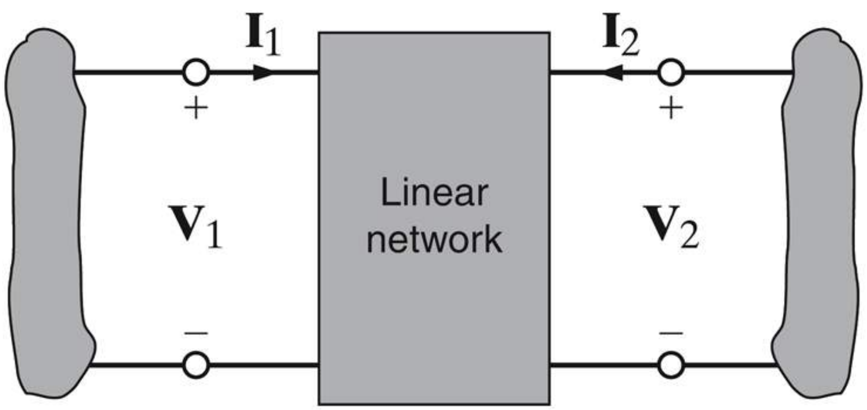
$$\mathbf{z}_{22} = \left.\frac{\mathbf{V}_2}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0}$$

Open-klem uitgangsimpedantie

$$\mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2$$
$$\mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2$$

$$\begin{bmatrix}\mathbf{V}_1\\ \mathbf{V}_2\end{bmatrix} = \begin{bmatrix}\mathbf{z}_{11} & \mathbf{z}_{12}\\ \mathbf{z}_{21} & \mathbf{z}_{22}\end{bmatrix}\begin{bmatrix}\mathbf{I}_1\\ \mathbf{I}_2\end{bmatrix}$$

Hybride parameters



Kortsluit-ingangsimpedantie

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$

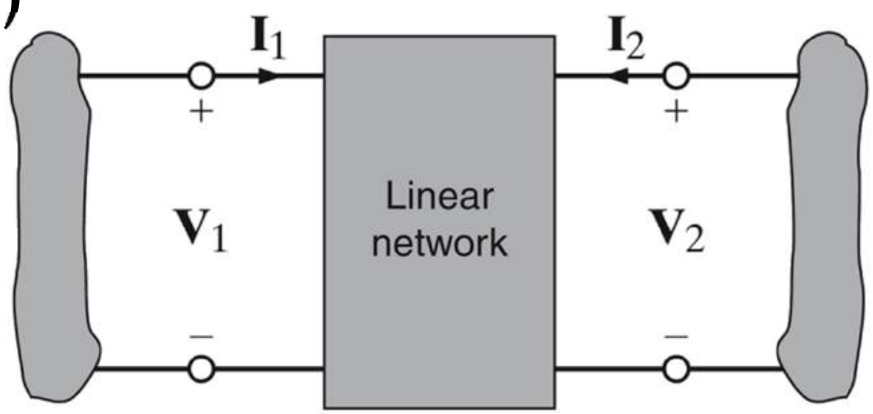
$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

Open-klem uitgangsadmittantie

$$V_1 = h_{11}I_1 + h_{12}V_2$$
$$I_2 = h_{21}I_1 + h_{22}V_2$$

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

Transmissie- of ketting-parameters (1)



$$k_{11} = A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$$

$$k_{12} = B = \left.\frac{V_1}{-I_2}\right|_{V_2=0}$$

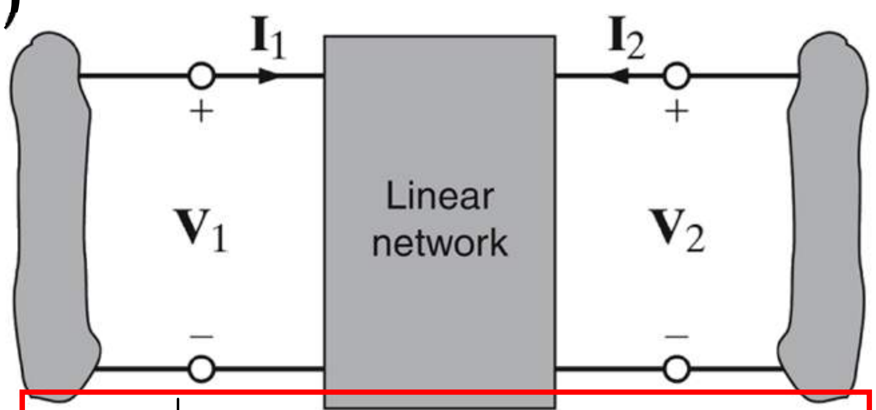
$$k_{21} = C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$$

$$k_{22} = D = \left.\frac{I_1}{-I_2}\right|_{V_2=0}$$

$$V_1 = k_{11}V_2 - k_{12}I_2$$
$$I_1 = k_{21}V_2 - k_{22}I_2$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

Transmissie- of ketting-parameters (2)



$$\mu = \left.\frac{V_2}{V_1}\right|_{I_2=0} = \text{spanningsversterkingsfactor}$$

$$\gamma = \left.\frac{I_2}{V_1}\right|_{V_2=0} = \text{transadmittantiefactor}$$

$$\zeta = \left.\frac{V_2}{I_1}\right|_{V_2=0} = \text{transimpedantiefactor}$$

$$\alpha = \left.\frac{I_2}{I_1}\right|_{V_2=0} = \text{stroomversterkingsfactor}$$

$$k_{11} = A = \frac{1}{\mu} = \left.\frac{V_1}{V_2}\right|_{I_2=0}$$

$$k_{12} = B = \frac{1}{\gamma} = \left.\frac{V_1}{-I_2}\right|_{V_2=0}$$

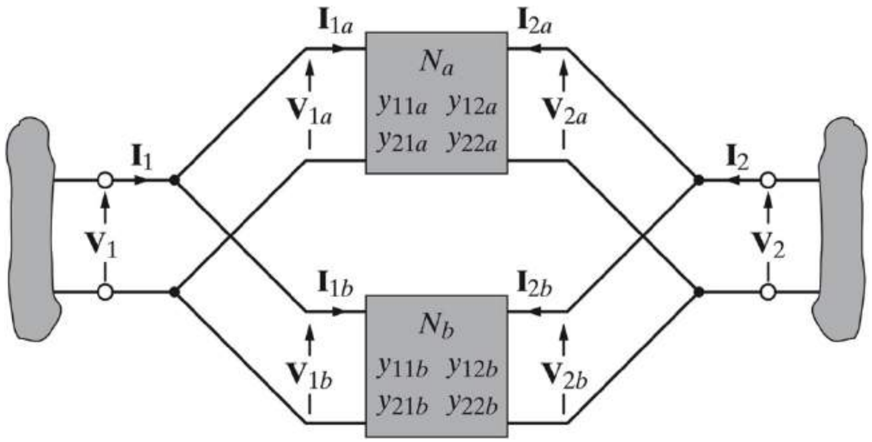
$$k_{21} = C = \frac{1}{\zeta} = \left.\frac{I_1}{V_2}\right|_{I_2=0}$$

$$k_{22} = D = \frac{1}{\alpha} = \left.\frac{I_1}{-I_2}\right|_{V_2=0}$$

TABLE 16.1 Two-port parameter conversion formulas

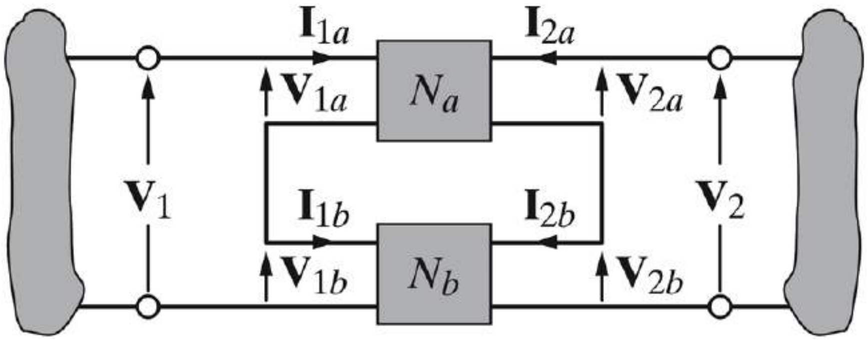
$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{y_{22}}{\Delta_Y} & \frac{-y_{12}}{\Delta_Y} \\ \frac{-y_{21}}{\Delta_Y} & \frac{y_{11}}{\Delta_Y} \end{bmatrix}$	$\begin{bmatrix} \frac{A}{C} & \frac{\Delta_T}{C} \\ \frac{1}{C} & \frac{D}{C} \end{bmatrix}$	$\begin{bmatrix} \frac{\Delta_H}{h_{22}} & \frac{h_{12}}{h_{22}} \\ \frac{-h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{bmatrix}$
$\begin{bmatrix} \frac{z_{22}}{\Delta_Z} & \frac{-z_{12}}{\Delta_Z} \\ \frac{-z_{21}}{\Delta_Z} & \frac{z_{11}}{\Delta_Z} \end{bmatrix}$	$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{D}{B} & \frac{-\Delta_T}{B} \\ -\frac{1}{B} & \frac{A}{B} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{h_{11}} & \frac{-h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{\Delta_H}{h_{11}} \end{bmatrix}$
$\begin{bmatrix} \frac{z_{11}}{z_{21}} & \frac{\Delta_Z}{z_{21}} \\ \frac{1}{z_{21}} & \frac{z_{22}}{z_{21}} \end{bmatrix}$	$\begin{bmatrix} \frac{-y_{22}}{y_{21}} & \frac{-1}{y_{21}} \\ \frac{-\Delta_Y}{y_{21}} & \frac{-y_{11}}{y_{21}} \end{bmatrix}$	$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$	$\begin{bmatrix} \frac{-\Delta_H}{h_{21}} & \frac{-h_{11}}{h_{21}} \\ \frac{-h_{22}}{h_{21}} & \frac{-1}{h_{21}} \end{bmatrix}$
$\begin{bmatrix} \frac{\Delta_Z}{z_{22}} & \frac{z_{12}}{z_{22}} \\ \frac{-z_{21}}{z_{22}} & \frac{1}{z_{22}} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{y_{11}} & \frac{-y_{12}}{y_{11}} \\ \frac{y_{21}}{y_{11}} & \frac{\Delta_Y}{y_{11}} \end{bmatrix}$	$\begin{bmatrix} \frac{B}{D} & \frac{\Delta_T}{D} \\ -\frac{1}{D} & \frac{C}{D} \end{bmatrix}$	$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$

Parallel-parallel schakeling van tweepoorten



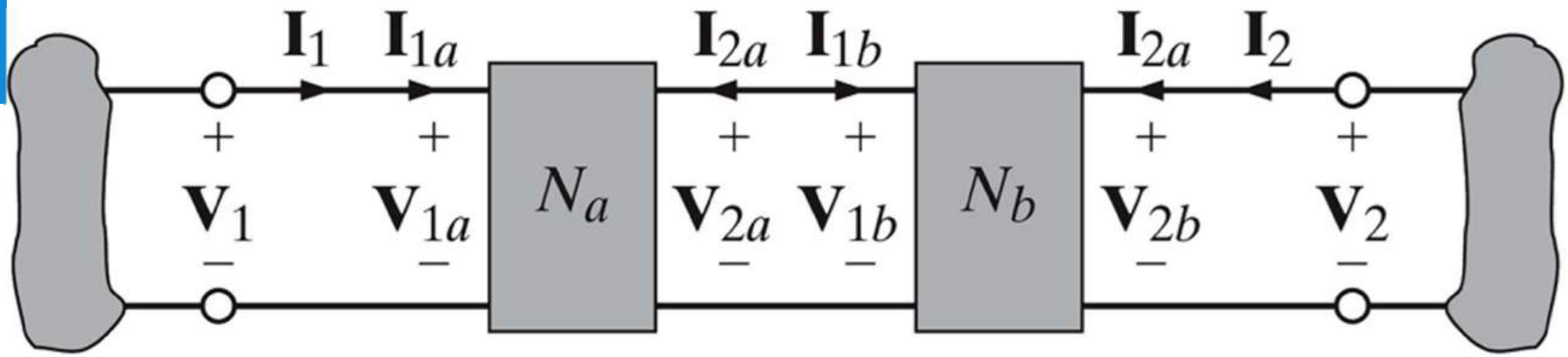
$$\begin{bmatrix} \mathbf{y}_{11} & \mathbf{y}_{12} \\ \mathbf{y}_{21} & \mathbf{y}_{22} \end{bmatrix} = \begin{bmatrix} \mathbf{y}_{11a} + \mathbf{y}_{11b} & \mathbf{y}_{12a} + \mathbf{y}_{12b} \\ \mathbf{y}_{21a} + \mathbf{y}_{21b} & \mathbf{y}_{22a} + \mathbf{y}_{22b} \end{bmatrix}$$

Serie-serie schakeling van tweepoorten



$$\begin{bmatrix} \mathbf{Z}_{11} & \mathbf{Z}_{12} \\ \mathbf{Z}_{21} & \mathbf{Z}_{22} \end{bmatrix} = \begin{bmatrix} \mathbf{Z}_{11a} + \mathbf{Z}_{11b} & \mathbf{Z}_{12a} + \mathbf{Z}_{12b} \\ \mathbf{Z}_{21a} + \mathbf{Z}_{21b} & \mathbf{Z}_{22a} + \mathbf{Z}_{22b} \end{bmatrix}$$

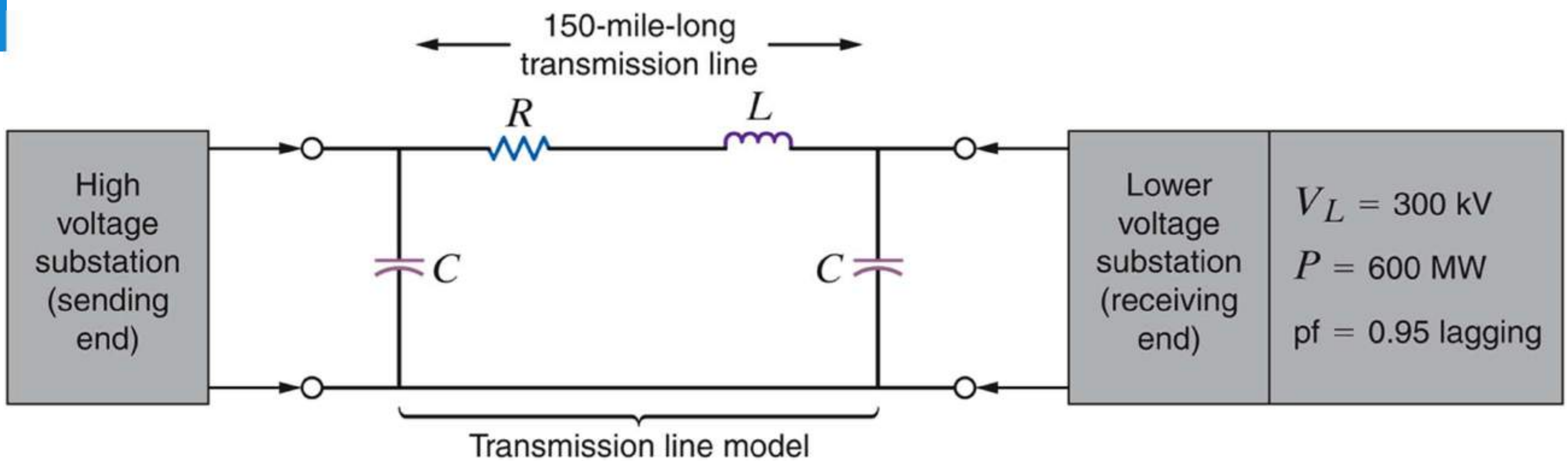
Cascade-schakeling van tweepoorten



$$\begin{bmatrix} \mathbf{k}_{11} & \mathbf{k}_{12} \\ \mathbf{k}_{21} & \mathbf{k}_{22} \end{bmatrix} = \begin{bmatrix} \mathbf{k}_{11a} & \mathbf{k}_{12a} \\ \mathbf{k}_{21a} & \mathbf{k}_{22a} \end{bmatrix} \begin{bmatrix} \mathbf{k}_{11b} & \mathbf{k}_{12b} \\ \mathbf{k}_{21b} & \mathbf{k}_{22b} \end{bmatrix}$$

Voorbeeld (Example 16.8), 1

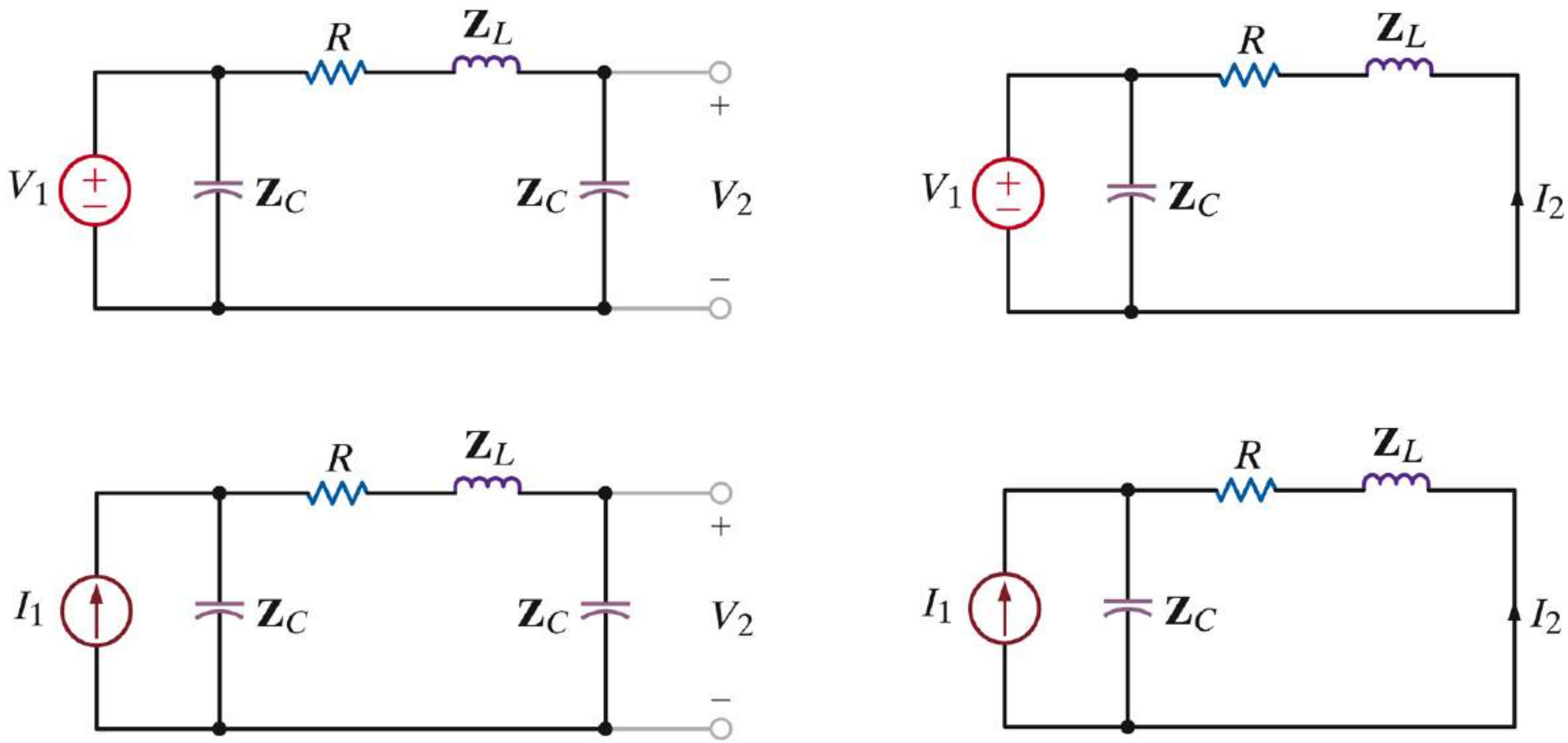
Gegeven:



Gevraagd: transmissie-parameters

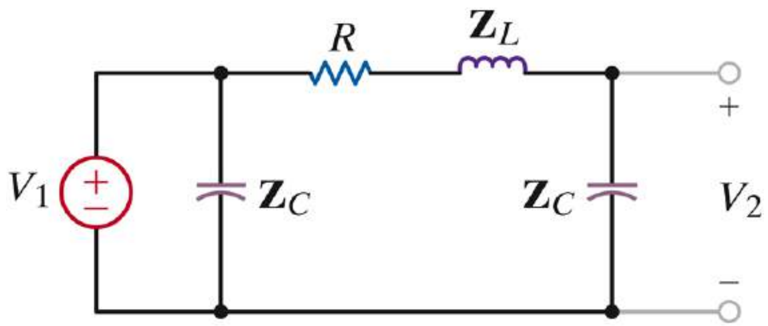
Voorbeeld (Example 16.8), 2

Transmissie-parameters



Voorbeeld (Example 16.8), 3

Transmissie-parameter: A

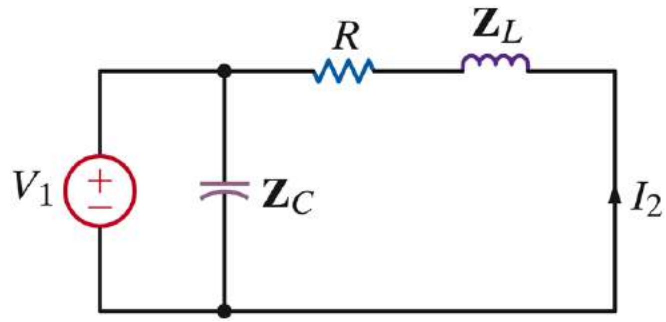


$$\mu = \left.\frac{V_2}{V_1}\right|_{I_2=0} = \frac{Z_C}{Z_C + Z_L + R}$$

$$k_{11} = A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{Z_C + Z_L + R}{Z_C}$$

Voorbeeld (Example 16.8), 4

Transmissie-parameters: B



$$\gamma = \left.\frac{-I_2}{V_1}\right|_{V_2=0} = \frac{1}{Z_L + R}$$

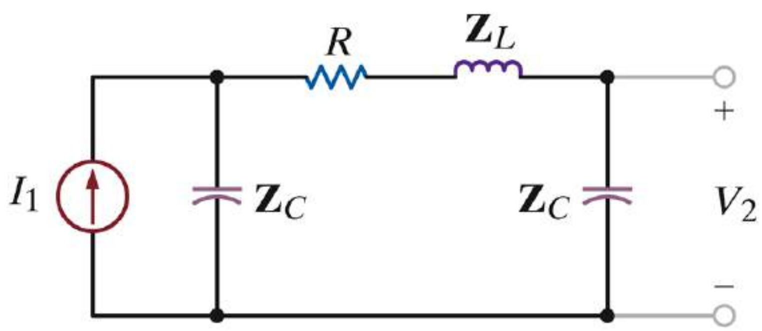
$$k_{12} = B = \left.\frac{V_1}{I_2}\right|_{V_2=0} = Z_L + R$$

Voorbeeld (Example 16.8), 5

Transmissie-parameters: C

$$\zeta = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \frac{Z_C^2}{2Z_C + Z_L + R}$$

$$k_{21} = C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{2Z_C + Z_L + R}{Z_C^2}$$

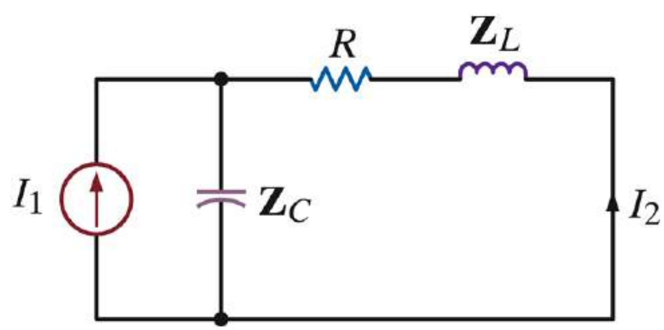


Voorbeeld (Example 16.8), 6

Transmissie-parameters: D

$$\alpha = \left.\frac{-I_2}{I_1}\right|_{V_2=0} = \frac{Z_C}{Z_C + Z_L + R}$$

$$k_{22} = D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} = \frac{Z_C + Z_L + R}{Z_C}$$



Voorbeeld (Example 16.8), 7

Transmissie-parameters:

$$\mathbf{V_1 = k_{11}V_2 - k_{12}I_2}$$
$$\mathbf{I_1 = k_{21}V_2 - k_{22}I_2}$$

$$\begin{bmatrix} \mathbf{V_1} \\ \mathbf{I_1} \end{bmatrix} = \begin{bmatrix} \mathbf{k_{11}} & \mathbf{k_{12}} \\ \mathbf{k_{21}} & \mathbf{k_{22}} \end{bmatrix} \begin{bmatrix} \mathbf{V_2} \\ \mathbf{-I_2} \end{bmatrix}$$

Volgende keer

- Onderwerpen: de leerstof in vogelvlucht en voorbeelden
- 7 januari
- Wouter Serdijn

Fijne kerstdagen en een succesvol en gelukkig 2013!