Linear Phase-Noise Model
Sub-Outline

- Generic Linear Phase-Noise Model
- Circuit-Specific Linear Phase-Noise Model
Generic Linear Phase-Noise Model - Outline

• Linear Oscillator Model
  • LC-Tank noise
  • active part noise
• (Phase) Noise Factor
• Phase-Noise Properties
Linear Oscillator Model

- transconductor noise
- LC-tank noise
- no tail-current source noise

- LC-tank impedance (noise shaping)

\[
Z(\omega_0 + \Delta\omega) \approx \frac{-j\omega_0 L}{2\Delta\omega / \omega_0} = \frac{-j}{2\omega_0 C} \frac{\omega_0}{\Delta\omega}
\]

\[
|Z(\omega_0 + \Delta\omega)|^2 \approx \frac{1}{4G_{tk}^2 Q^2} \left(\frac{\omega_0}{\Delta\omega}\right)^2 = \frac{R_{TK}^2}{4Q^2} \left(\frac{\omega_0}{\Delta\omega}\right)^2
\]

- LC-tank quality factor

\[
Q = \frac{1}{\omega_0 LG_{TK}}
\]

\[
Q = \frac{\omega_0 C}{G_{TK}}
\]
LC-Tank Noise

- tank resistance noise \((R_{TK}=1/G_{TK})\)
  \[ i_{GTK}^2 = 4KTG_{TK} \]

- tank contribution to the equivalent voltage noise spectral density
  \[ v_{GTK}^2 = i_{GTK}^2 \left| Z(\Delta\omega) \right|^2 = KT \frac{G_{TK}}{(\omega_0 C)^2} \left( \frac{\omega_0}{\Delta\omega} \right)^2 \]
Active Part Noise

- active part contribution to the equivalent voltage noise spectral density

\[ v_{-g}^2 = KT \frac{G_{TK}}{(\omega_0 C)^2} A \left( \frac{\omega_0}{\Delta\omega} \right)^2 \]

- active part noise factor \( A \)
  - excess negative conductance
  - additional noise of the active devices
  - ideally \( A=1 \) (i.e., \( g=G_{TK} \), and no excess noise from the active part)
Phase Noise

- total voltage noise spectral density

\[ v_{TOT}^2 = v_{GTK}^2 + v_{g}^2 \]

- resulting phase noise

\[ L(\Delta\omega) = \frac{2FkT}{P_S} \left( \frac{\omega_0}{2Q \Delta\omega} \right)^2 \]

\[ L(\Delta\omega) = \frac{1}{2} \frac{v_{TOT}^2}{v_s^2 / 2} = KT \frac{1}{v_s^2 G_{TK}} F \left( \frac{\omega_0}{Q \Delta\omega} \right)^2 \]

- oscillator noise factor \( F=1+A \)
Phase Noise Properties

• Leeson’s phase noise model

\[ L(\Delta \omega) = \frac{2FkT}{P_S} \cdot \left( \frac{\omega_0}{2Q \Delta \omega} \right)^2 \]

• inversely proportional to tank quality factor (square)
• inversely proportional to signal power
• -20dB/decade slope at mid frequencies (~MHz)
• directly proportional to oscillation frequency (square)
• phase-noise power consumption figure of merit

\[ FOM = L(\Delta \omega) \left( \frac{\Delta \omega}{\omega_0} \right)^2 V_C I_C \]
Phase Noise Plot

- 1/f noise
- Thermal, shot noise
- Noise floor due to active elements or instrumentation

Leeson’s modification to capture 1/f and flat noise part

\[
L(\Delta \omega) = \frac{2FKT}{P_s} \left[ 1 + \left( \frac{\omega_0}{2Q \Delta \omega} \right)^2 \right] \left( 1 + \frac{\omega_{1/f}}{\Delta \omega} \right)
\]
Circuit-Specific Linear Phase-Noise Model - Outline

- Spectral Noise Analysis
- Circuit Noise Analysis
Spectral Noise Analysis - Outline

- Phasor phase-noise model
- Oscillation condition
- LC-tank, $g_m$-cell, tail-current source noise
- (Phase-) Noise factor
VCO Noise Sources

LC-tank noise

transconductor noise

tail-current source noise

• $g_m$-cell transistors $Q_1$ and $Q_2$ always active

• current source noise always at the common mode
VCO Noise Sources

- LC-tank noise
  \[ i_N^2 (G_{TK}) = 2KT G_{TK} \]

- base-resistance thermal noise
  \[ v_N^2 (r_B) = 2KT r_B \]

- collector-current shot noise
  \[ i_N^2 (I_C) = qI_C = 2KT g_m / 2 \]

- base-current shot noise
  \[ i_N^2 (I_B) = qI_B = 2KT g_m / 2 \beta \]

- tail-current source output noise
  \[ i_N^2 (I_{TCS}) = 2KT \frac{g_{m,CS}}{2} \left[ 1 + 2r_{B,CS} g_{m,CS} + (r_{B,CS} g_{m,CS})^2 \left( \frac{1}{\beta_F} + \left( \frac{\omega}{\omega_T} \right)^2 \right) \right] \]
Collector-Current Shot Noise

- splitting of current noise sources
Collector-Current Shot Noise

- phase-related noise power:

\[ v_{PM}^2 (I_C) = \frac{1}{2} \frac{2i_n^2 (I_C) R_{TK}^2}{4} \]

- noise factor:

\[ F(2I_C) = \frac{2i_{PM}^2 (2I_C)}{4K T G_{TK}} = \frac{2K T g_m / 2}{4K T G_{TK}} \approx \frac{g_m}{4g_{mSUP}/2} = \frac{1}{2} \]
Base-Current Shot Noise

• splitting of current noise sources
• phase-related noise power:

\[
\begin{align*}
\nu_{PM}^2(I_B) &= \frac{1}{2} \frac{2i_n^2(I_B)R_{TK}^2}{4} \\
i_{PM}^2(2I_B) &= \frac{1}{2} \frac{i_n^2(I_B)R_{TK}^2}{2} = \frac{i_n^2(I_B)}{2}
\end{align*}
\]

• noise factor:

\[
F(2I_B) = \frac{2i_{PM}^2(2I_B)}{4KTG_{TK}} = \frac{2KTg_m/2\beta}{4KTG_{TK}} \approx \frac{g_m/\beta}{4g_{m-SUP}/2} = \frac{1}{2\beta}
\]

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Base-Resistance Thermal Noise

- voltage-to-current transformation
Base-Resistance Thermal Noise

- phase-related noise power:
\[ i_{PM}^2 (2r_B) = \frac{v_N^2 (r_B) g_m^2}{2} \]

- noise factor:
\[ F(2r_B) = \frac{2i_{PM}^2 (2r_B)}{4KG_{TK}} = \frac{g_m^2 2KTr_B}{4KG_{TK}} = \frac{g_m^2 r_B}{2G_{TK}} = c \]
Linear-Oscillator Phase Noise

- Linear-oscillator noise factor:

\[ F = F(R_{TK}) + F(2I_C) + F(2I_B) + F(2r_B) = 1 + \frac{1}{2} + \frac{1}{2\beta} + c \approx \frac{3}{2} + c \]

- Linear-oscillator phase noise:

\[ L(f + \Delta) = \frac{L(R_{TK}) + L(2I_C) + L(2I_B) + L(2r_B)}{(4\pi C_{TOT}\Delta)^2} = \frac{4KTG_{TK}F}{v_s^2(4\pi C_{TOT}\Delta)^2} \]

\[ L = \frac{4KTG_{TK}}{(4\pi C_{TOT}\Delta)^2(2V_T)^2} \left( \frac{3}{2} + c \right) \]
Linear-Oscillator Phase Noise

• LC-tank noise contribution ~ 1

• $g_m$-cell current shot noise contribution ~ $\frac{1}{2}$

• $g_m$-cell base-resistance noise contribution ~ $c$

• linear-oscillator noise factor ~ constant (independent of bias condition)

• But, linear “oscillator” (loop gain of 1) doesn’t oscillate, or at least doesn’t oscillate with a predictable amplitude!

• Oscillator with a loop gain of $\frac{2\pi}{(\pi+2)}=1.22$ oscillates and has ~3dB better phase noise than linear “oscillator”!