Linear Phase-Noise Model

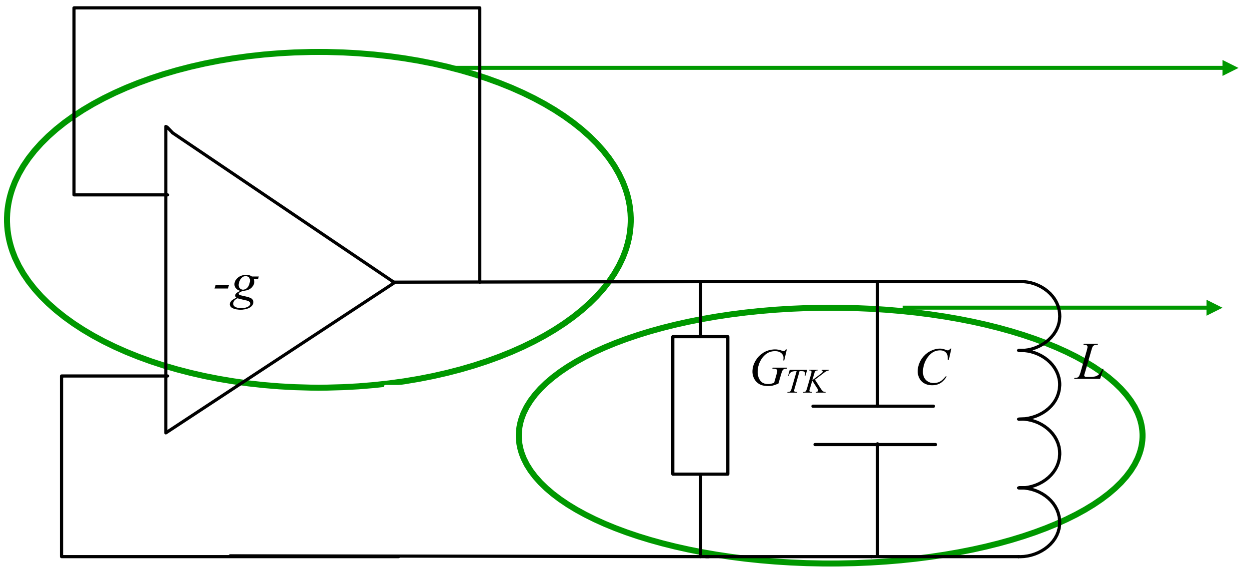
Sub-Outline

- Generic Linear Phase-Noise Model
- Circuit-Specific Linear Phase-Noise Model

Generic Linear Phase-Noise Model - Outline

- Linear Oscillator Model
    - LC-Tank noise
    - active part noise
- (Phase) Noise Factor
- Phase-Noise Properties

Linear Oscillator Model



- transconductor noise
- LC-tank noise
- no tail-current source noise

- LC-tank impedance (noise shaping)

$$Z(\omega_0+\Delta\omega)\cong\frac{-j\omega_0 L}{2\Delta\omega/\omega_0}=\frac{-j}{2\omega_0 C}\frac{\omega_0}{\Delta\omega}$$

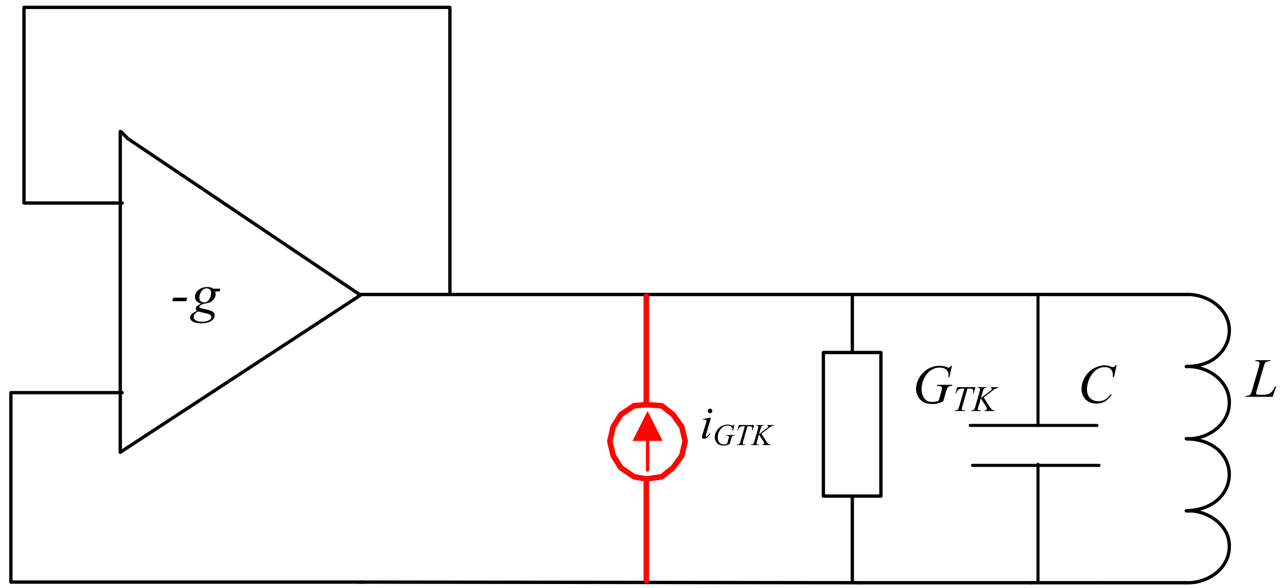
$$\left|Z(\omega_0+\Delta\omega)\right|^2\cong\frac{1}{4G_{TK}^2Q^2}\left(\frac{\omega_0}{\Delta\omega}\right)^2=\frac{R_{TK}^2}{4Q^2}\left(\frac{\omega_0}{\Delta\omega}\right)^2$$

- LC-tank quality factor

$$Q=\frac{1}{\omega_0 L G_{TK}}$$

$$Q=\frac{\omega_0 C}{G_{TK}}$$

LC-Tank Noise



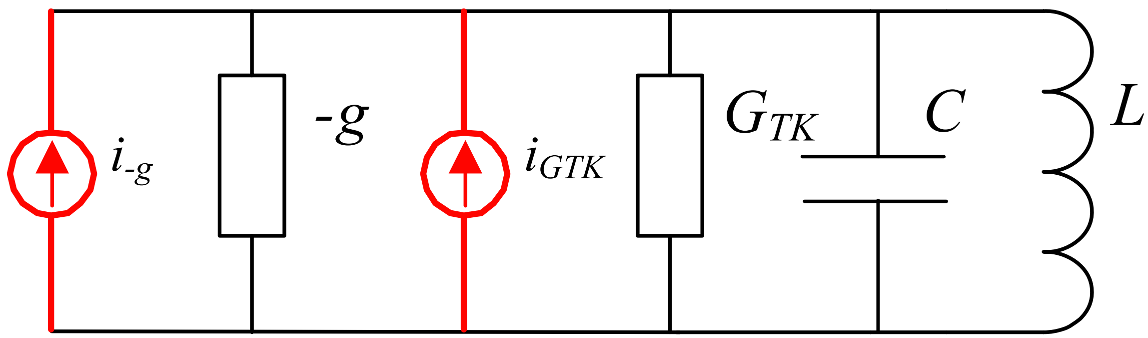
- tank resistance noise (R_{TK}=1/G_{TK})

$$i_{GTK}^2 = 4KTG_{TK}$$

- tank contribution to the equivalent voltage noise spectral density

$$v_{GTK}^2 = i_{GTK}^2 \left|Z(\Delta\omega)\right|^2 = KT \frac{G_{TK}}{(\omega_0 C)^2} \left(\frac{\omega_0}{\Delta\omega}\right)^2$$

Active Part Noise



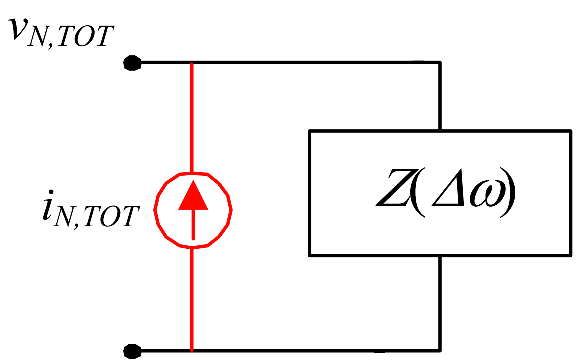
- active part contribution to the equivalent voltage noise spectral density

$$v_{-g}^2 = KT \frac{G_{TK}}{(\omega_0 C)^2} A \left(\frac{\omega_0}{\Delta\omega} \right)^2$$

- active part noise factor *A*
  - excess negative conductance
  - additional noise of the active devices
  - ideally A=1 (i.e., $g=G_{TK}$, and no excess noise from the active part)

Phase Noise

- total voltage noise spectral density



$$v_{TOT}^2 = v_{GTK}^2 + v_{-g}^2$$

$$v_{TOT}^2 = KT \frac{G_{TK}}{(\omega_0 C)^2} F \left(\frac{\omega_0}{\Delta\omega} \right)^2$$

- oscillator noise factor *F*=1+*A*

- resulting phase noise

$$\mathcal{L}(\Delta\omega) = \frac{1}{2} \frac{v_{TOT}^2}{v_S^2 / 2} = KT \frac{1}{v_S^2 G_{TK}} F \left(\frac{\omega_0}{Q \Delta\omega} \right)^2$$

$$\boxed{\mathcal{L}(\Delta\omega) = \frac{2FkT}{P_S} \cdot \left(\frac{\omega_0}{2Q\ \Delta\omega} \right)^2}$$

Phase Noise Properties

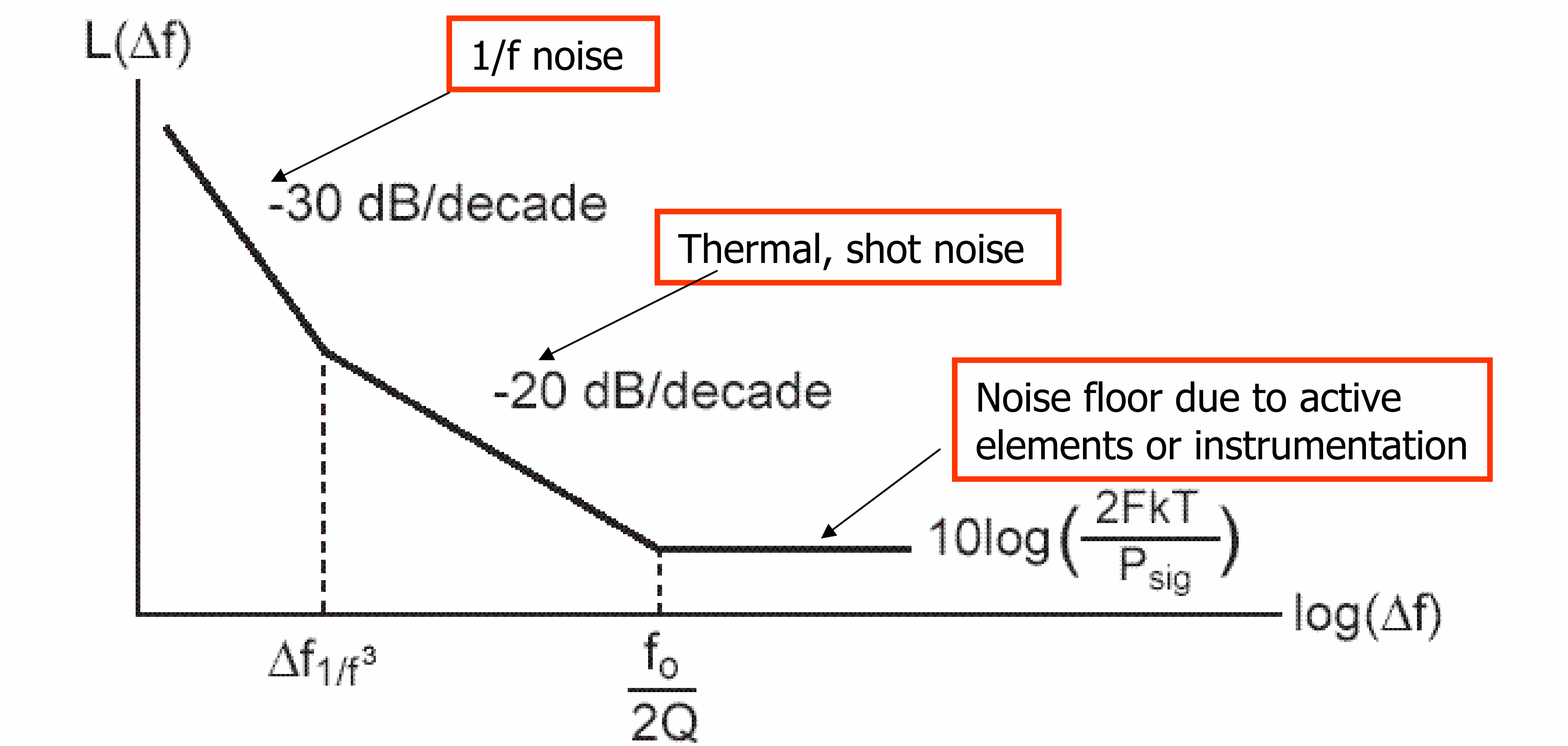
- Leeson's phase noise model

$$\mathcal{L}(\Delta\omega) = \frac{2FkT}{P_S} \cdot \left(\frac{\omega_0}{2Q\ \Delta\omega} \right)^2$$

- inversely proportional to tank quality factor (square)
- inversely proportional to signal power
- -20dB/decade slope at mid frequencies (~MHz)
- directly proportional to oscillation frequency (square)
- phase-noise power consumption figure of merit

$$FOM = \mathcal{L}(\Delta\omega) \left(\frac{\Delta\omega}{\omega_0} \right)^2 V_C I_C$$

Phase Noise Plot



- Leeson's modification to capture 1/*f* and flat noise part

$$\mathcal{L}(\Delta\omega) = \frac{2FkT}{P_S} \cdot \left[1 + \left(\frac{\omega_0}{2Q\,\Delta\omega}\right)^2\right]\left(1 + \frac{\omega_{1/f}}{\Delta\omega}\right)$$

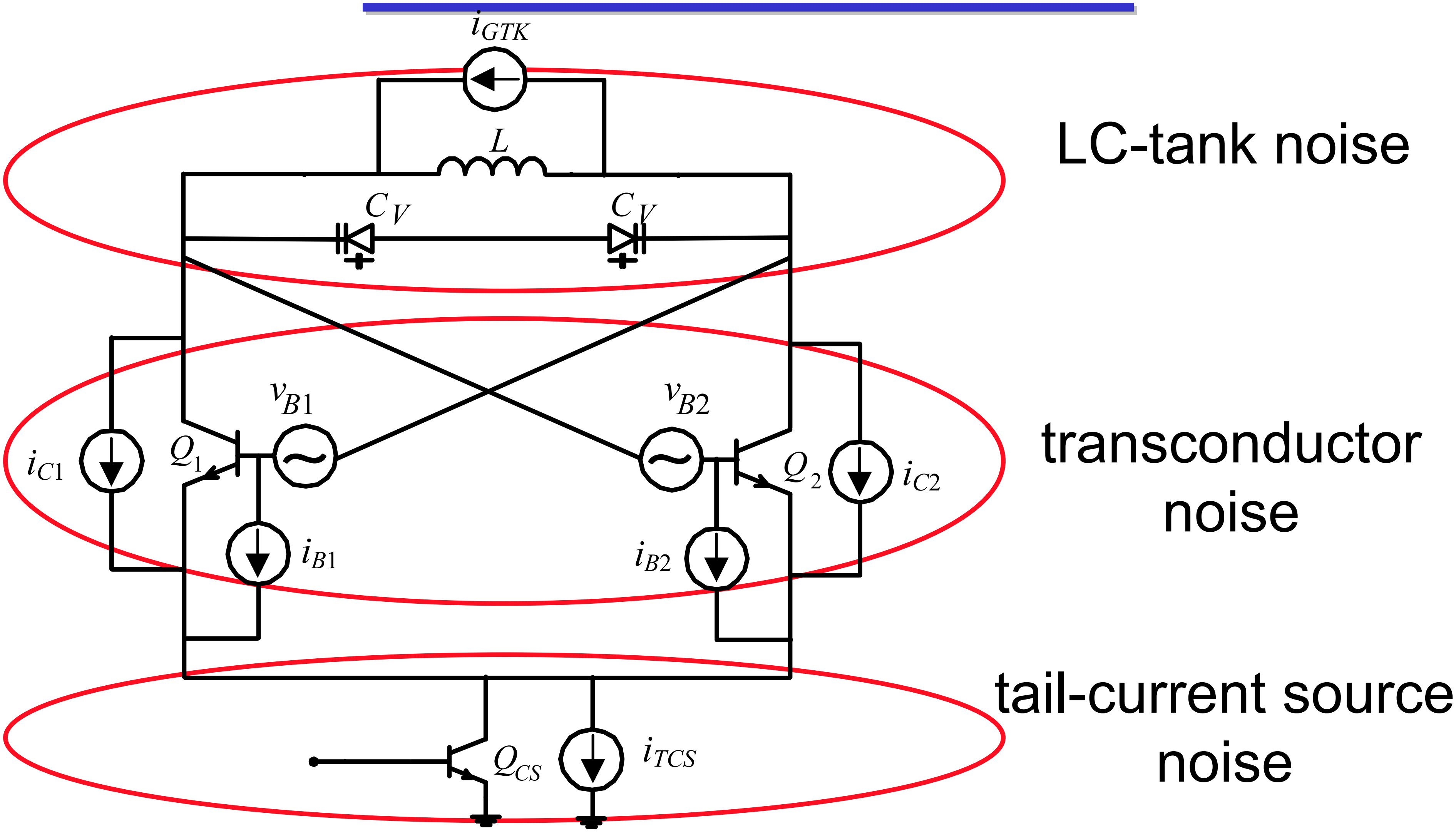
Circuit-Specific Linear Phase-Noise Model - Outline

- Spectral Noise Analysis
- Circuit Noise Analysis

Spectral Noise Analysis - Outline

- Phasor phase-noise model
- Oscillation condition
- LC-tank, g_m-cell, tail-current source noise
- (Phase-) Noise factor

VCO Noise Sources



- g_m-cell transistors Q_1 and Q_2 always active
- current source noise always at the common mode

VCO Noise Sources

- LC-tank noise

$$i_N^2(G_{TK}) = 2KTG_{TK}$$

- base-resistance thermal noise

$$v_N^2(r_B) = 2KTr_B$$

- collector-current shot noise

$$i_N^2(I_C) = qI_C = 2KTg_m/2$$

- base-current shot noise

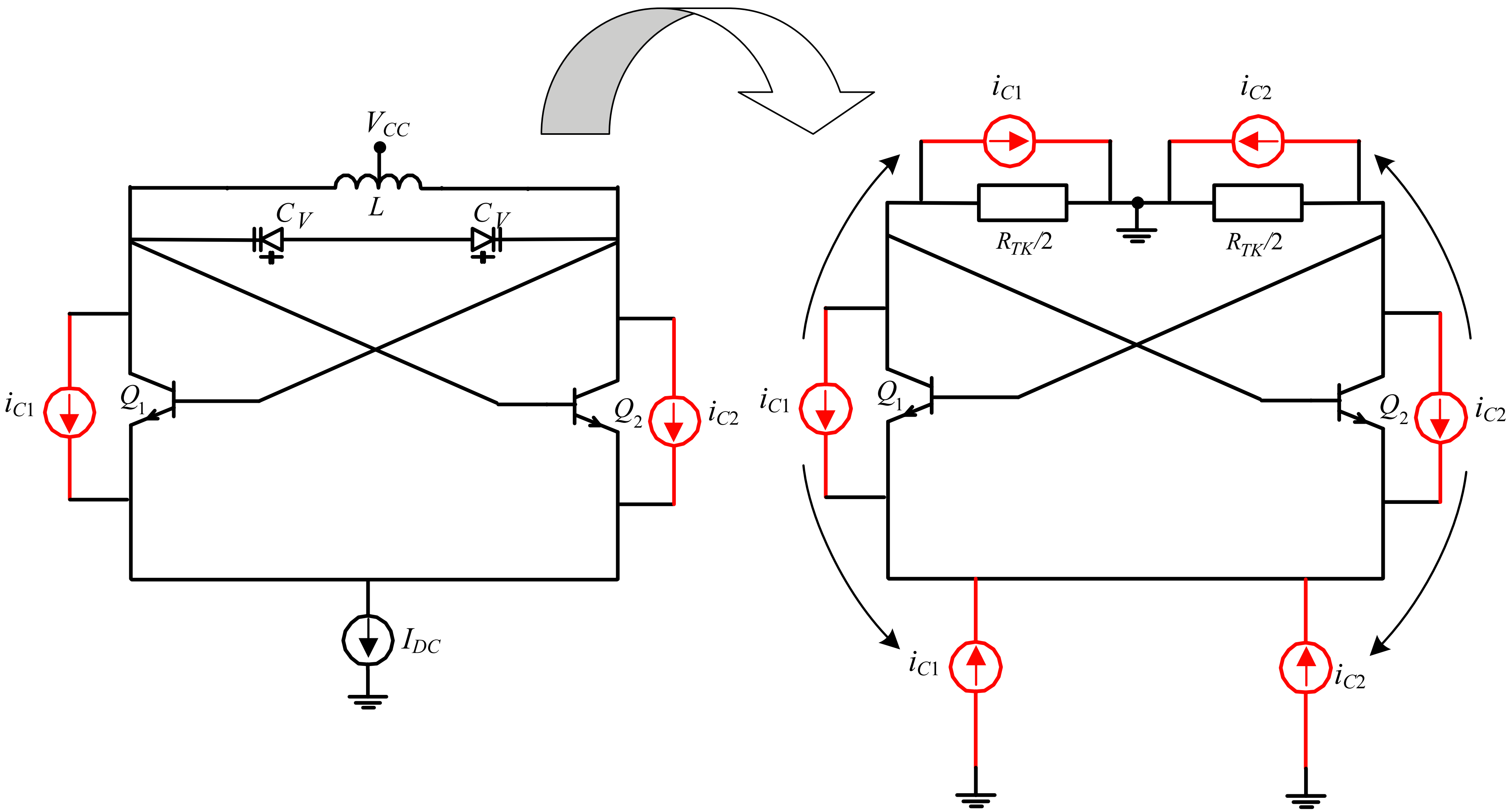
$$i_N^2(I_B) = qI_B = 2KTg_m/2\beta$$

- tail-current source output noise

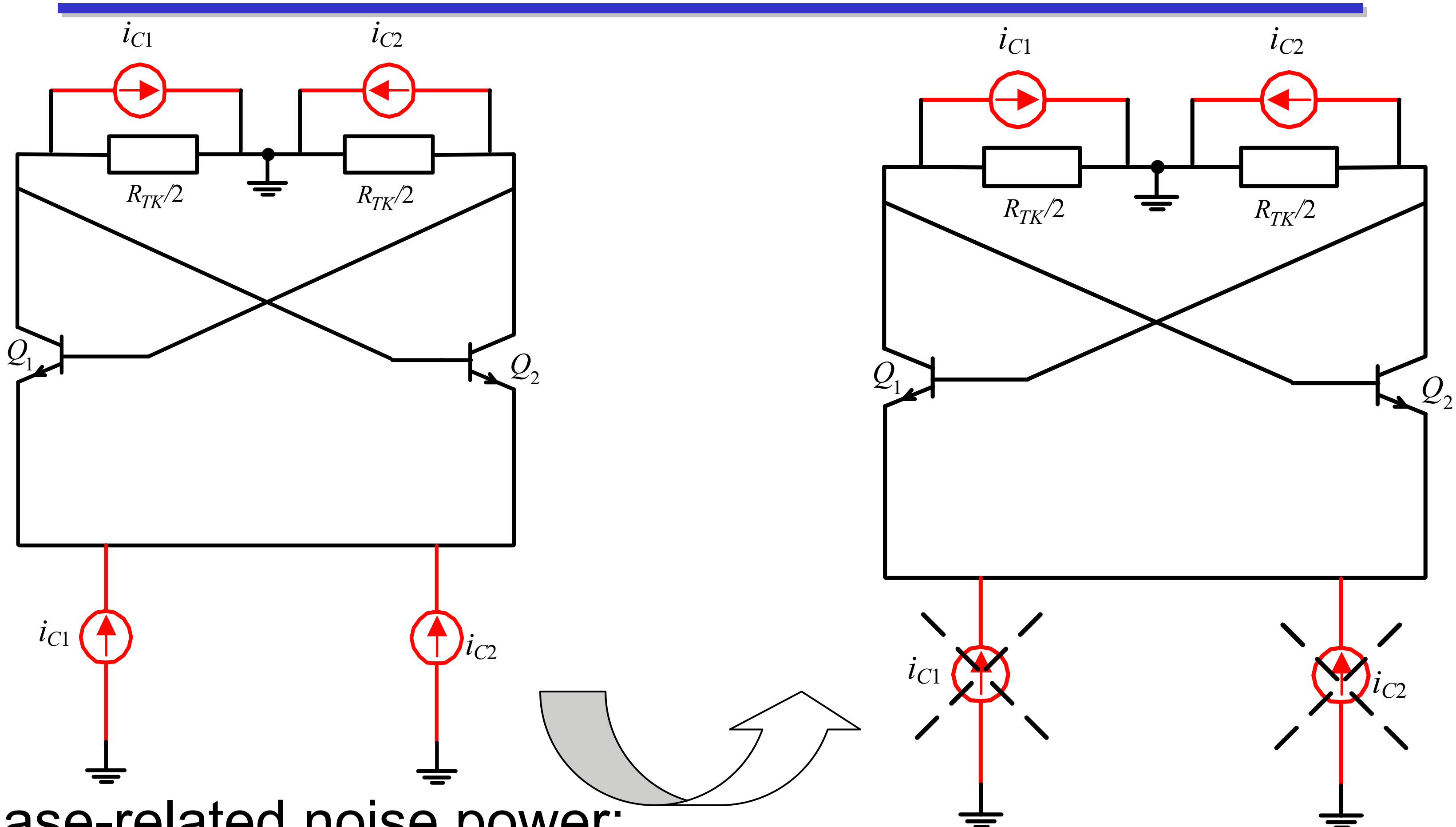
$$i_N^2(I_{TCS}) = 2KT\frac{g_{m,CS}}{2}\left[1 + 2r_{B,CS}g_{m,CS} + (r_{B,CS}g_{m,CS})^2\left(\frac{1}{\beta_F} + (\frac{\omega}{\omega_T})^2\right)\right]$$

Collector-Current Shot Noise

• splitting of current noise sources



Collector-Current Shot Noise



- phase-related noise power:

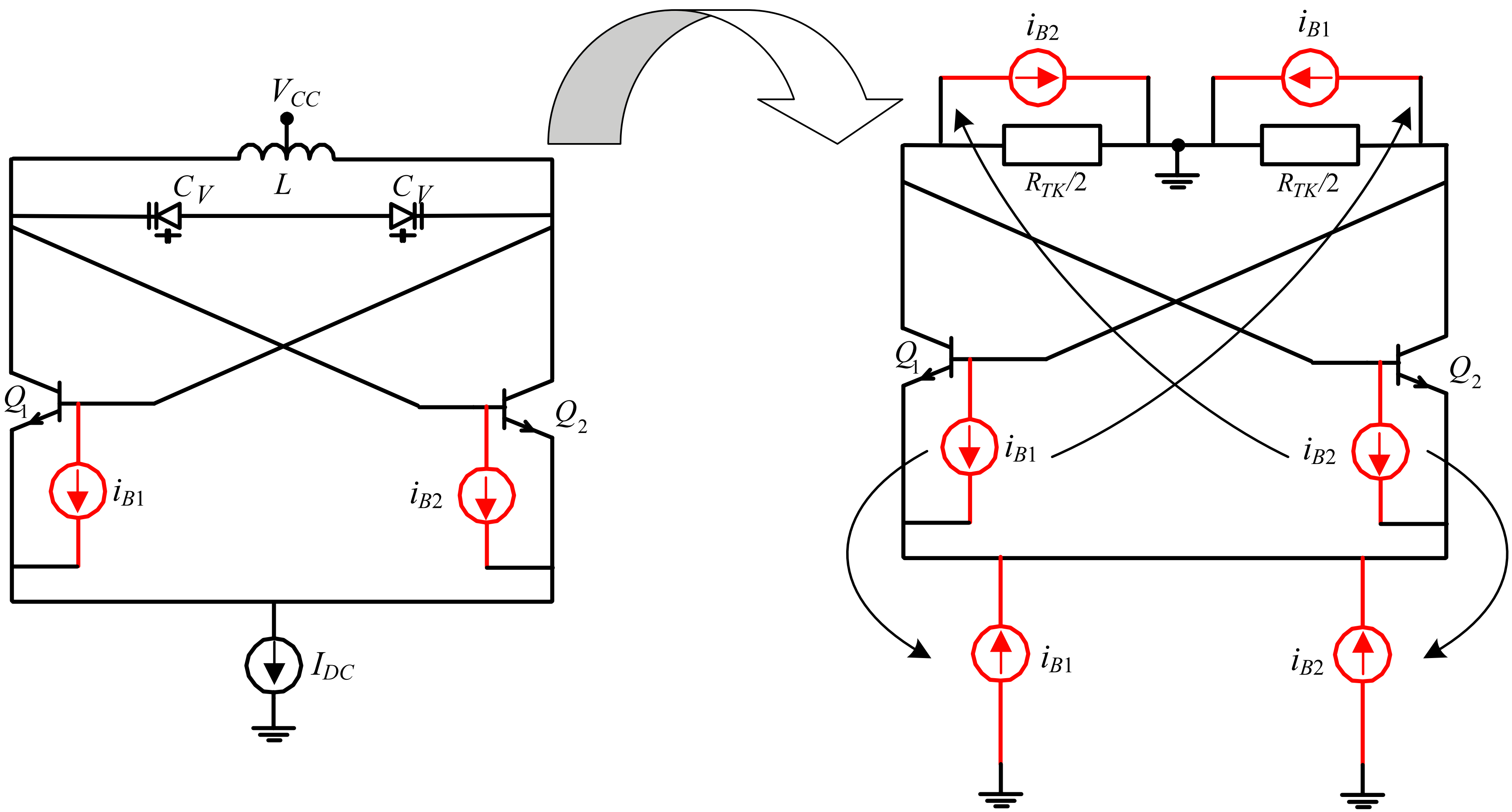
$$v_{PM}^2(I_C) = \frac{1}{2}\frac{2i_N^2(I_C)R_{TK}^2}{4} \qquad i_{PM}^2(2I_C) = \frac{1}{R_{TK}^2}\frac{i_N^2(I_C)R_{TK}^2}{2} = \frac{i_N^2(I_C)}{2}$$

- noise factor:

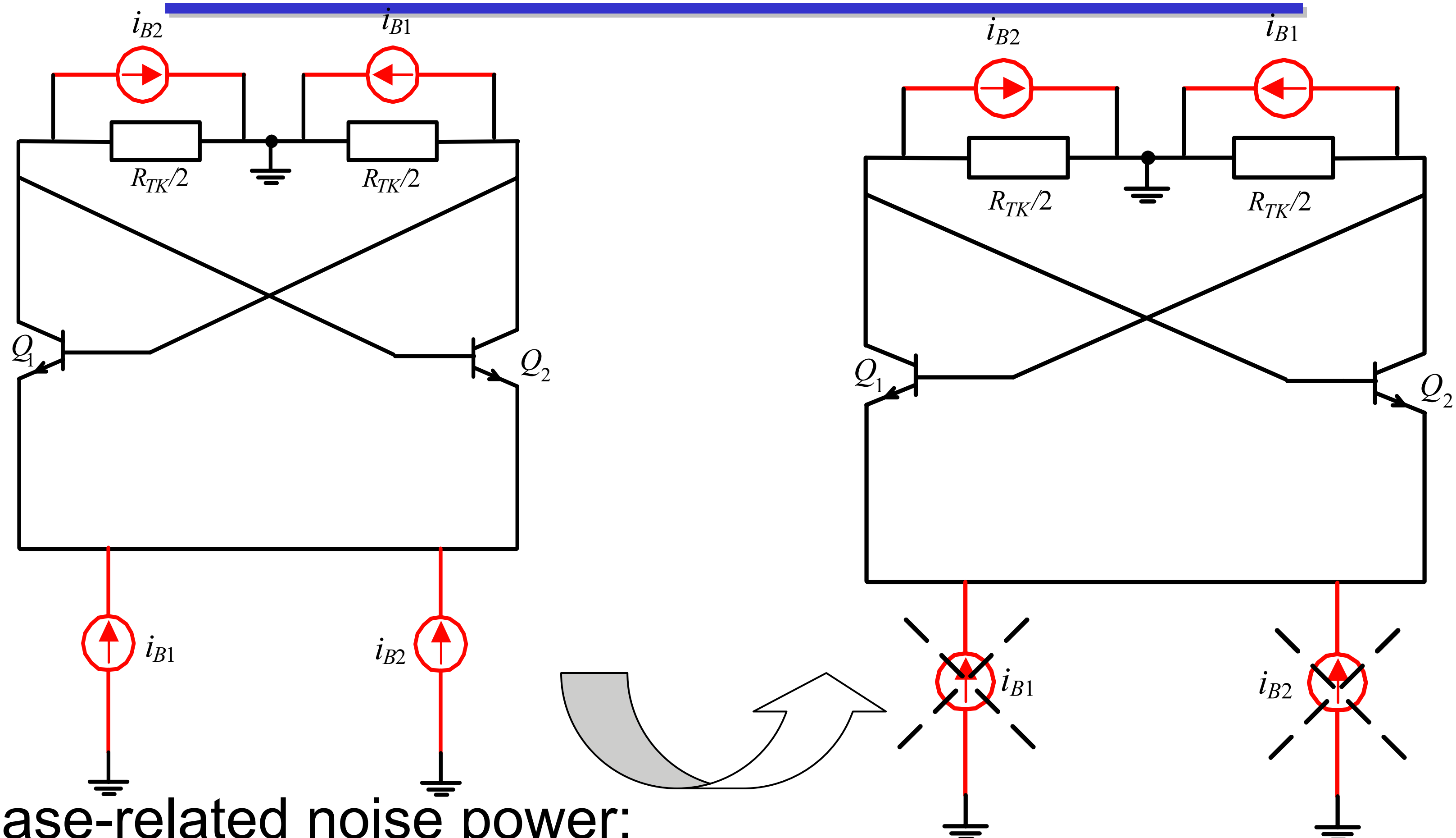
$$F(2I_C) = \frac{2i_{PM}^2(2I_C)}{4KTG_{TK}} = \frac{2KTg_m/2}{4KTG_{TK}} \cong \frac{g_m}{4g_{m-SUP}/2} = \frac{1}{2}$$

Base-Current Shot Noise

- splitting of current noise sources



Base-Current Shot Noise



- phase-related noise power:

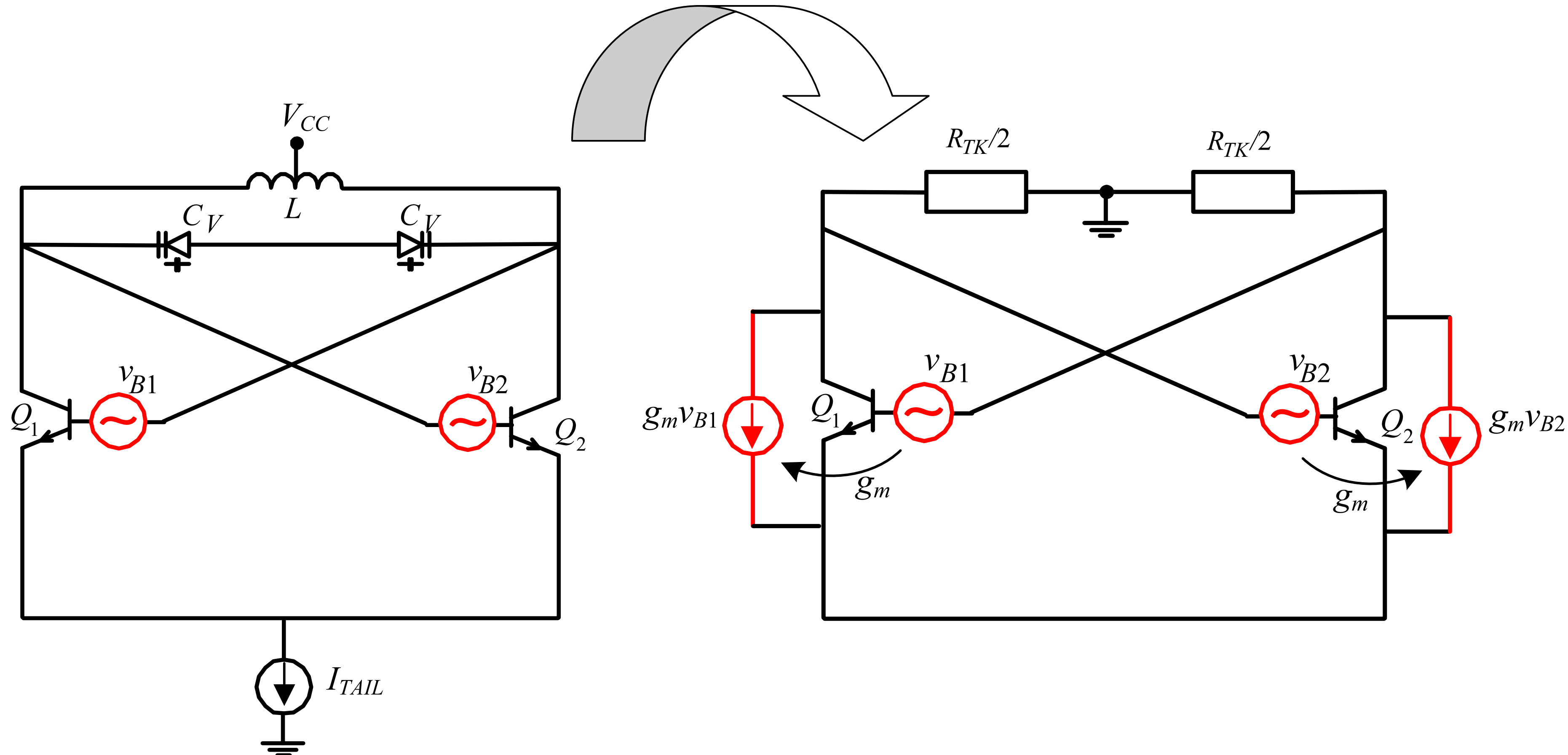
$$v^2_{PM}(I_B) = \frac{1}{2}\frac{2i^2_N(I_B)R^2_{TK}}{4} \qquad i^2_{PM}(2I_B) = \frac{1}{R^2_{TK}}\frac{i^2_N(I_B)R^2_{TK}}{2} = \frac{i^2_N(I_B)}{2}$$

- noise factor:

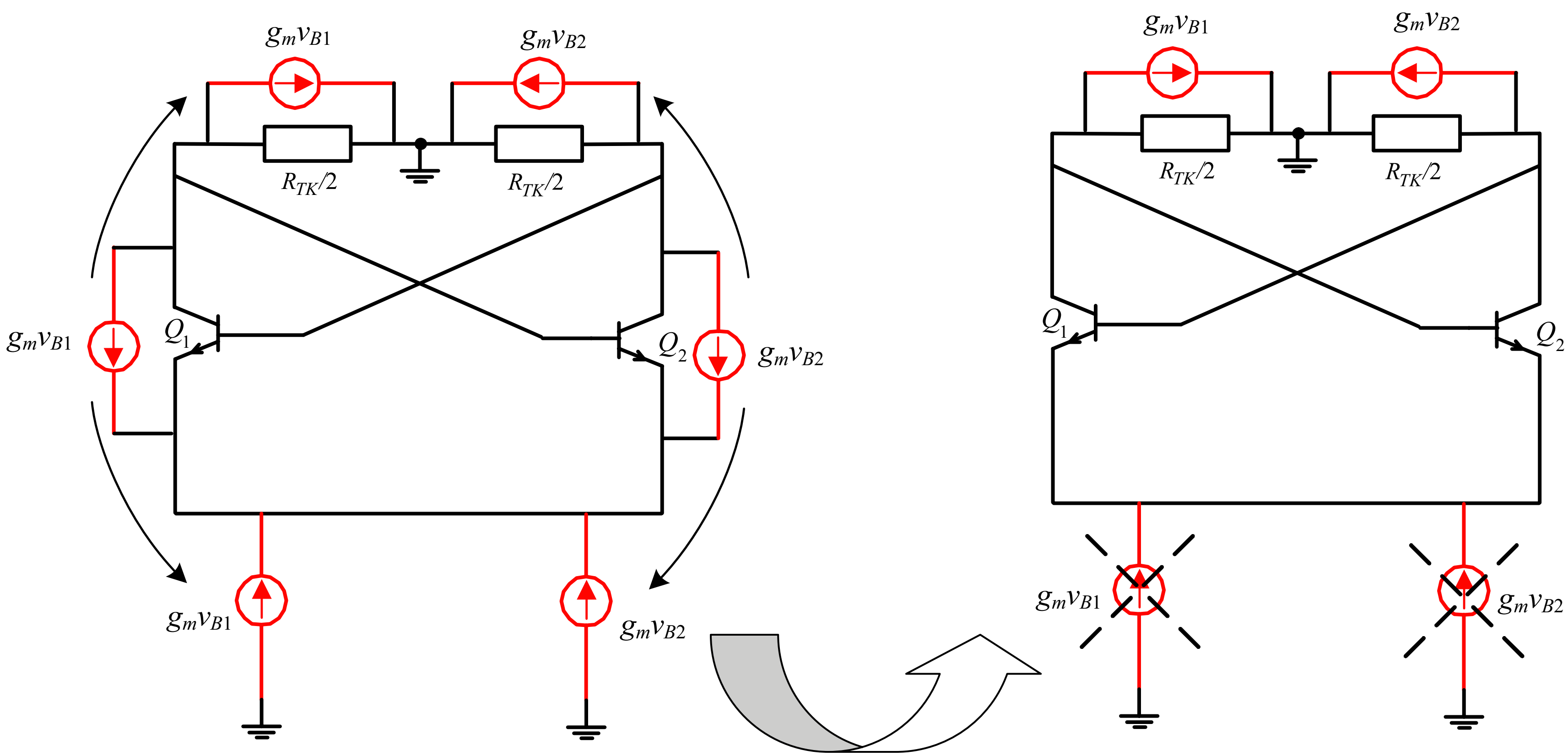
$$F(2I_B) = \frac{2i^2_{PM}(2I_B)}{4KTG_{TK}} = \frac{2KTg_m/2\beta}{4KTG_{TK}} \cong \frac{g_m/\beta}{4g_{m-SUP}/2} = \frac{1}{2\beta}$$

Base-Resistance Thermal Noise

- voltage-to-current transformation



Base-Resistance Thermal Noise



- phase-related noise power:

$$i^2_{PM}(2r_B) = \frac{v^2_N(r_B)g_m^2}{2}$$

- noise factor:

$$F(2r_B) = \frac{2i^2_{PM}(2r_B)}{4KTG_{TK}} = \frac{g_m^2 2KTr_B}{4KTG_{TK}} = \frac{g_m^2 r_B}{2G_{TK}} = c$$

Linear-Oscillator Phase Noise

- Linear-oscillator noise factor:

$$F = F(R_{TK}) + F(2I_C) + F(2I_B) + F(2r_B) = 1 + \frac{1}{2} + \frac{1}{2\beta} + c \cong \frac{3}{2} + c$$

- Linear-oscillator phase noise:

$$\mathcal{L}(f + \Delta) = \frac{\mathcal{L}(R_{TK}) + \mathcal{L}(2I_C) + \mathcal{L}(2I_B) + \mathcal{L}(2r_B)}{(4\pi C_{TOT}\Delta)^2} = \frac{4KTG_{TK}F}{v_s^2(4\pi C_{TOT}\Delta)^2}$$

$$\mathcal{L} = \frac{4KTG_{TK}}{(4\pi C_{TOT}\Delta)^2(2V_T)^2}\left(\frac{3}{2} + c\right)$$

Linear-Oscillator Phase Noise

- LC-tank noise contribution ~ 1
- g_m-cell current shot noise contribution ~ ½
- g_m-cell base-resistance noise contribution ~ c
- linear-oscillator noise factor ~ constant (independent of bias condition)

- But, linear "oscillator" (loop gain of 1) doesn't oscillate, or at least doesn't oscillate with a predictable amplitude!
- Oscillator with a loop gain of $2\pi/(\pi+2)=1.22$ oscillates and has ~3dB better phase noise than linear "oscillator"!