ME1633: Integration of Physics, Motion and Metrology

Assignment 4: Magnetism solutions

22 November 2013

- You can do this assignment on your own or in groups, as long as you hand in your own solutions and it becomes clear that you understand your solutions. Formulate your solutions step-by-step, carefully pointing out the logical structure of your answer, but keep your answers brief.

- *Only for the students who follow the PME track's course ME1633:* Your solution to this assignment must be submitted in the lecture or via Blackboard in pdf format *before* 4 December 2013, 23:59h.

- If in some question you happen to need an answer from a previous question that you don't know the answer to, assume an answer or at least explain the method you would use when you would have had that previous answer.

- Your solution must be formulated in English.

- Contact: (Problem 1-2) Murali K Ghatkesar (m.k.ghatkesar@tudelft.nl); Office hours (G-1-400): Tuesdays and Fridays between 12:30h and 14:00h or (Problem 3-5): Oscar van de Ven (F-2-330; o.s.vandeven@tudelft.nl)

Note about the answers: Some of the answers given in this document give some extra explanation or material. Therefore they are not necessarily representative for the solution we expect from you.
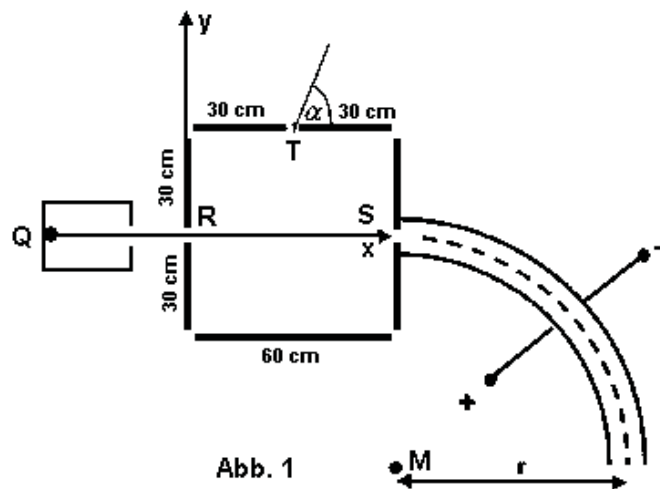

Problems 1 and 2

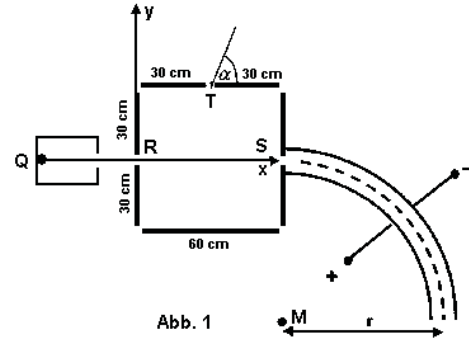For the solutions to problems 1 and 2: see the following slides.

Magnetism Assignment

Questions 1 and 2

Murali Krishna Ghatkesar

Abb. 1

1a. Calculate the electric potential required to accelerate the electrons from rest to v_0 (from Q to R in Figure 1).

$$E_{kin} = \frac{1}{2}mv_o^2 = qV$$

$$V = \frac{mv_o^2}{2q} = \frac{9.1 * 10^{-31} * (4 * 10^7)^2}{2 * 1.6 * 10^{-19}} = 4550V$$

Abb. 1

1b. In the following experiment, there are simultaneously both non-vanishing magnetic and electrical fields present inside the cube (E $\neq$ 0; B $\neq$ 0). The electrons enter the box again at R with v_0 and leave at S with the same velocity. There are two field configurations possible to achieve this. Explain one solution how the fields have to be oriented in order to accomplish this. Which criterion do you select to define the respective strengths?
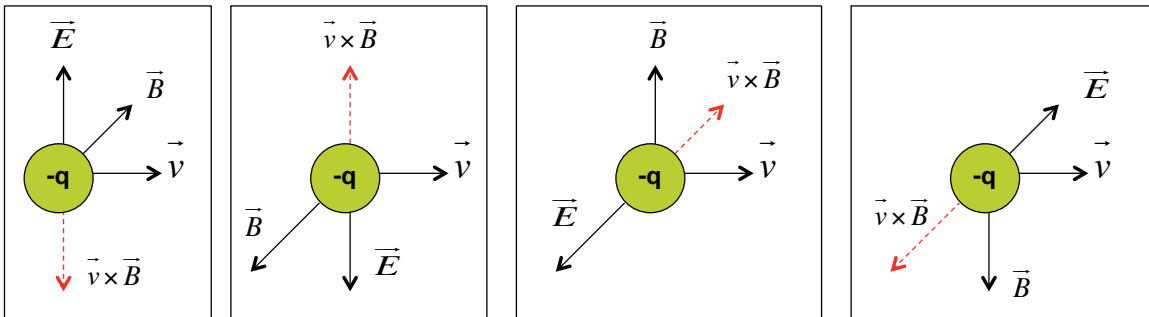
Criteria: Forces have to cancel each other both in magnitude and direction

1c. An electrical potential of 1.5kV is applied to the condenser formed by two sidewalls of the cube. Calculate the electrical and magnetic field strength required to meet the conditions outlined in **1b**.

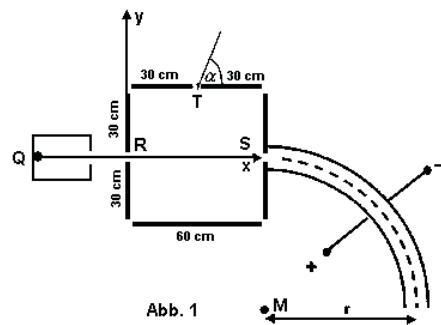$$E = \frac{U}{d} = \frac{1500}{0.6} = 2.5 kV/m$$
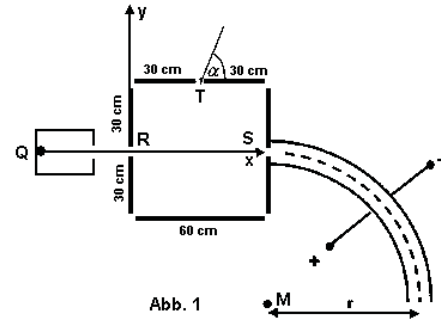
$$eE = evB$$

$$B = \frac{E}{v} = \frac{2500}{4 \times 10^7} = 62.5 \mu T$$

1 d   Only static magnetic field is present in the next experiment configuration. The electron beam must again be rectilinear between R and S. For which B-field is this possible? What is the influence on the speed of the electrons? Motivate your answer.
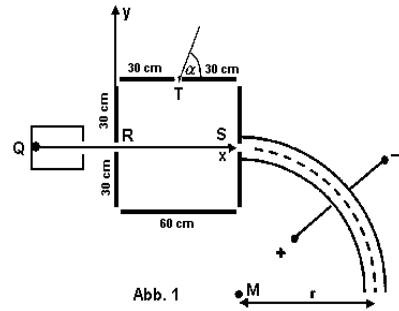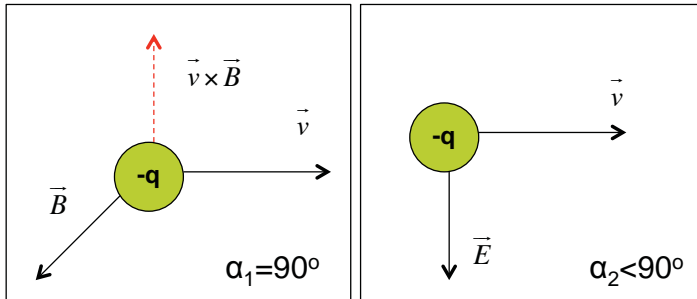
$$\vec{F} = q\vec{v} \times \vec{B} = qvB\sin\theta$$

B field must be homogeneous and parallel to velocity.

No change observed because Lorentz Force on the electrons is zero.

1e. In the final experiments there is either a magnetic or an electric field inside the cube. The electrons shall leave the cube at T. There are two different situations observed. The electrons either leave under i) $\alpha_1 = 90°$ or ii) $\alpha_2 < 90°$. Explain how these two cases can be produced. Calculate the magnitude of the field strengths for the case where there is only an electrical field. iii) What is the speed of the electron in the two cases?

Abb. 1

II)

$$x_R = 0; x_T = 30cm$$

$$t = (x_T - x_R)/v = 7.5 \times 10^{-9} s$$

Abb. 1

$$y_T = \frac{1}{2}at^2$$

$$F = eE = ma$$

$$E = \frac{ma}{e} = \frac{2my_T}{et^2} = 60.7kV/m$$

1e. In the final experiments there is either a magnetic or an electric field inside the cube. The electrons shall leave the cube at T. There are two different situations observed. The electrons either leave under i) $\alpha_1 = 90°$ or ii) $\alpha_2 < 90°$. Explain how these two cases can be produced. Calculate the magnitude of the field strengths for the case where there is only an electrical field. iii) What is the speed of the electron in the two cases?
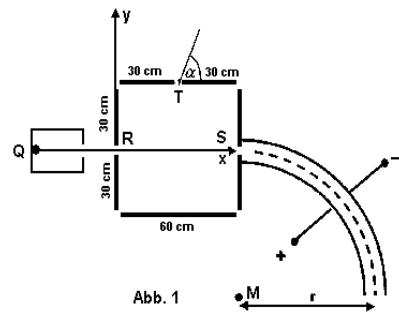
III) With only magnetic field there is no change in speed

Abb. 1

With only Electric field

$$v_x = 4 \times 10^7 m/s$$

$$v_y = at = \frac{eE}{m} \frac{(x_T - x_R)}{v_x} = 26.7 \times 10^7 m/s$$

$$v = \sqrt{v_x^2 + v_y^2} = 27 \times 10^7 m/s$$
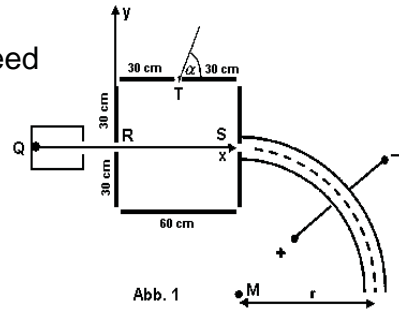
1f. The electrons leave the cube in S again having a velocity v_0. They then enter a circularly curved condenser (see figure 1). The electrical field strength is such that the electrons follow the dashed line. The field strength on the trace is 6.0 kV/m. Calculate the radius of curvature.

$$F_{electric} = F_{centripetal}$$

$$eE = \frac{mv^2}{R}$$

$$R = \frac{mv^2}{eE} = 1.52m$$

Abb. 1

2a. In the class, you have seen that magnet dropped through a metal tube took more time to reach the other side compared to the magnet dropped through a plastic tube. Explain why?
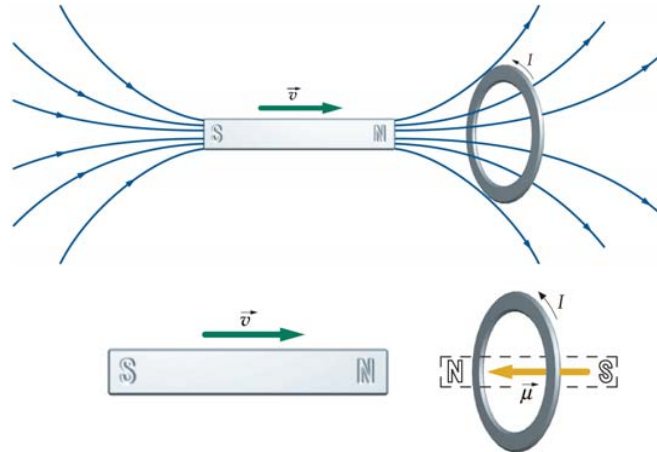
Faraday's Law: $\varepsilon = -\dfrac{d\phi_m}{dt}$

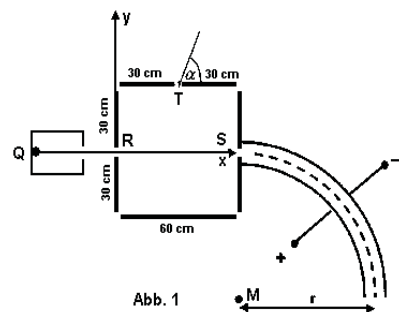Lenz's Law: The induced *emf* is in such a direction as to oppose, the change that produces it.

2b. If a metal wire loop was wound on the plastic tube. Make a graph of the magnetic flux through the loop and current induced in the loop as a function of time. Explain your graph. Indicate different time points on your plot and any assumptions made.
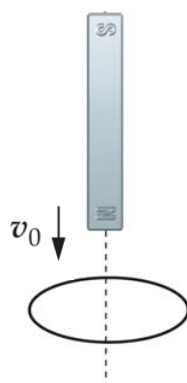
Assume: From top, positive direction for the current is clockwise
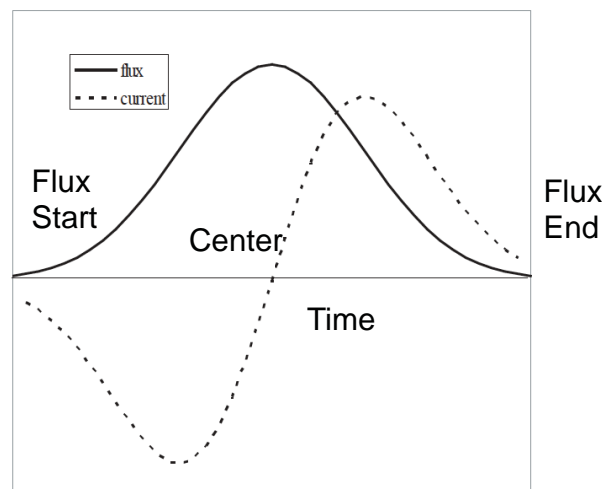
2c. Propose a method to determine acceleration due to gravity using a plastic tube, metal wire and the data of induced current in a loop.

Magnet

d

Coils

$$y = \frac{1}{2}gt^2$$

t^2

(Zero Crossing Time at Each loop)

Slope $= \dfrac{2}{g}$

y (Distance of each loop from top)

— flux
···· current

Flux Start

Center

Flux End

Time

2d. Magnetic field strengths can be measured by a rotating coil gaussmeter. This device uses a small multi-turn coil rotating at a high speed on an axis perpendicular to the magnetic field. This coil is connected to an ac voltmeter by means of slip rings, like those shown in the **Figure 2**. In one specific design, the rotating coil has 400 turns and an area of 1.40 cm2. The coil rotates at 180 rev/min. If the magnetic field strength is 0.450 T, find the maximum induced emf in the coil and the orientation of the normal to the plane of the coil relative to the field for which this maximum induced emf occurs.
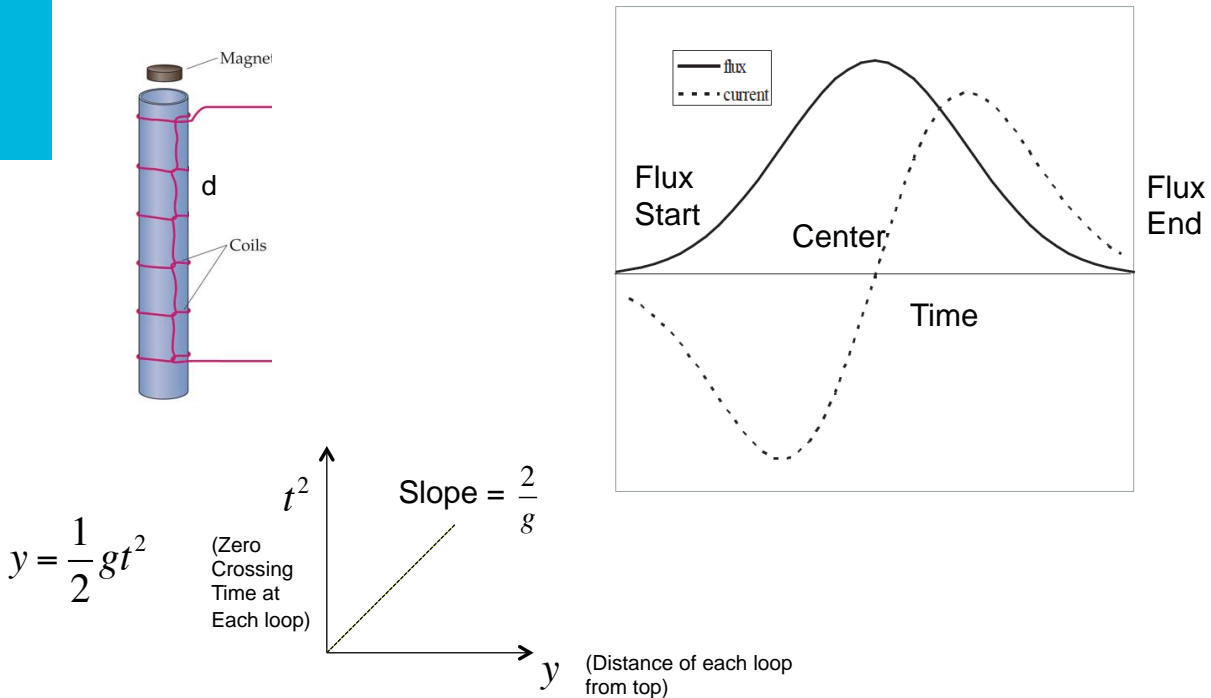
b

N turns

ω

$\vec{B}$

a

(a)

$\theta = \omega t$

(b)
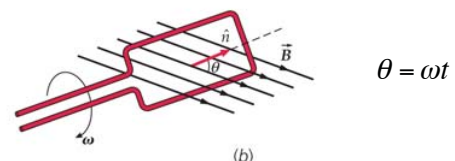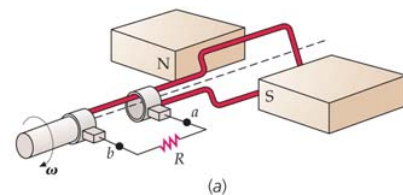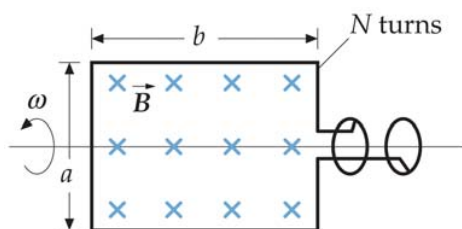
$$\varepsilon = -\frac{d\phi_m}{dt}$$

$$\phi_m(t) = NBA\cos\omega t$$

$$\varepsilon = -\frac{d}{dt}NBA\cos\omega t = NBA\omega\sin\omega t$$

$$\varepsilon_{max} = NBA\omega = 0.475V$$

Introduction to the HDD actuator

The hard disk drive (HDD), introduced by IBM in 1956, is still a cost-effective way of storing digital data. Therefore this mechanical way of storing data remains popular. An HDD consists of one or more rigid ('hard') rapidly rotating disks (platters) with magnetic heads arranged on a moving actuator arm to read and write data to the surfaces.[1] In this assignment we will study the magnetic actuator that rotates the arm around a pivot point. This arm is indicated in the left picture in Figure 1.

To analyze the magnetic actuator we will assume that the system can be represented by the 2-dimensional schematic shown in Figure 2, this can be seen as the cross-section indicated in Figure 1. The numeric values for the relevant properties are listed in Table 1.

Problem 3: Hard-disk-arm traction

(8 points)

(3p) **3a.** Which of the basic electromagnetic principles of actuation generates the rotational motion of the arm? Explain why this is the case in at most 4 sentences.

Answer

- This actuator is a Lorenz actuator, because we are dealing with a current carrying wire in a magnetic field.

- Motion in x-direction of the coil does not generate a flux change in the magnetic circuit. Therefore there is no reluctance force contribution.

We want to determine the motor constant of this actuator. For this we have to study the flux in the magnetic circuit, excluding the field due to the coil.

(5p) **3b.** Explain clearly why it is not necessary to include the field generated by the coil to determine the force on the coil in this specific situation.

Answer

- Concerning the Lorentz force: We are interested in the force *between the coil and the stator*. The force on the current in the coil due to the magnetic field generated by the current in the coil does not give a net force on the coil, but only an internal force that does no mechanical work. As the field moves along with the coil $\frac{\partial \Phi}{\partial x} = 0$.

- The reluctance force due to the field generated by the coil is negligible because the reluctance change as a function of position is small due to symmetry (and exactly zero in the center position).

_____

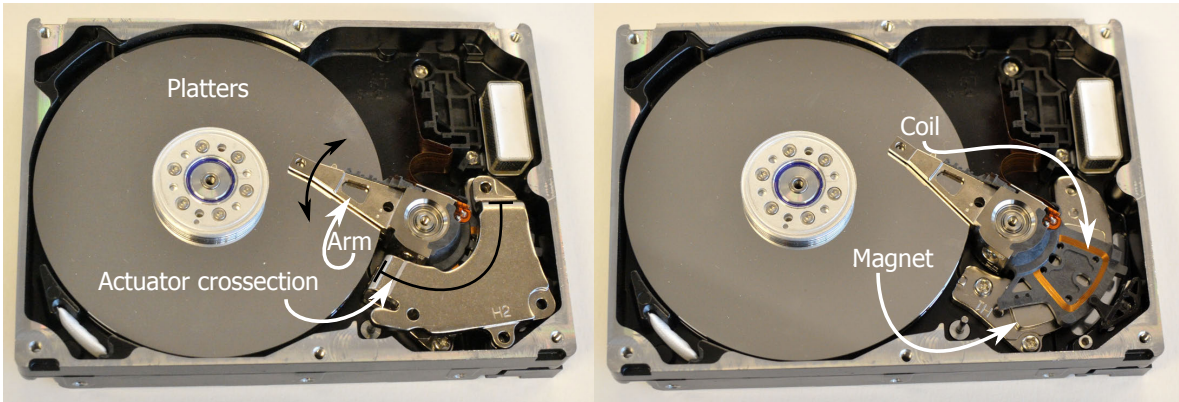[1]http://en.wikipedia.org/wiki/Hard_disk_drive

Figure 1: A harddisk with the cover off showing the head actuator (left). In the right picture one of the magnets is removed to uncover the coil. The indicated cross-section is schematically shown in Figure 2.
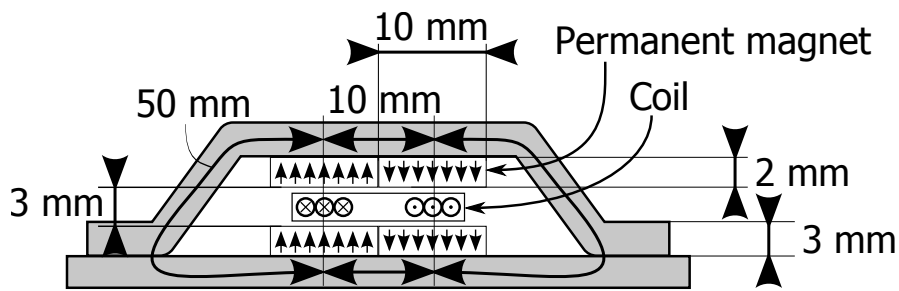
Figure 2: Actuator schematic. The coil (the wires in the different directions are indicated by $\otimes$ and $\odot$) is supposed to move in horizontal direction, and is suspended in the other directions. Small arrows indicate the magnetization direction of the permanent magnets. The relevant dimensions are indicated in the drawing and in Table 1.

Table 1: Properties of the hard disk actuator.

Property	Value
Depth (orthogonal to the drawing) (d)	10 mm
Magnet width (horizontal) (w_{m})	10 mm
Magnet thickness (vertical) (t_{m})	2 mm
Air-gap thickness (vertical) (t_{air})	3 mm
Core length left & right (horizontal & vertical) ($l_{\mathrm{c,lr}}$)	50 mm
Core length top & bottom (horizontal) ($l_{\mathrm{c,tb}}$)	10 mm
Core thickness (horizontal & vertical) (t_{c})	3 mm
Magnet remanent flux density (B_{r})	1.35 T
Core relative permeability ($\mu_{\mathrm{r,core}}$)	4000
Air relative permeability ($\mu_{\mathrm{r,air}}$)	1
Magnet relative permeability ($\mu_{\mathrm{r,m}}$)	1

Problem 4: Magnetic circuit

(24 points)

In order to determine the magnetic field in the air gap of the actuator, we will start with a system with only 2 out of 4 magnets, as shown in the upper 2 drawings in Figure 3 (situations A and B).

(6p) **4a.** Draw the magnetic field-lines as you expect them in the top and the middle drawing in Figure 3 (Situations A and B). Pay attention to the flux line density and indicate the flux direction. (To hand in your solutions digitally, please draw on the printed paper, scan your solution and include it in the document you hand in through Blackboard.)

Answer The flux lines are indicated in Figure 3. Note that the second image is the first image mirrored and with the flux direction inverted. The flux lines bulge out in the air gap and there are more flux lines in the short iron part than in the long one. In a more or less equivalent FEM simulation this looks as follows:
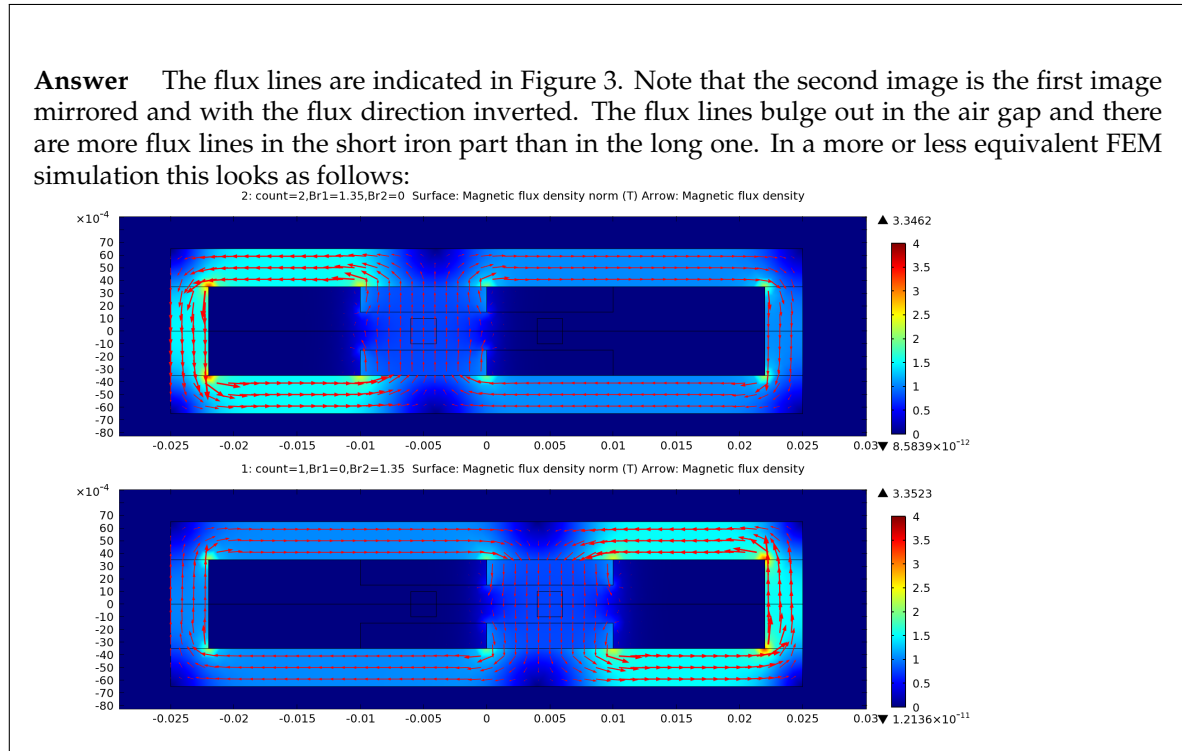
The Hopkinson's law equivalent magnetic circuit is a representation of the magnetic system in the shape of an electrical circuit with as basic elements the magnetic reluctance, $\mathcal{R}$; the magnetomotive force, $\mathcal{F}$; and the circuit flux, Φ.

(7p) **4b.** For Situation A (the top drawing in Figure 3): Make a schematic drawing of the equivalent circuit according to Hopkinson's law using reluctances and magnetomotive force sources. Make sure you include all relevant flux paths and leave out the irrelevant or negligible flux paths. Stray flux paths may be neglected.

Answer

Situation A:
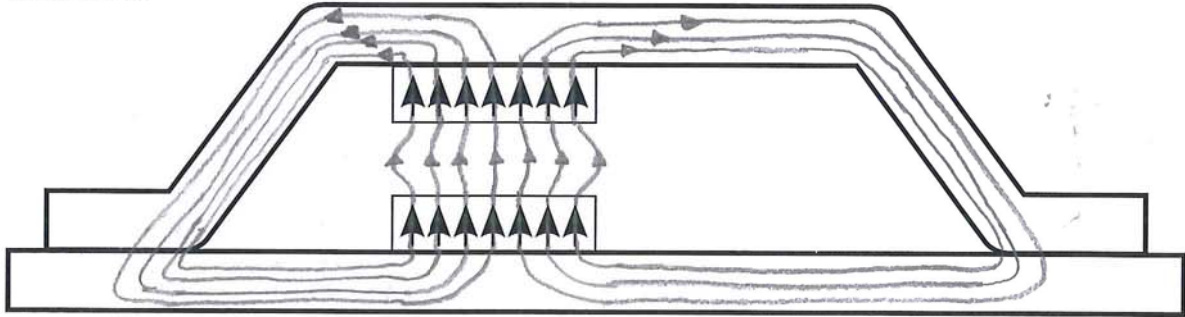
Situation B:
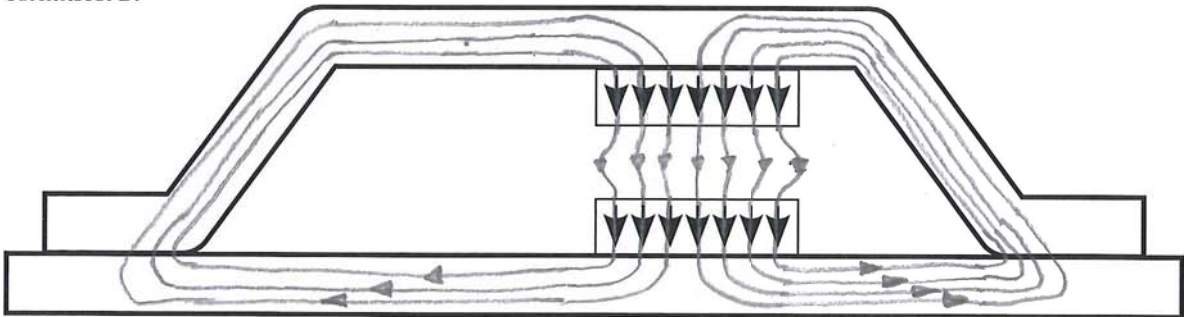
Situation C:

Figure 5: Schematic drawings to draw the magnetic field lines in the partial situations A and B and in the full actuator, situation C..

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- The Hopkinsons law equivalent looks like this:

- This may be further simplified to:
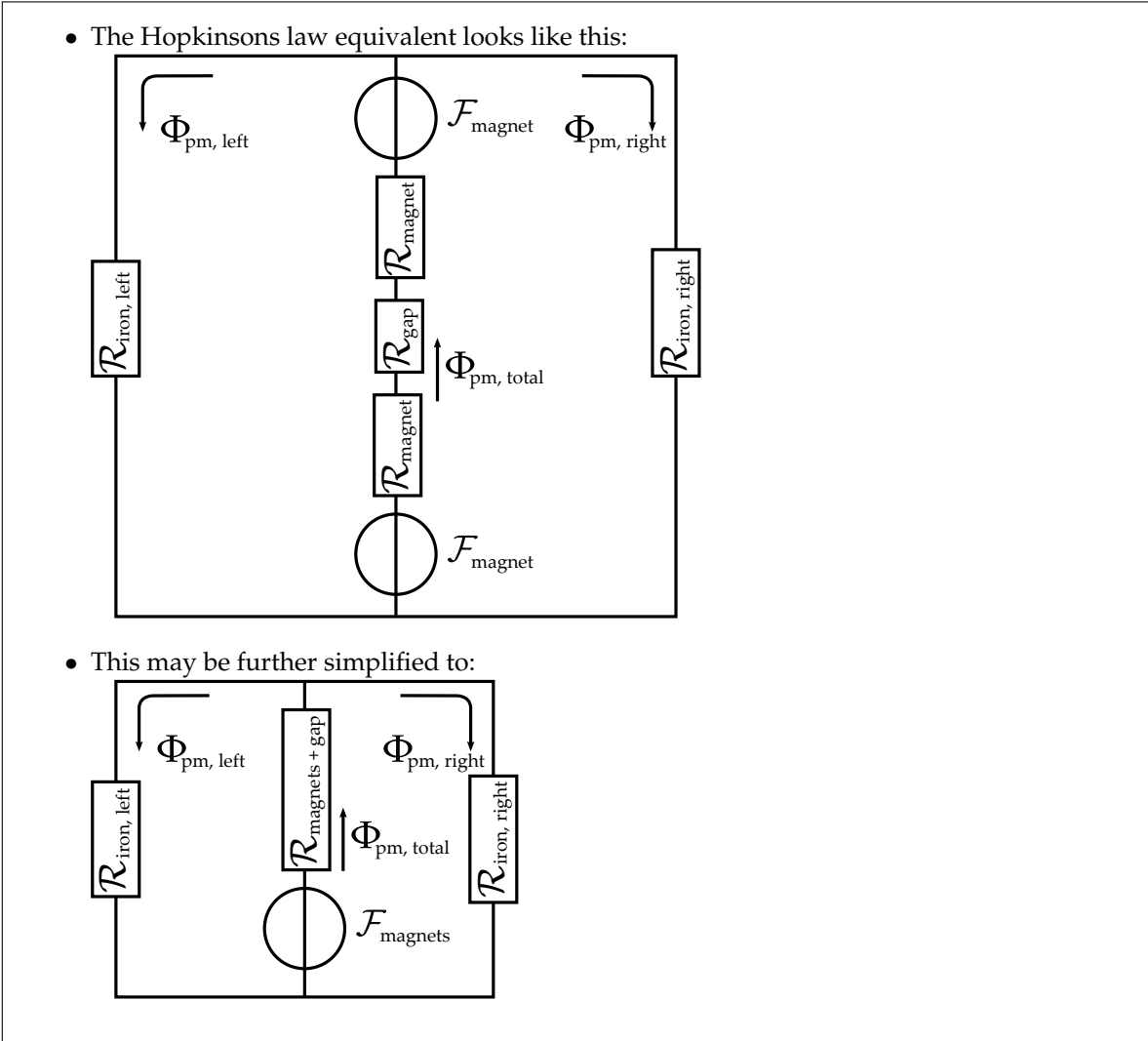
(8p) **4c.** For the same situation as in Question 4b (Situation A): use the Hopkinson's law equivalent circuit to determine the *flux* through the different flux paths and the *flux-density* in the airgap.[2] Use $\lambda = 0.8$ to take into account the stray flux in the air gap.

Answer

- The reluctances and magnetomotive forces in the equivalent circuit are:

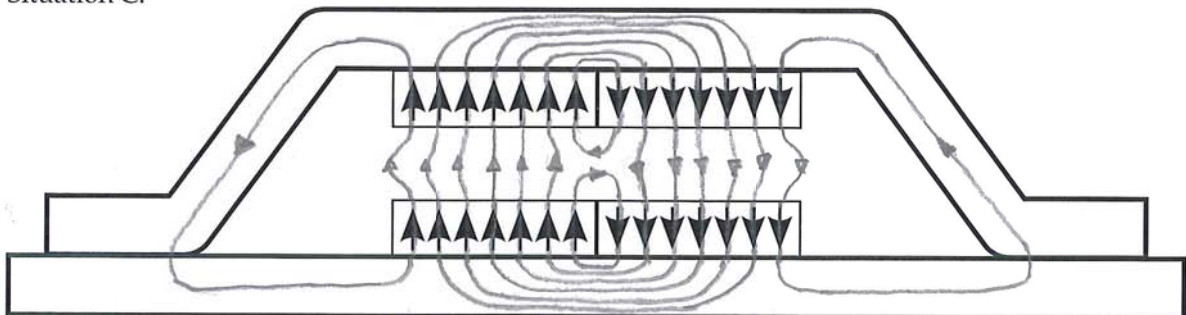$$\mathcal{R}_{\text{magnet}} = \frac{l}{\mu_0 \mu_r A} = \frac{2 \cdot 10^{-3}}{4\pi \cdot 10^{-7} \cdot 1 \cdot (10 \cdot 10^{-3} \cdot 10 \cdot 10^{-3})} = 1.59 \cdot 10^7 \text{ A/Wb}$$

$$\mathcal{R}_{\text{gap}} = \frac{l}{\mu_0 \mu_r A} = \frac{3 \cdot 10^{-3}}{4\pi \cdot 10^{-7} \cdot 1 \cdot (10 \cdot 10^{-3} \cdot 10 \cdot 10^{-3})} = 2.39 \cdot 10^7 \text{ A/Wb}$$

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[2]Note that the equivalent reluctance of two parallel reluctances is computed in the same way as the equivalent resistance of two parallel resistances.

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$$\mathcal{R}_{\text{iron,left}} = \frac{l}{\mu_0 \mu_r A} = \frac{50 \cdot 10^{-3}}{4\pi \cdot 10^{-7} \cdot 4000 \cdot (3 \cdot 10^{-3} \cdot 10 \cdot 10^{-3})} = 3.32 \cdot 10^5 \,\text{A/Wb}$$

$$\mathcal{R}_{\text{iron,right}} = \frac{l}{\mu_0 \mu_r A} = \frac{(10 + 10 + 50) \cdot 10^{-3}}{4\pi \cdot 10^{-7} \cdot 4000 \cdot (3 \cdot 10^{-3} \cdot 10 \cdot 10^{-3})} = 4.64 \cdot 10^5 \,\text{A/Wb}$$

$$h_c = \frac{B_r}{\mu_0 \mu_r} = \frac{1.35}{4\pi \cdot 10^{-7} \cdot 1} = 1.07 \cdot 10^6 \,\text{A/m}$$
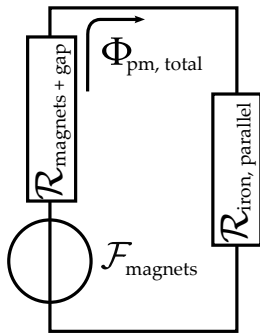
$$\mathcal{F}_{\text{magnet}} = h_c l_m = 1.07 \cdot 10^6 \cdot 2 \cdot 10^{-3} = 2.15 \cdot 10^3 \,\text{A}$$

- The $\mathcal{F}$ of the 2 permanent magnets and the $\mathcal{R}$ of the magnets and the airgap can be taken together:

$$\mathcal{R}_{\text{magnets+gap}} = 2 \cdot \mathcal{R}_{\text{magnet}} + \mathcal{R}_{\text{gap}} = 5.57 \cdot 10^7 \,\text{A/Wb}$$

$$\mathcal{F}_{\text{magnets}} = 2 \cdot \mathcal{F}_{\text{magnet}} = 4.30 \cdot 10^3 \,\text{A}$$

- To determine the total flux, compute the equivalent $\mathcal{F}$ and $\mathcal{R}$ for:

- The parallel reluctance is:

$$
\begin{aligned}
\mathcal{R}_{\text{iron,parallel}} &= \frac{1}{\frac{1}{\mathcal{R}_{\text{iron,left}}} + \frac{1}{\mathcal{R}_{\text{iron,right}}}} \\
&= \frac{\mathcal{R}_{\text{iron,left}} \cdot \mathcal{R}_{\text{iron,right}}}{\mathcal{R}_{\text{iron,left}} + \mathcal{R}_{\text{iron,right}}} \\
&= \frac{3.3 \cdot 10^5 \cdot 4.6 \cdot 10^5}{3.3 \cdot 10^5 + 4.6 \cdot 10^5} = 1.93 \cdot 10^5 \,\text{A/Wb}
\end{aligned}
$$

- The total flux is:

$$\Phi_{\text{pm,total}} = \frac{\mathcal{F}_{\text{magnets}}}{\mathcal{R}_{\text{iron,parallel}} + \mathcal{R}_{\text{magnets+gap}}} = 7.69 \cdot 10^{-5} \,\text{Wb}$$

Note that the iron reluctance is small compared to the other reluctances and may thus be neglected.

- Therefore the flux density in the airgap is: (Eq. 5.31 in the book)

$$B_{\text{gap}} = \frac{\Phi_{\text{pm,total}} \lambda}{A_{\text{gap}}} = \frac{7.69 \cdot 10^{-5} \cdot 0.8}{(10 \cdot 10^{-3} \cdot 10 \cdot 10^{-3})} = 0.615 \,\text{T}$$

- The total flux is divided over the right and the left part of the iron core. The MMFs (equivalent to voltage) over both parts of the core are equal and the flux is divided proportional to the inverse of the reluctance.

$$\begin{aligned} \mathcal{F}_{\text{iron,parallel}} &= \Phi_{\text{pm,total}} \mathcal{R}_{\text{iron,parallel}} = 14.87\,\text{A} \\ \Phi_{\text{iron,left}} &= \frac{\mathcal{F}_{\text{iron,parallel}}}{\mathcal{R}_{\text{iron,left}}} = 4.48 \cdot 10^{-5}\,\text{Wb} \\ \Phi_{\text{iron,right}} &= \frac{\mathcal{F}_{\text{iron,parallel}}}{\mathcal{R}_{\text{iron,right}}} = 3.20 \cdot 10^{-5}\,\text{Wb} \end{aligned}$$

Where the positive directions are as indicated in the schematic.

(3p) **4d.** Is the reluctance of the iron parts significant for the fluxes and the flux-density in computed in Question 4c? Give a brief explanation.

Answer

- For computing the total flux (through the airgap) the iron reluctance is only a few percent of the total and may thus be neglected.

- To determine how the flux is distributed over the left and the right iron part, their relative reluctance is used. The reason that there is flux in the left and right iron parts is the difference in iron reluctance. Therefore the iron reluctance is relevant in this case.

Problem 5: Magnetic superposition

(28 points)

(6p) **5a.** Superimpose (add) the fields you have found in Question 4a to form the field of the total system. Draw the combined field in the bottom diagram of Figure 3 (Situation C, the system with 4 magnets). Again, pay attention to the flux line density and the flux direction.

> **Answer** The combined field is shown in Figure 3. Or in the simulation:
>
>
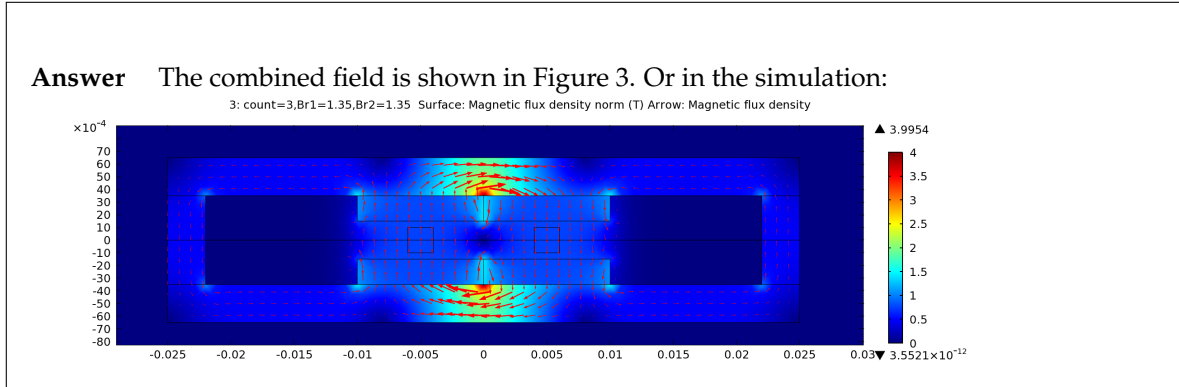
(6p) **5b.** In the flux-pattern you have found in the Question 5a, one path is dominant. Explain briefly why it is a valid assumption that all flux follows this path. Support your reasoning by a flux calculation. Hint: you may use the fluxes computed in Question 4c for this.

> **Answer**
>
> - Superposition: fluxes from both magnets cancel approximately out at the sides. The fluxes in the iron at both sides are of the same order of magnitude and for the different magnet pairs the flux is mirrored around the vertical axis (left and right are interchanged) and the direction is opposite. The flux in the core is:
>
> $$\Phi_{A,iron} = \left[\begin{array}{c} \Phi_{A,iron,left} \\ \Phi_{A,iron,right} \end{array} \right]$$
>
> $$\Phi_{B,iron} = \left[\begin{array}{c} \Phi_{B,iron,left} \\ \Phi_{B,iron,right} \end{array} \right] = \left[\begin{array}{c} -\Phi_{A,iron,right} \\ -\Phi_{A,iron,left} \end{array} \right]$$
>
> $$\Phi_{total,iron} = \Phi_{A,iron} + \Phi_{B,iron}$$
>
> $$= \left[\begin{array}{c} \Phi_{A,iron,left} \\ \Phi_{A,iron,right} \end{array} \right] + \left[\begin{array}{c} \Phi_{B,iron,left} \\ \Phi_{B,iron,right} \end{array} \right]$$
>
> $$= \left[\begin{array}{c} \Phi_{A,iron,left} - \Phi_{A,iron,right} \\ \Phi_{A,iron,right} - \Phi_{A,iron,right} \end{array} \right] = \left[\begin{array}{c} 1.28 \cdot 10^{-5} \\ -1.28 \cdot 10^{-5} \end{array} \right] \text{Wb}$$
>
> where the flux through the gap was relatively large:
>
> $$\Phi_{pm,total} = 7.69 \cdot 10^{-5} \text{Wb}.$$
>
> The flux in the dominant path is therefore:
>
> $$\Phi_{dominant} = \Phi_{pm,total} - \Phi_{total,iron,left} = 6.41 \cdot 10^{-5} \text{Wb}.$$
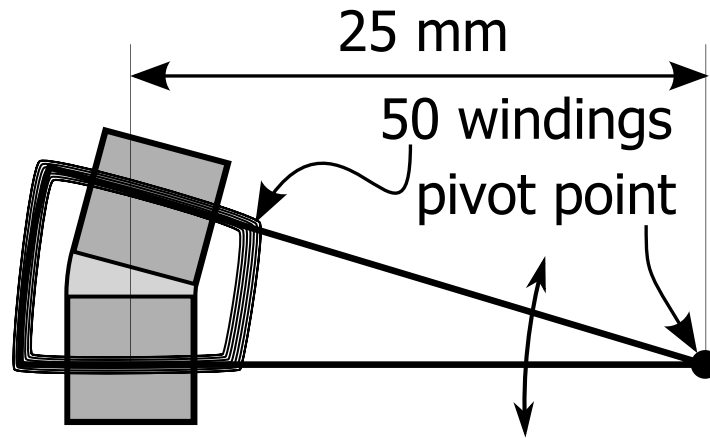
Figure 4: Schematic top view of the HDD arm actuator.

(6p) **5c.** Given that number of windings, $n = 50$, and that the coil is at an effective distance $l_{\text{pivot}} = 25\,\text{mm}$ from the pivot point, as is shown in Figure 4, determine the motor torque constant of the actuator.

Answer

- The flux density in the gap is computed in question 4c (note that superposition does not change this significantly):
$$B_{\text{gap}} = 0.615\,\text{T}$$

- As the reluctance of the magnetic circuit does not change as a function of position x this is a pure Lorentz force, given by:

$$
\begin{aligned}
F &= nI\frac{\text{d}\Phi}{\text{d}x} \\
&= nI\left(B_{\text{gap}}d - -B_{\text{gap}}d\right) \\
&= nI\left(2B_{\text{gap}}d\right)
\end{aligned}
$$

where d is the actuator depth and the factor 2 is caused by the fact that on one side of the coil positive flux is added and at the other side the same amount of negative flux is subtracted.

- The linear motor constant is the force per unit of current:

$$
\begin{aligned}
k_{\text{m,lin}} &= \frac{F}{I} \\
&= 2nB_{\text{gap}}d = 0.62\,\text{N/A}.
\end{aligned}
$$

- With a length of $l_{\text{pivot}} = 25 \cdot 10^{-3}\,\text{m}$ the motor torque constant becomes:

$$k_{\text{m,rot}} = k_{\text{m,lin}}l_{\text{pivot}} = 0.015\,\text{Nm/A}.$$

When you try to remove the upper 2 magnets and the top of the core from the magnetic system (this motion is indicated by the large arrow in Figure 5) you notice that you need a large force for that.
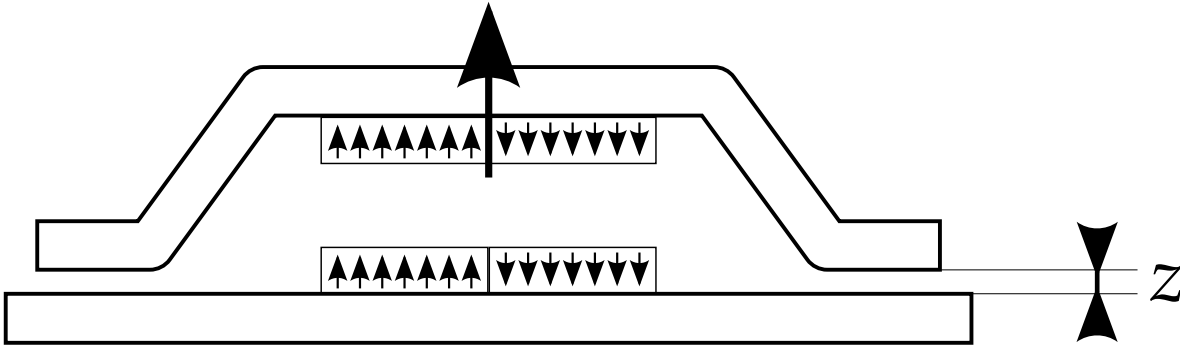
Figure 5: Removing one of the magnet assemblies.

We will compute the 'sticktion force' of this magnet configuration. Note that the coil can be entirely left out for now, as we are interested in the force due to the permanent magnets only.

(3p) **5d.** Which basic physical/electromagnetic principle generates this force? Explain why this is the case in at most 4 sentences.

Answer

- This is a reluctance force.

- This is the case because the derivative dphi/dx is a function of the displacement and this is caused by a change of the reluctance of the magnetic circuit.

- Also there is no coil or current present, so there is no Lorentz force contribution.

We will assume that all flux flows through the dominant flux path discussed in Question 5b; however, the size of the airgap increases with displacement z, as is indicated in Figure 5. Note that there are permanent magnets and there is no current carrying coil in this magnetic circuit. Therefore, the most straightforward method to compute the 'sticktion force' is using the spatial derivative of the magnetic energy in the system. In this case the magnetic force is given by:[3]

$$F_{\text{mag}} = \frac{\text{d}}{\text{d}z} \left(-E_{\text{mag}} \right),$$

where the magnetic energy can be approximated in the Hopkinson's law parameters:

$$E_{\text{mag}} = \frac{1}{2} \int_V \frac{B^2}{\mu} \text{d}V \approx \frac{1}{2} \Phi^2 \mathcal{R} = \frac{1}{2} \frac{\mathcal{F}_{\text{m}}^2}{\mathcal{R}}.$$

(7p) **5e.** Use the the stored magnetic energy in the system as a function of distance z (indicated in Figure 5) to derive an expression for the force in z-direction as a function of z, and compute the force at distance $z = 0$.

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[3]If you follow the entire derivation in Section 5.3.2 of the book, we assume to have only non-zero power storage and mechanical output, so that: $P_{\text{storage}} - P_{\text{mech}} = 0$.

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Answer

- The magnetomotive force of the circuit is the combined MMF of all 4 magnets:

$$\mathcal{F}_{\mathrm{m}} = 4\mathcal{F}_{\mathrm{magnet}} = 4h_{\mathrm{c}}l_{\mathrm{magnet}}$$

- The circuit reluctance consists of iron, magnet and air:

$$\mathcal{R} = 2\mathcal{R}_{\mathrm{Iron}} + 2\mathcal{R}_{\mathrm{Gap}}(z) + 4\mathcal{R}_{\mathrm{Magnet}}$$

where all reluctances are as given before. The iron reluctance may be neglected.

- The expression for the magnetic energy found before can be filled in in:

$$\begin{aligned}
F_{\mathrm{mag}}(z) &= \frac{\mathrm{d}}{\mathrm{d}z}\left(-E_{\mathrm{mag}}\right) \\
&= \frac{\mathrm{d}}{\mathrm{d}z}\left(-\frac{1}{2}\frac{\mathcal{F}^2}{\mathcal{R}(z)}\right) \\
&= -\frac{1}{2}\mathcal{F}^2\frac{\mathrm{d}}{\mathrm{d}z}\left(\frac{1}{\mathcal{R}(z)}\right)
\end{aligned}$$

- Let's define:

$$\begin{aligned}
\mathcal{R} &= \mathcal{R}_0 + \mathcal{R}_1(z) \\
\mathcal{R}_0 &= 2\cdot\mathcal{R}_{\mathrm{Iron}} + 2\cdot\mathcal{R}_{\mathrm{Gap},0} + 4\cdot\mathcal{R}_{\mathrm{Magnet}} = 1.11\cdot10^8 \,\mathrm{A/Wb} \\
\mathcal{R}_1(z) &= \frac{2z}{\mu_{\mathrm{Gap}}A_{\mathrm{Gap}}} = 1.59\cdot10^{10}\cdot z \,\mathrm{A/Wb}
\end{aligned}$$

so that

$$\begin{aligned}
F_{\mathrm{mag}}(z) &= -\frac{1}{2}\mathcal{F}_{\mathrm{m}}^2\frac{\mathrm{d}}{\mathrm{d}z}\left(\frac{1}{\mathcal{R}_0 + \mathcal{R}_1(z)}\right) \\
&= -\frac{1}{2}\mathcal{F}_{\mathrm{m}}^2\left(\frac{-\frac{\mathrm{d}}{\mathrm{d}z}(\mathcal{R}_1(z))}{(\mathcal{R}_0 + \mathcal{R}_1(z))^2}\right) \\
&= -\frac{1}{2}\mathcal{F}_{\mathrm{m}}^2\left(\frac{-\frac{2}{\mu_{\mathrm{Gap}}A_{\mathrm{Gap}}}}{\left(\mathcal{R}_0 + \frac{2z}{\mu_{\mathrm{Gap}}A_{\mathrm{Gap}}}\right)^2}\right) \\
&= -\frac{1}{2}\left(4\cdot h_{\mathrm{c}}l_{\mathrm{m}}\right)^2\left(\frac{-\frac{2}{\mu_{\mathrm{Gap}}A_{\mathrm{Gap}}}}{\left(2\cdot\mathcal{R}_{\mathrm{Iron}} + 2\cdot\mathcal{R}_{\mathrm{Gap},0} + 4\cdot\mathcal{R}_{\mathrm{Magnet}} + \frac{2z}{\mu_{\mathrm{Gap}}A_{\mathrm{Gap}}}\right)^2}\right)
\end{aligned}$$

- Then at $x = 0$

$$F_{\mathrm{mag}}(0) = -47.3\,\mathrm{N}$$

- The force-displacement graph is given below: