Examination + answers:

Course name: Mechatronic System Design
Course code: WB2414-08
Date: April 7th, 2011
Time: 9:00 - 12:00
Contents: 4 Questions (10 points (+1))

Remark: Like the January examination maximum 10 points can be earned, excluding the given point for an empty paper. One point can be seen as a bonus point to compensate other flaws.

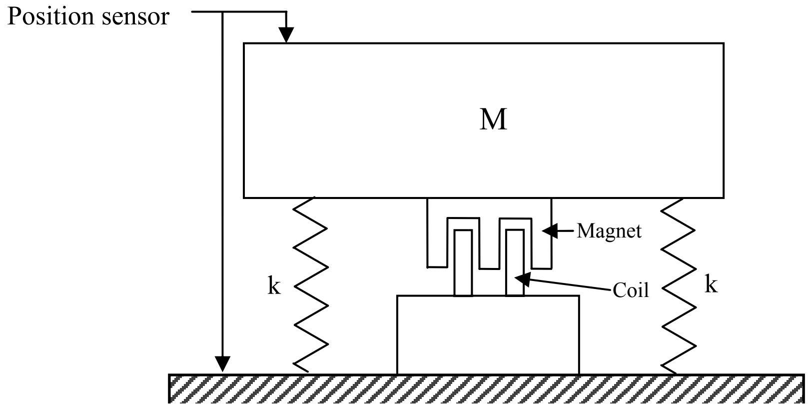


Figure 1: Vibration table

Question 1 (3.5 points) (motion control)

A vibration table is used to test the sensitivity of several objects for vibrations as shown schematically in Figure 1. The system is position controlled to achieve a constant position amplitude over a frequency range from 0 Hz until 1 kHz (+/- 3 dB).
The system consists of:

- A mass M of 3kg.
- 3 springs with a stiffness of k = 10^3 N/m each between the mass and the ground.
- An amplifier-Lorentz actuator combination with a total combined frequency independent transfer function of 10 N/V between the mass and the ground.
- A position sensor between the mass and the ground of 10^3 V/m.
- An optimally tuned PD-feedback controller between the sensor and the amplifier-Lorentz actuator (damping and overshoot Q~1.3, ζ~0.7).

Question 1A (2 points)

Draw the open loop bode plot (magnitude and phase) of the transfer function of the total system. Include relevant values for resonance frequency, slope and level of the lines. Draw the bode plot in three parts:

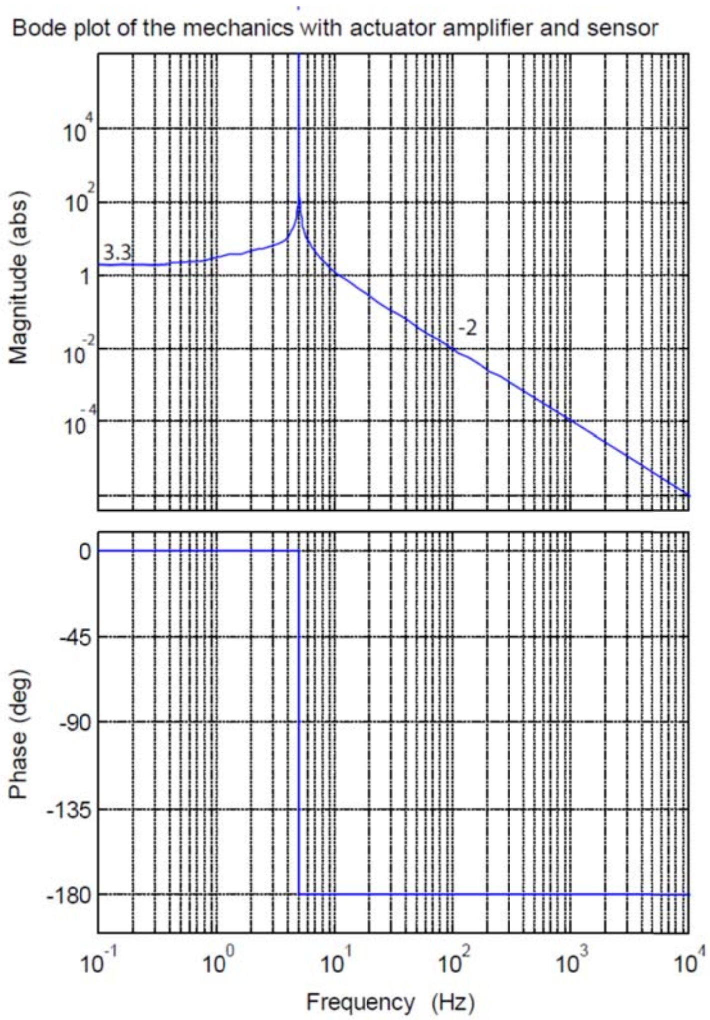
1. The mechanics including the amplifier, Lorentz actuator and sensor (0.7 pts)
2. The controller (0.7 pts)
3. The combined total system (0.6 pts)

Question 1B (1.5 points)

An object under test with a mass of 9 kg is placed on the table. This mass is rigidly connected to the table. The side effects of the static load due to gravity may be neglected.
To compare the effect of the added mass first draw an approximation of the amplitude bode plot of the closed loop without the added object en then in the same plot add the approximated closed loop amplitude bode plot of the system with the added mass. Comment the differences caused by the additional mass in respect to the bandwidth, damping and magnitude of the relevant lines (slope and level).
Propose improvements in the controller, when needed for optimal stability and/or bandwidth with the higher mass. Also comment the robustness these improvements, when the mass is be taken off again under closed loop control.

Answers Question 1:

Q1a 1 Mechanics with actuator, amplifier and sensor



The open loop bode plot of the mechanics is the plot of an undamped mass-spring system. The key values which should be shown in the plot are:

- The height of the spring line which is given by: $\frac{1}{k} \cdot \text{Kp}$, where Kp follows from the force factor of the actuator-amplifier combination and the gain of the position sensor: $\text{Kp}=10 \cdot 10^3=10^4$. So the value of the spring line is: $\frac{1}{3000} \cdot 10^4 = 3.3$

- The resonancepeak at::

$$f_{res} = \frac{\omega_0}{2\pi} = \frac{\sqrt{\frac{k}{m}}}{2\pi} = \frac{\sqrt{\frac{3000}{3}}}{2\pi} = 5Hz$$

- The -2 slope after the resonance frequency.

-The phase starting at 0 and jumping to -180 at the resonance frequency.

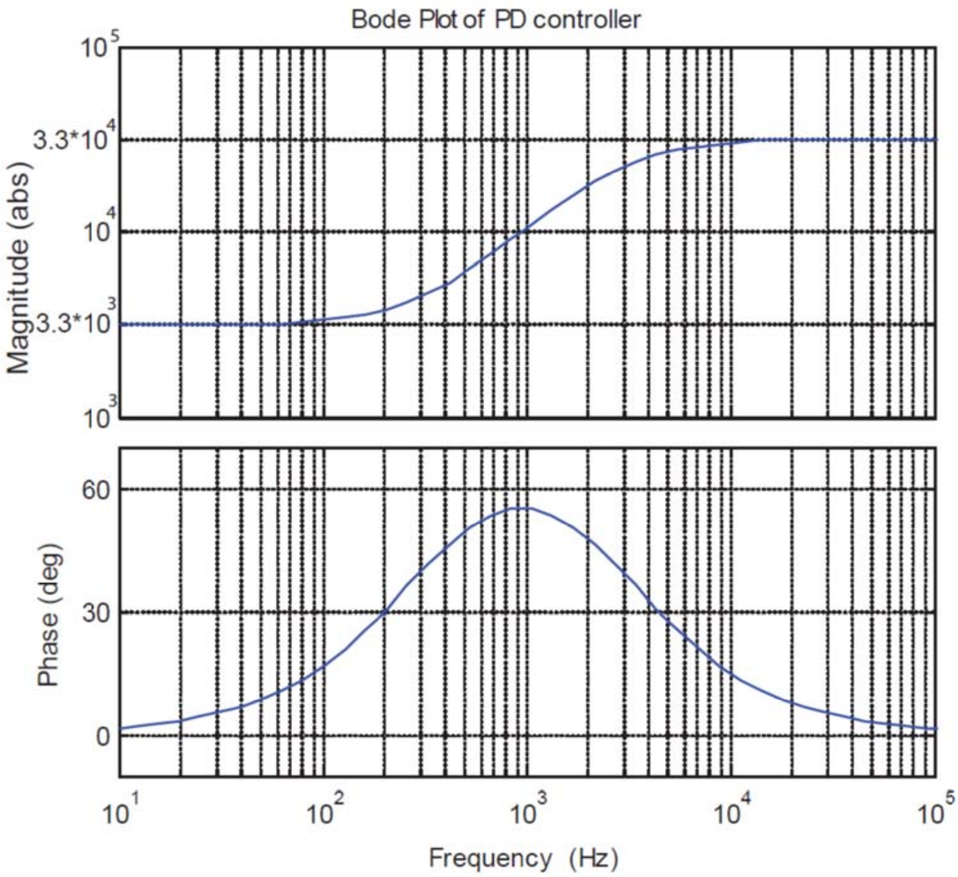
Q1a 2. controller

The (optimal tuned) PD-controller gives the system a bandwidth of 1 kHz. To achieve this, the controller must add additional phase to the system at the bandwidth frequency and enough gain to 'lift' the bode plot to a gain of 1 at 1kHz. The standard PD-controller has a transfer function of: $G(s) = k_p \frac{\tau_1 s + 1}{\tau_2 s + 1}$. The values for τ_1 and τ_2 follow from:

$$\tau_1 = \frac{1}{\omega_1} = \frac{1}{0.3\omega_{bw}} = \frac{1}{0.3 \cdot 1000 \cdot 2\pi} = 5.3 \times 10^{-4}$$

$$\tau_2 = \frac{1}{\omega_2} = \frac{1}{3\omega_{bw}} = \frac{1}{3 \cdot 1000 \cdot 2\pi} = 5.3 \times 10^{-5}$$

To lift the bode plot to 1 @ 1kHz, 10^4 gain (@1kHz) is required. (The mass-line with -2 slope crosses 1 at 10Hz, 1000Hz is 2 decades further, so the gain of the mechanical system at 1kHz is 10^{-4}) So the bode plot of the controller should look like:



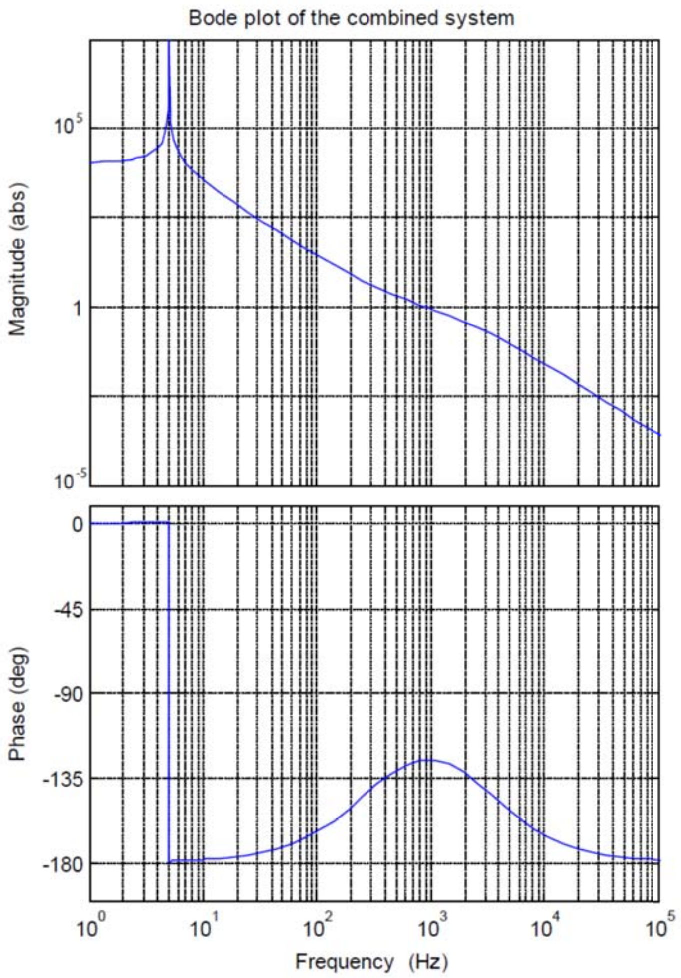
Q1a. 3 The combined system

The open loop bode plot of the total system is the combination of the first two. Adding these two plots results in the required plot. Elements that should be clearly shown in your plot are:

- The amplitude is lifted by the controller, so the value at low frequencies (the spring-line) is now $3.3*3.3*10^3=10^4$ and the graph crosses 1 at the bandwidth frequency of 1kHz.

- The mass-line starts with a -2 slope after the resonance peak but changes into a -1 slope between approximately 3.10^2 and 3.10^3 Hz. After 3.10^3 Hz, it changes back to a -2 slope.

- The phase starts a 0 degrees, jumps to -180 at the resonance frequency, is gradually lifted (≈50dB) around 1kHz and goes gradually back to -180 degrees.



Q1b:

The closed loop amplitude bode plot of the system without the added object has a constant amplitude from 0Hz to 1kHz, with a small overshoot (Q~1.3 ζ~0.7). After 1 kHz, the gain goes down with a gradually to -2 changing slope.

With the object on the table, the total mass is increased from 3 to 12kg. As a result the resonance frequency of the mechanical system is lower by a factor 2. The plot 'shifts' to the left, and the kp of the controller is no longer large enough to keep the bandwidth at 1kHz resulting in a reduced bandwidth. Also the overshoot is increased because the lead-lag filter is not optimally tuned anymore. The -2 slope of closed loop amplitude plot is also 'shifted' to the left as shown in the figure (also the phase is plotted here, this was not required in your answer).

Improvements:

When only the stability needs to be improved, the time constants of the lead-lag filter should be increased with a factor 2, shifting the frequencies of the lead-lag filter to a proportional lower value. In that case however the bandwidth is still half the original value.

When the bandwidth of the system should remain equal the Kp of the controller should be increased with a factor 4 and the time constants of the lead lag filter should remain unchanged.

When the mass is taken away again both improvements need to be corrected again. especially the factor 4 higher kp will result in marginal stability at 2 kHz

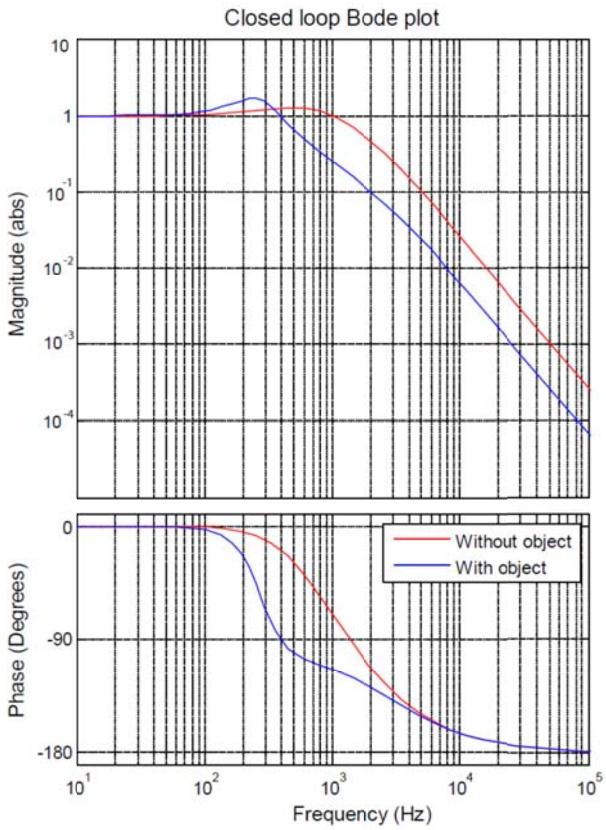


Figure 2: Linear motor driven positioning system

Question 2 (2 points) positioning systems and actuators

A linear motor driven positioning system (Figure 2) has to make a displacement of 20 mm in not more than 80 ms from standstill to standstill.
The moving mass of the system is 12kg.

Question 2A: (0.5 points)

With what minimal acceleration level can the displacement be made in the specified time in an ideal situation when all dynamic factors are neglected. Draw a graph of the acceleration, velocity and position as a function of time for this ideal situation? Make a drawing with values.

Question 2B: (1 points)

The motor is a linear Lorentz actuator with the following non ideal properties:

R_{motor} : 10 Ω
L_{motor} : 10 mH
F_{motor} : 10 N/A

Draw a graph of the voltage over the motor coil terminals during the step as a function of time, when driven with a current, that would be necessary for the acceleration profile calculated at question A. Comment the different factors that contribute to this voltage.

Question 2C: (0.5 points)

The amplifier can deliver maximum +/-200V. What measure should be taken to prevent the amplifier from clipping? This means that the output voltage of the amplifier is not allowed to exceed the power supply voltage.

Answers to question 2

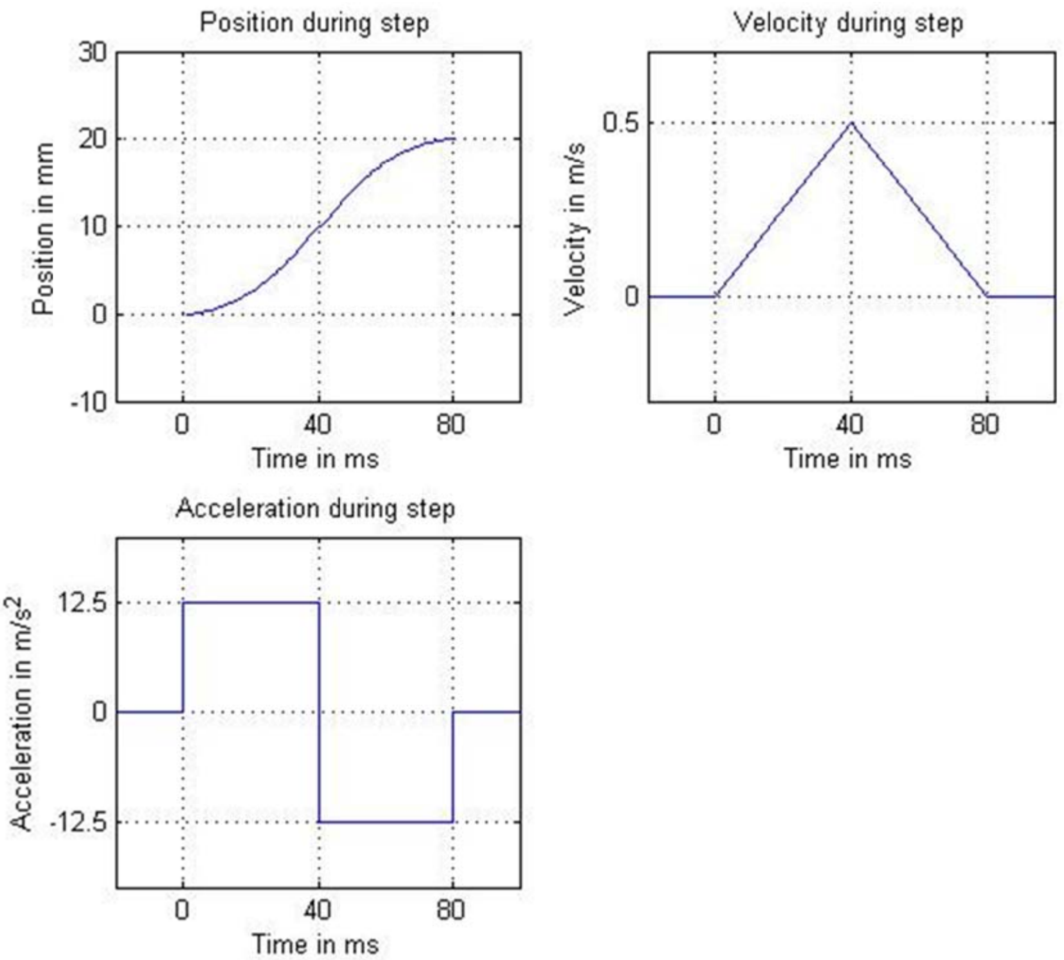
Q1a

$$v_{\text{average}} = \frac{x}{t} = \frac{20 \cdot 10^{-3}}{80 \cdot 10^{-3}} = 0.25 \, {}^{m}\!/_{s}$$

The velocity and the acceleration must both be zero at the end of the step. So the time to accelerate is the half of the total step time and the maximum velocity is reached at half of the step. The maximum velocity is twice the average velocity.

$$v_{\max} = 2 \cdot 0,25 = 0,5 \, {}^{m}\!/_{s}$$

$$s(t) = 10mm = 0.01m = \frac{1}{2}at^2 \;\rightarrow\; a = a_{\min} = \frac{0.01 \cdot 2}{0.04^2} = 12,5 \, {}^{m}\!/_{s^2}$$



Q1b

$$V_{\text{motor}} = I_{\text{motor}} \cdot R_{\text{motor}} + V_{\text{Emk}} + L_{\text{motor}} \cdot \frac{\mathrm{d}I}{\mathrm{d}t}$$

$$F = m \cdot a = 12 \cdot 12{,}5 = 150N$$

$$I_{\text{motor}} = \frac{150}{10} = 15A$$

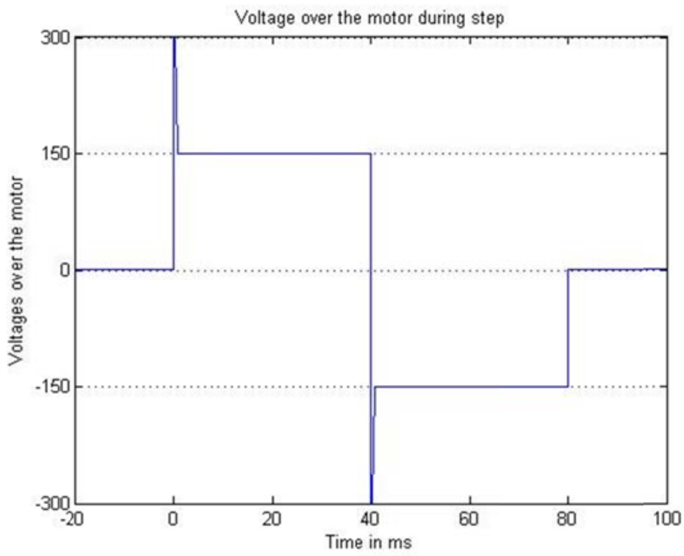
$\frac{dI}{dt} = \infty$ *at the moment the acceleration starts and -*∞ *at the moment the acceleration changes sign. Between those moments,* $\frac{dI}{dt} = 0$

$$F_{\text{motor}} = B \cdot L \cdot I \rightarrow B \cdot L = 10 \left[\mathrm{N}/\mathrm{A} \right]$$

$$V_{\text{Emk.max}} = B \cdot L \cdot \frac{\mathrm{d}x}{\mathrm{d}t} = 10 \cdot 0{,}5 = 5 \quad [\mathrm{V}]$$

This is the maximum voltage induced by the speed. Considering that it is the maximum value and compared with the voltage required to accelerate this is negligible.

$$V_{\text{motor}} = 15 \cdot 10 + 0{,}01 \cdot \frac{\mathrm{d}I}{\mathrm{d}t} = 150 + 0{,}01 \cdot \frac{\mathrm{d}I}{\mathrm{d}t} \quad [\mathrm{V}]$$



Q1C

To prevent the amplifier from clipping, the voltage required for the acceleration needs to be lower than the voltage which can be maximal delivered by the amplifier. This can be done by limiting the jerk with for example a motion profile.

$V_{\max} = 200\ \ [\mathrm{V}] = 150 + 0{,}01 \cdot \frac{\mathrm{d}I}{\mathrm{d}t}$ so there is 50 V available for the acceleration.

$\frac{\mathrm{d}I}{\mathrm{d}t} = \frac{50}{0{,}01} = 5 \cdot 10^3 \left[\mathrm{A}/\mathrm{s}\right]$ with this, the time needed to achieve the required acceleration is:

$t_{\mathrm{acc}} = \frac{15\ \ [\mathrm{A}]}{5 \cdot 10^3\ \ \left[\mathrm{A}/\mathrm{s}\right]} = 0{,}003\ \ [\mathrm{s}] = 3\ \ [\mathrm{ms}]$, so if the actuator is controlled with a motion profile where the acceleration of 12,5 m/s is build up in 3ms, no clipping occurs. (neglecting V_{emk})

Question 3 Electronics (3.5 points)

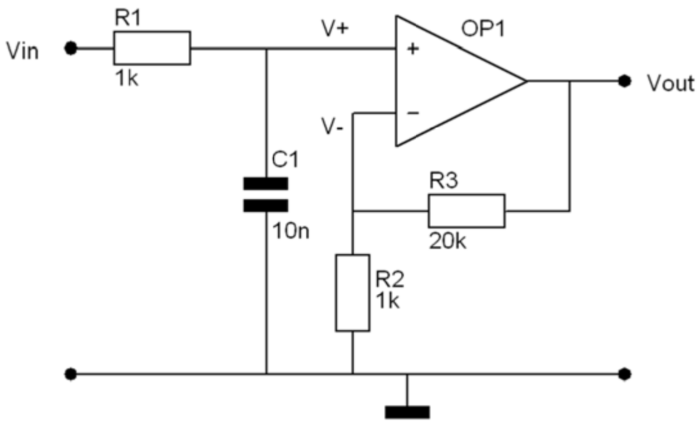


Figure 3. Filter with opamp

Consider the filter circuit given in Figure 3. You are going to determine the properties of this circuit. In the first part of the analysis you may assume that the operational amplifier (opamp) is ideal. To the input a generator is connected that gives a sine wave output, of which we can vary the amplitude and the frequency.

Question 3A: (0.3 points)

What is the input impedance of this circuit, that the generator connected to the input of this circuit would "see" at very low frequencies?

Question 3B: (0.3 points)

What is the input impedance that the generator connected to the input of this circuit would see at very high frequencies?

Question 3C: (0.5 points)

Describe in words what this circuit does as a function of frequency.

Question 3D: (0.4 points)

Sketch a magnitude Bode plot as a function of frequency for the node V+. Indicate the slopes as e.g. +1 or -1. What is the corner frequency?

Question 3E: (0.4 points)

Sketch a magnitude Bode plot as a function of frequency for the node V-. Indicate the slopes as e.g. +1 or -1. What is the corner frequency?

Question 3F: (1 points)

Now you will calculate in three steps the full transfer function from the input to the output, to determine what the corresponding Bode plots for the output are.

Step one: Calculate the full transfer function Vout/Vin with the resistors and capacitor as general complex impedances Z_1 to Z_4. Hint: start by determining the voltage at node V+, then at node V-. Then you know the current through resistor R3, and hence also through R2, and hence the output voltage.

Step 2: Put in the frequency-dependent impedances for the capacitor and the resistors. Rewrite your transfer function in such a way, that the time constant involved is clearly recognizable. Also write it out individually, as in e.g. $\tau_1 = R_3C_1$. Calculate the corner frequency in kHz.

Step 3: Based on your analysis of the circuit and the equation you obtained after step 2 sketch the magnitude Bode plot for the full transfer function Vout/Vin. Indicate the slopes as e.g. +1 or -1, the corner frequency, and the amplitude gain at low frequencies. You don't have to convert the gain to dB, just indicating the gain is enough

The last part of the question deals with the non-ideal properties of such a circuit.

Question 3G: (0.3 points)

Why do you need a power supply (not indicated in the schematic) for this circuit?

Question 3H: (0.3 points)

At very high frequencies, the opamp circuit of Fig 3 always suffers from the fact that the feedback resistor has some very small but noticeable parasitic capacitance Cp (~1pf) across it. Please draw a schematic like Fig. 3, including this capacitor and
Sketch the corresponding magnitude Bode plot. Indicate the slopes as e.g. +1 or -1, and indicate the position of the (now 2) corner frequencies.

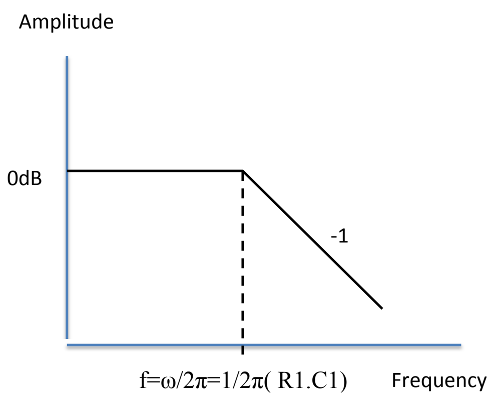
Answers to Question 3

3a. At very low frequencies, the capacitor behaves as an open circuit. The source hence sees the series resistance of R1 and the positive input of the opamp, which is infinitely high. Hence the input impedance is infinitely high at low frequencies.

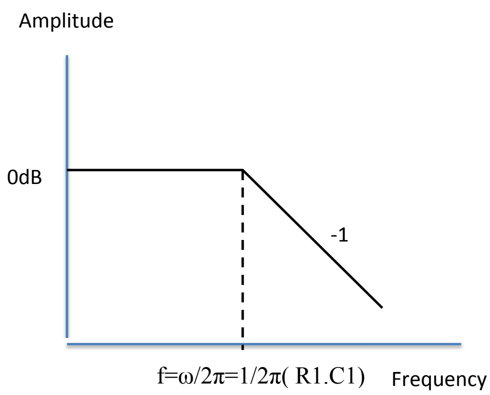
3b. At very high frequencies, the capacitor behaves as a short-circuit. Hence the generator sees R1 connected to ground. The input impedance is hence resistive, and 1kΩ

3c. R1 and C1 together form a low-pass filter which is not loaded at all by the infinitely high input impedance of the opamp, so it behaves as an ideal RC lowpass filter. The opamp itself is a non-inverting amplifier. So at low frequencies the signal is amplified, and above the cutoff frequency determined by R1and C1, the signal decreases in amplitude.

3d. The voltage at V+ follows an ideal first-order RC low pass filter curve, because the opamp input draws no current and hence has no influence.



3e. The voltage at V- follows the voltage at V+ perfectly, because the opamp keeps both inputs at the same potential. The graph and the time constant are exactly the same as in 1d.



3f Step 1:
Let Z1 = R1, Z2 = 1/jωC1, Z3 = R2, and Z4 = R3. Because the opamp will change its output to do whatever it can to make V+ and V- identical, we have:

$\frac{V_+}{Vin} = \frac{Z2}{Z1+Z2}$, and also $\frac{V_-}{Vin} = \frac{Z2}{Z1+Z2}$.

The current through Z3 is then:

$$I_{R2} = \frac{Z2}{Z1+Z2} Vin \cdot \frac{1}{Z3}$$

Because $Vout = V_- + I_{R2}Z4$ we can write it out as

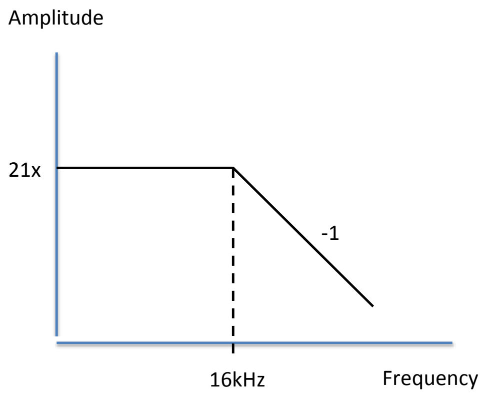
$$\frac{Vout}{Vin} = \frac{Z2}{Z1+Z2}\left(1+\frac{Z4}{Z3}\right)$$

3f, step 2 . Z1= R1, Z2 = 1/jωC1, Z3 = R2, and Z4 = R3.

$$\frac{Vout}{Vin} = \frac{\frac{1}{j\omega C1}}{R1+\frac{1}{j\omega C1}}\left(1+\frac{R4}{R3}\right) = \frac{1}{1+j\omega R1C1}\left(1+\frac{R4}{R3}\right).$$

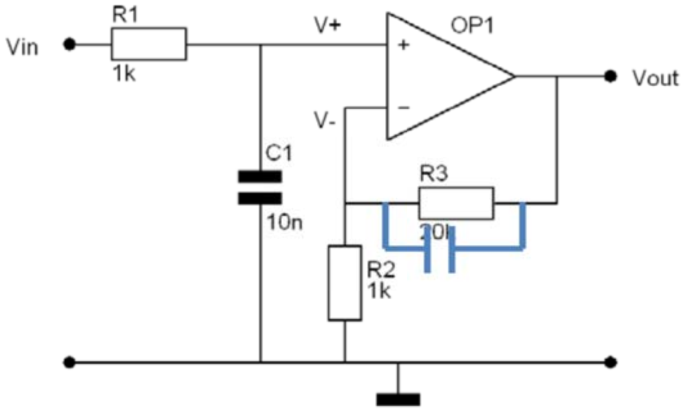
The first order low-pass filter has $\tau_1 = C1R1 = 1\cdot 10^{-5}$ seconds, f(-3dB) = $\omega/2\pi = ½\pi\tau_1$ = 16kHz.

3f,step 3. The amplification at low frequencies is 1+R4/R3 = 1 +20 = 21 times.

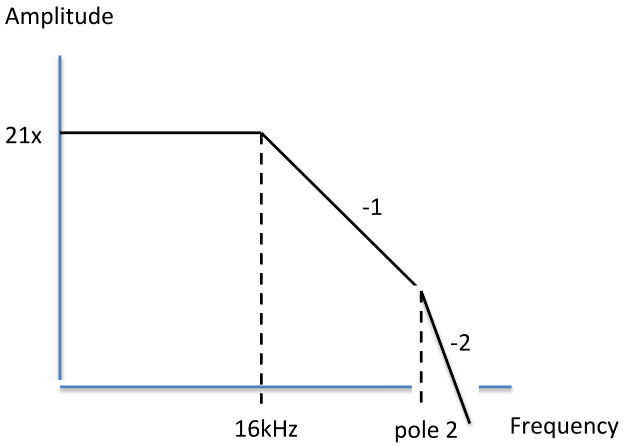


3g The inputs of the opamp draw no current. Still, the output of the opamp has to provide a voltage and a current, and the corresponding power has to come from somewhere. Therefore a power supply is required.

3h.



The parasitic capacitor makes a second pole, which is typically much higher in frequency than the pole calculated for R1C1.



Remark:
At very high frequencies the second pole would be cancelled again as the non inverting amplifier would then become a unity gain amplifier (gain =1). In reality however that happens at frequencies where the open loop gain of the opamp is very low or even below one.

Question 4 Lorentz actuator (1 point)

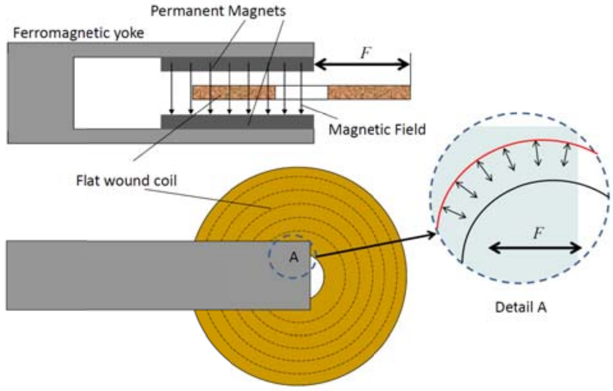


Figure 4 Basic Lorentz type actuator

The basic Lorentz type actuator of Figure 4, for use in a position controlled system, suffers from a lot of inefficiency. One part is due to the fact that most of the coil is outside the air gap and during the lectures solutions were suggested to reduce these problems.
One other problem is the fact that the air gap between the permanent magnets is quite large as it needs to accommodate the coil. Optimising the design of this type of actuator always deals with the right geometry for all the parts.

The following questions need to be answered using common sense and a qualitative knowledge of the material from the course

Question 4A: (0.5 point)

A sometimes suggested solution for the air gap problem is to make the coil of a ferromagnetic material, like isolated wound wires of iron. This wire is normally used for heating because iron has a factor 6 higher resistivity then copper, but its relative magnetic permeability is a factor 100 higher than copper, which could be a benefit in this configuration.

Explain why you think this option is never used in practice. Only a qualitative answer is needed as it should show to what extent you understand requirements on actuation and the principle of this actuator type. Your answer should not be more than 10 sentences and needs to address the following aspects:

- Magnetic flux density
- Saturation
- Magnet size and configuration
- Efficiency (Energy consumed vs force)
- Force behaviour in general (Force to current and stiffness)

Question 4B: (0.5 points)

As an alternative one might consider creating a coil with copper windings interleaved with iron parts, where 50% of the coil volume consists of iron and 50% of copper. Is this a configuration, that can give a more optimal result? Only a rough estimated reasoning of less than 10 sentences is required using the same aspects as for Q4A.

Answers to Question 4

Q4a

The ferromagnetic material will reduce the reluctance of the permanent magnet, thereby increasing the flux density. Due to saturation the flux density is limited to about 2 T. In practice this is about a factor 2-4 higher than with a normal air gap. This means that an equal force would require proportionally less current, reducing the power consumption with the current ratio squared. This would approximately compensate the increase in power consumption due to the factor 6 higher resistivity of the wire ($P=I^2R$). So there is no real advantage. The disadvantage is that the magnet circuit needs to be designed with pole shoes to concentrate the flux density of the permanent magnets. This increases the size. Further the ferromagnetic wire will show a positive stiffness in the driving direction and a negative stiffness in the orthogonal direction.

Even more important (and in fact the really right answer as was given by a few students in the examination) is that the iron coil will concentrate the flux on the coil irrespective of the position of the coil inside the air gap, so there is no or only a little flux change as function of the coil displacement, which means there is hardly any force aside of the negative stiffness force.

Q4b:

Replacing a part of the coil by 50% iron will reduce the magnetic reluctance with just less than a factor 2 increasing the flux density approximately also with less than a factor two (very rough estimation). The saturation limits the flux density to 2T in the iron which is in average 1T in the coil due to the 50% filling This means the current in the copper coil can be just more than half the current without iron, depending on the initial situation without iron. The coil volume is half of the original value, which means the resistance for the same amount of windings is twice as high. As a result the power consumption for the same force will be a bit lower depending on the real numbers. Roughly speaking very little advantage is left and the additional stiffness is sufficient reason for not using this modification.