¹ Chapter 6 Beyond elasticity: plasticity, yielding and ductility

6.1 Introduction and synopsis

- Yield strength the stress beyond which a material becomes plastic; yielding is both useful (forming, absorbing energy) and dangerous.
- This chapter focuses on yielding and plasticity and thus on metals – the plasticity of metals explains their historic and ongoing dominance as structural materials.



Dislocations in the intermetallic compound Ni₃Al

6.1 Strength, plastic work and ductility: definition and measurement

- Yield properties and ductility are measured with standard tensile tests taken to failure.
- The *yield stress* σ_y (or elastic limit, σ_{el}) requires careful definition. In metals the onset of plasticity is not distinct define the 0.2% proof (or offset yield) stress.
- Beyond the yield point most metals work harden causing the rise to the maximum called the *tensile* strength.
- Followed by non-uniform deformation (*necking*) and fracture.



Figure 6.1 Stress–strain curve for a metal.

- For polymers, σ_y is the stress at which the stress-strain curve becomes markedly nonlinear; typically a strain of 1%; beyond yielding behavior depends on T.
- Well below T_g brittle
- Approaching T_g some plasticity
- At T_g cold drawing (large extension at constant stress, molecules gradually align, harden, and then fracture.
- Higher T viscous flow, moldable (thermosets become rubbery then decompose).
- σ_y for a polymer matrix composite defined by a set deviation, often 0.5%.



Figure 6.2 Stress–strain curves for a polymer at T below and above the glass temperature.

• Plastic strain, ε_{pl} is the permanent strain resulting from plasticity, the total minus the recoverable elastic part:

 $\varepsilon_{pl} = \varepsilon_{tot} - \sigma/E$



- Ductility measure of how much plastic strain can be tolerated; in a standard tensile test it is measured by the elongation ε_f (tensile strain at break in %). Strictly speaking it depends on sample dimensions and is not a material property.
- Plastic work is done in deforming a material permanently by yield or crushing. The increment of plastic work done for a small permanent extension or compression dL under force F per unit volume V = AL₀ is

 $dW_{pl} = FdL/V = FdL/A_0L_0 = \sigma d\epsilon_{pl}$, thus work from $\epsilon = 0$ to $\epsilon = \epsilon^*$: $W_{pl} = \int_{0 \to \epsilon^*} \sigma d\epsilon_{pl}$

• Again the area under the stress-strain curve

Example

Example 6.1

A metal rod with a yield strength $\sigma_y = 500$ MPa and a density $\rho = 8000$ kg/m³ is uniformly stretched to a strain of 0.1 (10%). How much plastic work is absorbed per unit volume by the rod in this process? Assume that the material is elastic perfectly-plastic, that is it deforms plastically at a constant stress σ_y . Compare this plastic work with the elastic energy calculated in Example 4.3.

- Ceramics and glasses are brittle at RT they have yield strengths but they are never reached as they fracture first; even in compression they crush before they yield.
- Sometimes a practical measure, the compressive crushing strength, is used. It is not a true yield called the elastic limit, σ_{el} .



Figure 6.3 Stress–strain curve for a ceramic.



- Hardness test: pyramidal diamond or hardened steel ball pressed into surface; the size of the permanent indent is a measure of resistance to plastic deformation. Often more convenient than compression or tensile test (destructive).
- H = F/A; A is the area of the indent projected onto a plane perpendicular to the load.
- The indented region is constrained by the surrounding material and thus $H > \sigma_y$. Expect $H \sim 3\sigma_y$ in MPa but illogical scales are used in hardness; for example, the Vickers hardness (kg/mm²), $H_v \sim \sigma_y/3$.



Figure 6.4 The hardness test. The Vickers test uses a diamond pyramid; the Rockwell and Brinell tests use a steel sphere.



Figure 6.5 Common hardness scales compared with the yield strength.



Some extra data on hardness (not for exam)

6.3 The big picture: charts for yield strength

- The strength-density chart
- Again the members of each family cluster



Figure 6.6 The strength-density chart.

Example Mohs Question Vickers 9 — Approx. yield strength, MPa Is this the best way to identify Rockwell C 2000 -----1900 — 80 a material? 6000 — 1800 -----**Rockwell A** 1700 — 5500 -----1600 -----90 — 8 — 5000 -----1500 -----1400 -----4500 — 1300 -----4000 -----1200 -----7 — Brinell 1100 -----70 -----3500 -----1000 -----900 -----3000 — 800 -----85 — 6 — 800 — 700 -----2500 -----60 -----700 -----80 -----600 -----600 -----2000 — 5 -----500 -----50 -----75 — 500 -----1500 -----400 -----400 — 70 -----40 -----4 _____ 300 — 30 20 10 65 <u>—</u> 300 -----1000 — 200 — 3 -----200 -----500 ----incorrect in book, p. 117, should be: Rockwell C 2 _____ 100 -----100 — 1 -----0 — 0 -----Example 6.2

An unidentified metal from a competitor's product is measured to have a 'Rockwell A' hardness of 50 and a density of approximately 8800 kg/m³. Estimate its Vickers hardness and its yield strength. Locate its position on the strength–density chart (Figure 6.6). What material is it likely to be?

- Comparing to modulus-density chart
- For a given material, the modulus typically has a narrow range; the strength can span a decade (see metals). Strength is manipulable, modulus hardly.



Figure 6.6 The strength-density chart.

Compare with this one Figure 4.6 The modulus–density chart.

- The modulus-strength chart
- Allows examination of the yield strain, σ_y/E , the strain at which the material ceases to be linearly elastic. On log axes the contours of constant yield strain are a family of parallel lines.



Figure 6.7 The Young's modulus–strength chart. The contours show the strain at the elastic limit, σ_v/E .

6.4 Drilling down: the origins of strength and ductility

- Bonds, like springs, have a stress-strain curve. Remember we approximate σ = F/a₀². Stretch from initial length a₀ to a, giving a strain (a a₀)/a₀.
- The peak strength is the bond strength.
- The bond is 'broken' when stretched more than ~ 10% of original length (ε ~ 0.1).
- Bond stiffness S = F/ δ and δ = $a_0/10$ so F to break a bond is ~ S $a_0/10$.



Figure 6.8 The stress–strain curve for a single atomic bond (it is assumed that each atom occupies a cube of side a_0).

- The "ideal" strength should then be $\sigma_{ideal} \sim F_{max}/a_0^2 = S/10a_0 = E/10$.
- Remembering $E = S_0/a_0$, $\sigma_{ideal}/E \sim 1/10$. More refined: 1/15.

Why don't we observe this? Why are materials in reality so weak?



Figure 6.9 The ideal strength is predicted to be about *E*/15, where *E* is Young's modulus. The figure shows σ_{v}/E with a shaded band at the ideal strength.

- Crystalline imperfection: defects in metals and ceramics
- Often organized by dimensionality: point defects (vacancies, impurities – substitutional, interstitial), dislocations, grain boundaries
- Real materials are riddled with imperfections (with few exceptions) and these defects explain diffusion, strength, conductivity, ...
- Vacancies do not influence strength, others do
- For strength, dislocations are the key players



Figure 6.10 Defects in crystals. (a) Vacancies–missing atoms. (b) Foreign (solute) atom on interstitial and substitutional sites. (c) A dislocation–an extra halfplane of atoms. (d) Grain boundaries. Different colors do not mean different kinds of atoms (except in (b))

- Dislocations and plastic flow
- Engineering materials have strengths << than E/15 explained by the presence of dislocations only in the mid-20th century.
- *Edge* dislocation extra 'half plane'. The dislocation line separates the part of the plane that has slipped from the part that has not.



Figure 6.11 (a) Making a dislocation by cutting, slipping and rejoining bonds across a slip plane.
This type of "making" is not how dislocations are made in reality, but it helps understanding them.
(b) The atom configuration at an edge dislocation in a simple cubic crystal. The configurations in other crystal structures are more complex but the principle remains the same.

 The slip vector (Burger's vector) b describes the direction and magnitude of the relative motion and results in a shear strain γ at the end of the process.



Figure 6.12 An initially perfect crystal is shown in (a). Effect of deformation (shear stress): The passage of the dislocation across the slip plane, shown in the sequence (b), (c) and (d), shears the upper part of the crystal over the lower part by the slip vector **b**. When it leaves the crystal has suffered a shear strain γ.

• The *screw* dislocation – displacement parallel to the edge of the cut rather than perpendicular. Geometry: similar to a corkscrew. As it sweeps through the crystal the lattice is displaced parallel to the line.



- Dislocations can also be *mixed* made up of little segments of both edge and screw – b is fixed while the line wanders.
- It is far easier to move a dislocation through a crystal, breaking and reforming bonds along a line, than simultaneously breaking all the bonds in a plane. Moving a carpet by pushing a fold.

- Dislocations move more easily on some planes and in some directions than others – slip planes, slip directions.
- These planes and directions depend on crystal structure.
- Individual slip displacements are tiny with many dislocations moving on many planes macroscopic shape changes can occur.
- Volume is unchanged.



Figure 6.14 Dislocation motion causes extension.

Why does a shear stress make a dislocation move?

- Crystals resist dislocation motion with a friction-like resistance f per unit length – yielding occurs when an external stress exceeds f.
- Let one dislocation move distance L_2 shifting upper half of crystal by b. The shear τ acts on area L_1L_2 giving shear force $F_s = \tau L_1L_2$. The work done for displacement b: $W = \tau L_1L_2b$.
- The work is done against the resistance f per unit length times the length L₁ over a displacement L₂ giving fL₁L₂.
- Equating the two work terms: $\tau b = f$; if $\tau > f/b$ the dislocations will move.



Figure 6.15 The force on a dislocation. (a) Perspective view. (b) Plan view of slip plane.

Line tension.

- Dislocations attempt to keep their length as short as possible they behave as if they had a line tension, T, like a rubber band.
- The line tension is an energy per unit length (note: an elastic quantity):
- $T \sim Eb^{2}/2$

Where does the lattice resistance come from?

- Contributions to f: f_i the intrinsic resistance of the lattice to shear due to bonding – lower in metals, very high in ceramics. However, metals can be further strengthened while ceramics are prone to fracture.
- Obstacles to slip in metals and alloys: solid solution hardening, \mathbf{f}_{ss} ; precipitation hardening, \mathbf{f}_{ppt} , work hardening via new dislocations, \mathbf{f}_{wh} , and grain boundaries, \mathbf{f}_{ab} .
- As we will see, all can be manipulated.

Plastic flow in polymers

- Above ~ 0.75 T_g polymers become plastic. During *drawing* the chains slide over each other, unravel, and align. The drawn material is stiffer and stronger than before (x 8) but geometries are limited (fibre, sheet).
- PE, PP, and nylon draw at RT; PMMA requires higher T. Drawing at too low T leads to crazing – incipient cracks with ligaments (verbindingen) crossing the craze surfaces – if continued a crack develops followed by fracture.
- Crazing may limit tensile elongation but large plastic strains may be accommodated in compression by shear banding.



6.5 Manipulating strength



(a) Perfect lattice, resistance f_i

Solute atoms

(b) Solution hardening, resistance f_{ss}



Figure 6.17 A 'dislocation-eye' view of the slip plane across which it must move. "Hardening" = providing obstacles for the dislocations.

Strengthening metals

- Impede dislocation motion. Obstacles increase f and improve strength – how much? we need spacing and strength of interaction.
- For a distance between obstacles in the slip plane of L, the number of obstacles touching unit length of dislocation line is $N_L = 1/L$.
- Each obstacle exerts pinning force p, so the contribution to the resistance is f = p/L; the added contribution to the shear is $\Delta \tau = p/bL$.
- The pinning is an elastic effect due to local distortions and p scales as Eb².
- The shear stress needed to force the dislocation through the field of obstacles has the form $\tau = \alpha Eb/L$ where α is dimensionless and characterizes the obstacle strength.



Solution hardening

- Deliberate addition of impurities alloying. Every element has a slightly different size – this locally strains the lattice and and 'roughens' the plane and interferes with dislocation motion.
- Concentration (atom fraction) $c = b^2/L^2$; thus $L = b/c^{1/2}$.
- This gives (through $\tau = \alpha Eb/L$): $\tau_{ss} = \alpha Ec^{1/2}$.



Figure 6.18 (a) Solution hardening.



Figure 6X1. (a) Liquid copper and liquid nickel are completely soluble in each other. (b) Solid coppernickel alloys display complete solid solubility, with copper and nickel atoms occupying random lattice sites. (c) In copper-zinc alloys containing more than 30% Zn, a second phase forms (**'precipitation'**) because of the limited solubility of zinc in copper. Also: Cu in Al \rightarrow CuAl₂ particles.

Dispersion and precipitate strengthening

- Dispersed small, strong particles either in a MMC (e.g. SiC particles in liquid AI) or via an in situ precipitation process (exceeding the solubility limit).
- Dislocation bows between particles it cannot cut to a radius minimum given by L/2 (after that it expands under lower stress).
- The critical configuration is the semicircle where the total force τbL on line segment of length L is just balanced by the force 2T due to line tension. Dislocation escapes when τ_{ppt} = 2T/bL ~ Eb/L.
- Obstacles exert a resistance $f_{ppt} = 2T/L$.



Figure 6.18 (b) Precipitation or dispersion hardening.



Figure 6.19 Successive positions of a dislocation as it bypasses particles that obstruct its motion. The critical configuration is that with the tightest curvature, shown in (b).

Example

Example 6.3

A polycrystalline aluminum alloy contains a dispersion of hard particles of diameter 10^{-8} m and an average centre-to-centre spacing of 6×10^{-8} m, measured in the slip planes. The Young's modulus of aluminum is E = 70 GPa and b = 0.286 nm. Estimate the contribution of the hard particles to the yield strength of the alloy.

Work hardening

• Dislocations are generated by plastic deformation (cause and effect).





- Dislocation density, ρ_d (line length per volume).
- Even a well annealed sample has 10¹⁰m/m³ and when deformed this can reach 10¹⁷m/m³.
- A moving dislocation runs into a forest of dislocations with average spacing $L = \rho_d^{-1/2}$. and creates a 'jog' (**stapje**) of length b in each forest dislocation each exerting a pinning force of p = Eb²/2.
- This yields $\tau_{wh} \sim Eb(\rho_d)^{1/2}/2$





How dislocations are generated by a Frank-Read source (only two obstacles) Figure 6.18 (c) Forest hardening (work hardening).

Example

Example 6.4

A cube of the aluminum in Example 6.3 has a side length 1 cm and a dislocation density of 10^8 mm^{-2} . Assuming a square array of parallel dislocations, estimate: (a) the total length of dislocation in the sample; (b) the distance between dislocations; (c) the contribution of the dislocations to the strength of the crystal.

Grain boundary hardening

- Grain boundaries disrupt dislocation motion.
- New dislocations have to nucleate across the grain boundary, with the same total slip vector b_{tot} as the original b.
- It is found that $\tau_{gb} \sim k_p / (D)^{1/2}$
- Where k_p is the Petch constant and D is the grain size not significant unless the grains approach 1 μ m or smaller.





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Relation between dislocation strength and yield stress

- To first approximation: strengthening mechanisms add up.
- Total yield strength then becomes

 $\tau_{y} = \tau_{i} + \tau_{ss} + \tau_{ppt} + \tau_{wh} + \tau_{gb}$

• One issue remains: we have calculated *one crystal* loaded *in shear*. How about a polycrystal loaded in tension? Some kind of average?

Polycrystalline material

 A uniform tensile stress σ creates a shear on planes at an angle to the tensile axis – dislocations move first on planes where the shear is maximum. The shear stress is given by

 $\tau = Fsin\theta/(A/cos\theta) = \sigma sin\theta cos\theta$ where $\sigma = F/A$. Max: $\tau = \sigma/2$.

- In an aggregate of crystals some will be favorably oriented, at 45°, others will not – this gives a reduction factor of 1.5.
- Result: tensile stress to cause yielding in a polycrystal is about 3 times the shear strength of the single crystal; $\sigma_y = 3\tau_y$.



Figure 6.20 The resolution of stress. A tensile stress σ gives a maximum shear stress $\tau = \sigma/2$ on a plane at 45° to the tensile axis.

Question Can we understand that there is a maximum by simple reasoning?

PI question

E 6.4. The lattice resistance of copper, like that of most FCC metals, is small. When 10% of nickel is dissolved in copper to make a solid solution, the tensile strength of the alloy is 150 MPa. What would you expect the tensile strength of an alloy with 20% nickel to be?

- 1. 75 MPa
- 2. 106 MPa
- 3. 212 MPa
- 4. 300 MPa



$$\tau_{ss} = \alpha Ec^{1/2}$$

 $\sigma_y = 3\tau_y$

Alloy	Typical uses	Solution hardening	Precipitation hardening	Work hardening
Pure Al Pure Cu Cast Al, Mg	Kitchen foil Wire Automotive parts	<i>\\\</i>	✓]]]]]]
Bronze (Cu–Sn), Brass (Cu–Zn) Non-heat-treatable wrought Al Heat-treatable wrought Al	Marine components Ships, cans, structures Aircraft, structures	 	s 555	\
Low-carbon steels	Car bodies, structures, ships, cans	$\sqrt{\sqrt{2}}$		$\sqrt{\sqrt{2}}$
Low alloy steels Stainless steels Cast Ni alloys	Automotive parts, tools Pressure vessels Jet engine turbines	\ \	\	\$ \$\$\$

Table 6.1. Metal alloys with typical applications, indicating the strengthening mechanisms used

Symbols: $\checkmark \checkmark \checkmark =$ Routinely used. $\checkmark =$ Sometimes used.

Note: wrought means built, created

Strength and ductility of alloys

- Manipulating the strength of metals has been the most explored area of materials science.
- Hardening mechanisms are often combined to optimize properties.
- Strength is paid for with ductility.



Figure 6.21 Strengthening mechanisms and the consequent drop in ductility (copper alloys). The mechanisms are frequently combined. The greater the strength, the lower the ductility (the elongation to fracture, ε_f). Strengthening of polymers

- Dislocations are not relevant we think of relative slippage of polymer chain segments or shear of a molecular cluster in a glass.
- Strengthening is achieved by impeding this slippage: blending, drawing, cross-linking, reinforcement.
- *Blend*: mixture of two polymers
- *Drawing*: alignment by stretching (mylar film, fishing line, ...).
- Cross-linking (replace Van der Waals forces with strong bonds): Vulcanized rubber, epoxies.
- *Reinforcement*: cheap fillers or engineered fibers.