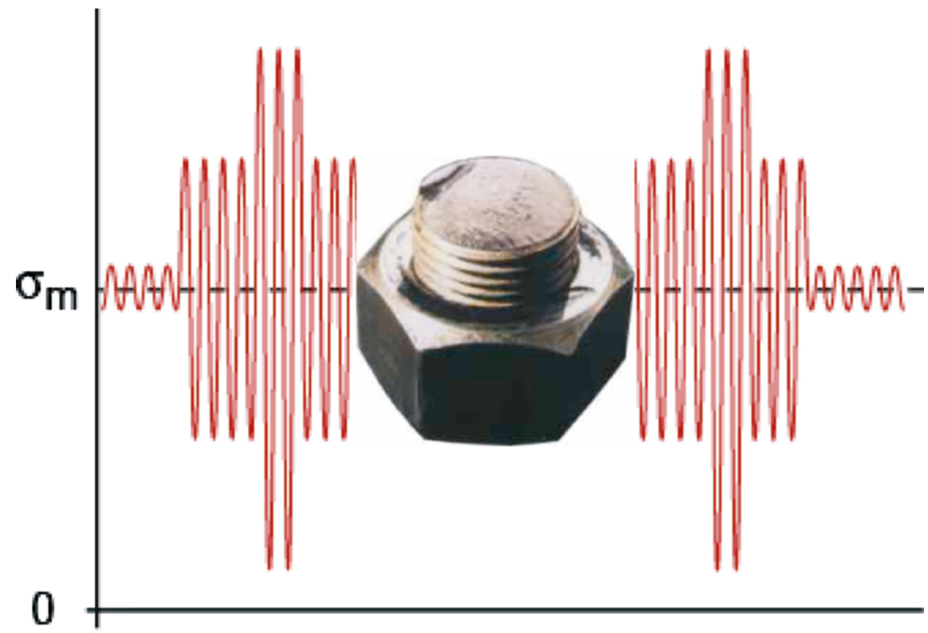
Chapter 9
Shake, rattle and roll: cyclic loading, damage and failure

9.1 Introduction and synopsis

- Fatigue failure is insidious (**sluipend**) – damage accumulates over a long time with little sign that anything is happening. Then sudden failure.
- We review low amplitude cyclic loading, energy dissipation (shown by damping), damage accumulation, and cracking.

A bolt that has failed by fatigue.

The crack initiated at the root of a thread which acts as a notch, concentrating stress.



9.2 Vibration and resonance: the damping coefficient

- Metals, glasses, and ceramics have low material damping (internal friction) while leather, wood, foams, elastomers, and polymers have high damping. We are not talking about plastic deformation here, but about ‘elastic’ deformation.

- We have supposed that elastic loading is completely recoverable – not fully 100% true and becomes especially important for vibrations (lots of load-unload cycles) - energy loss becomes obvious.

- Mechanical loss coefficient (damping coefficient), η (a dimensionless quantity), measures the degree to which a material dissipates vibrational energy. η is the fraction of stored elastic energy not returned on unloading. For a bell we want a small value, for damping we want a large value.

The problem of fatigue

- Low amplitude vibration causes no permanent damage – at higher amplitudes fatigue begins. Cyclic stress can harden the material, causing dislocations to entangle, from which a crack may nucleate.
- Fatigue motions: waves, rotation, reciprocation, vibration – all are risky.
- In the real world the loading cycle is often complex and changes with time – e.g., underside of wing *(f* ~ 1 Hz). During lifetime of a wing millions of cycles occur.

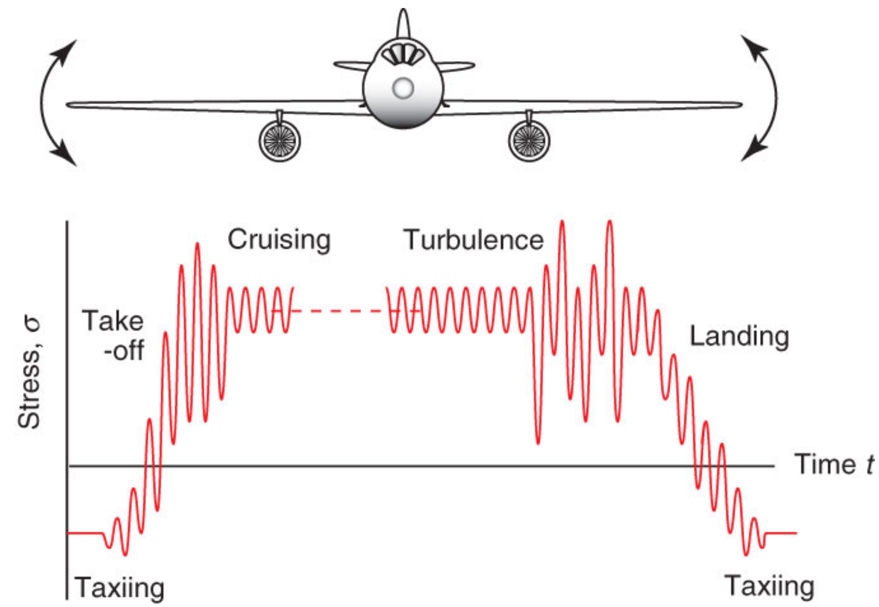


Figure 9.2 Schematic of stress cycling on the underside of a wing during flight.

Question
How can we see that this is the *underside* of a wing?

- Fatigue failure of a metal food can lid, credit card – low cycle fatigue, failure after a small number of cycles (above yield, below tensile).
- High cycle is more significant for most engineering designs – below yield but still leads to failure after many cycles.
- In both cases we consider samples that are initially undamaged – most of the lifetime of the part is spent generating the crack which then fails fairly quickly – *initiation-controlled*.
- If a crack is present or assumed to be present the lifetime is *propagation-controlled*. Different design approach.

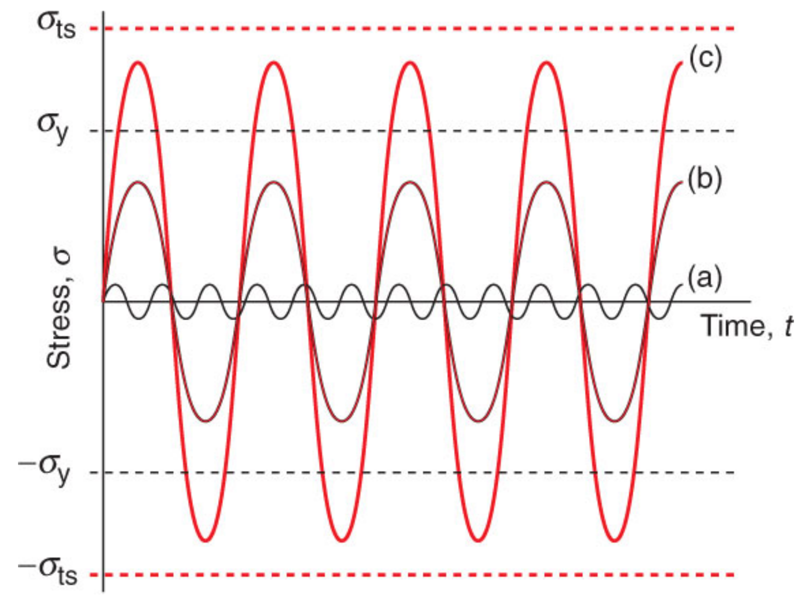


Figure 9.1 Cyclic loading. (a) Very low amplitude acoustic vibration. (b) High-cycle fatigue: cycling well below general yield, σ_y. (c) Low cycle fatigue: cycling above general yield (but below the tensile strength σ_{ts}).

High-cycle fatigue and the S-N curve

- Cyclic stress of amplitude $\Delta\sigma/2$ about the mean σ_m and the number of cycles to fracture N_f is plotted.
- Most tests use a sinusoidal variation with stress amplitude $\sigma_a = \Delta\sigma/2 = (\sigma_{max} - \sigma_{min})/2$ and a mean stress of $\sigma_m = (\sigma_{max} + \sigma_{min})/2$.
- Usually reported for a specified value of $R = \sigma_{min}/\sigma_{max}$
- $R = -1$ means the mean stress is zero; $R = 0$ means the stress cycles from 0 to σ_{max}.

- # cycles to failure increases as the stress amplitude decreases.
- The endurance limit σ_e is the stress amplitude about zero mean stress below which fracture does not occur until at least 10^7 cycles ($N_f > 10^7$).
- In design we treat this as strength-limited but use σ_e instead of σ_y.

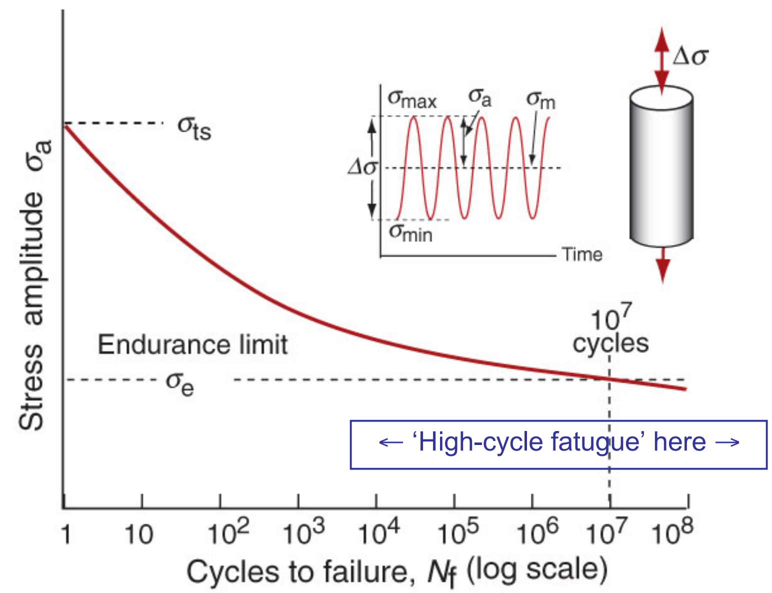


Figure 9.3 Cyclic stress amplitude vs cycles to failure. The fatigue strength at 10^7 cycles is called the endurance limit, σ_e.

- High-cycle fatigue life is approximately given by **Basquin's law**: $\Delta\sigma\, N_f^b = C_1$. C_1 and b are constants with b small (0.07 – 0.13).
- Dividing by E gives the strain range $\Delta\varepsilon = \Delta\sigma/E = (C_1/E)/N_f^b$
- Taking logs: $\log(\Delta\varepsilon) = -b \log(N_f) + \log(C_1/E)$
- When plotted we get the high-cycle fatigue part with slope (–b).

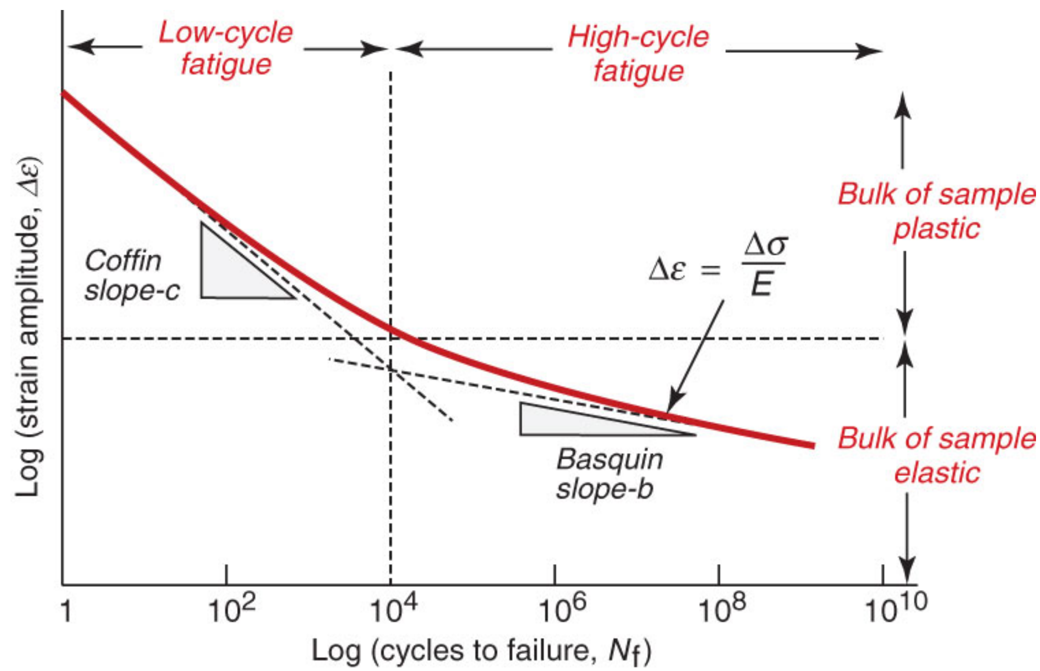


Figure 9.4 The low- and high-cycle regimes of fatigue and their empirical description of fatigue.

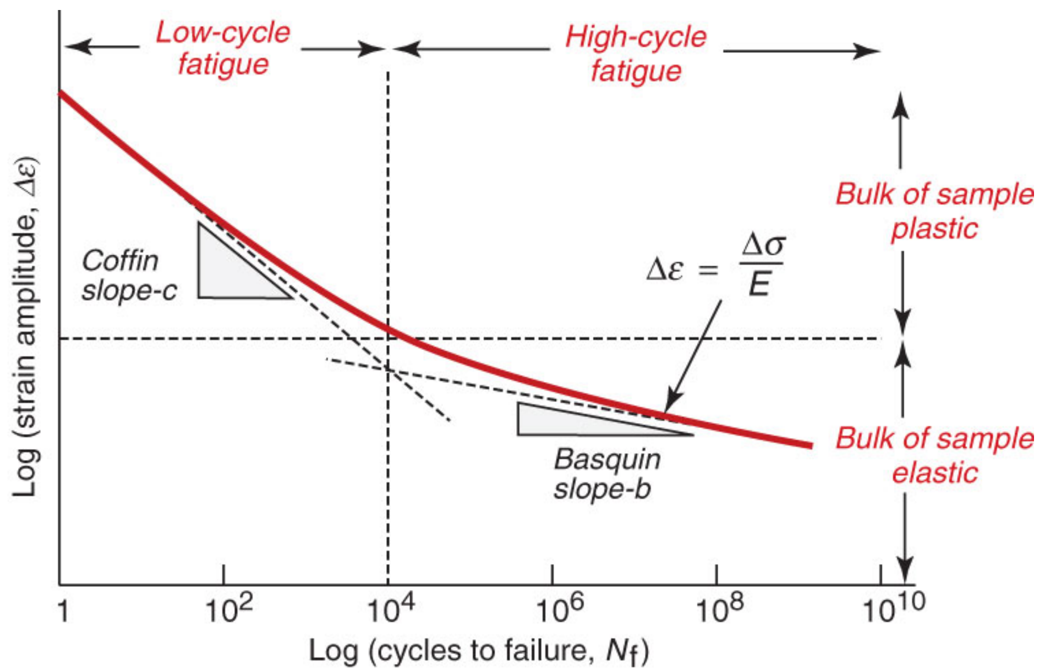
Example

Example 9.1

The fatigue life of a component obeys Basquin's law, equation (9.4), with $b = 0.1$. The component is loaded cyclically with a sinusoidal stress of amplitude 100 MPa (stress *range* of 200 MPa) with zero mean, and has a fatigue life of 200 000 cycles. What will be the fatigue life if the stress amplitude is increased to 120 MPa (stress *range* = 240 MPa)?

Low-cycle fatigue

- Peak stress exceeds yield – entire sample is initially plastic until work hardening raises strength.
- Another empirical law (**Coffin**): the plastic strain range (total minus elastic) $\Delta\varepsilon^{pl} = C_2/N_f^c$; c ~ 0.5
- Ignoring the distinction between strains we can plot as left-hand branch.



- More complex variations? **Goodman** relates stress range for failure $\Delta\sigma_{\sigma m}$ under a mean stress σ_m to that for failure at zero mean stress $\Delta\sigma_{\sigma 0}$.
- $\Delta\sigma_{\sigma m} = \Delta\sigma_{\sigma 0}\ (1 - \sigma_m/\sigma_{ts})$ where σ_{ts} is the tensile stress.
- Increasing σ_m causes a small stress range to be as damaging as a larger one with zero mean. The corrected stress range can be plugged into Basquin.

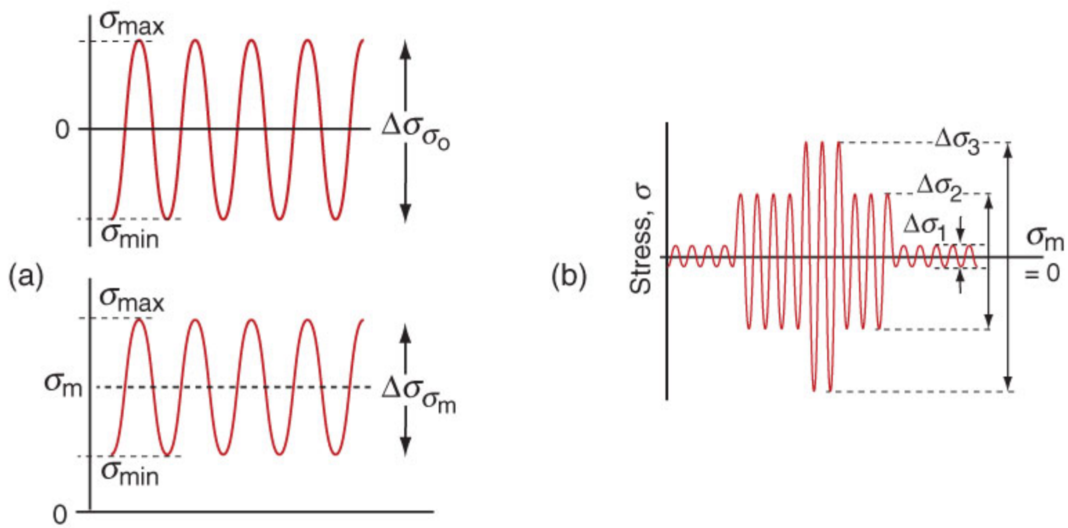


Figure 9.5 (a) The endurance limit refers to a zero mean stress. Goodman's law scales of the stress range to a mean stress σ_m. (b) When the cyclic stress amplitude changes, the life is calculated using Miner's cumulative damage rule.

Example

Example 9.2

The component in Example 9.1 is made of a material with a tensile strength σ_{ts} = 200 MPa. If the mean stress is 50 MPa (instead of zero), and the stress amplitude is 100 MPa, what is the new fatigue life?

- **Miner's rule** of cumulative damage (for variable amplitude situations) assumes damage accumulates at different stress ranges $\Delta\sigma_i$, i = 1..n according to:

  $$\sum_{i=1..n} N_i/N_{f,i} = 1$$

  At each stress level, a fraction $N_i/N_{f,i}$ of the available life is used up.

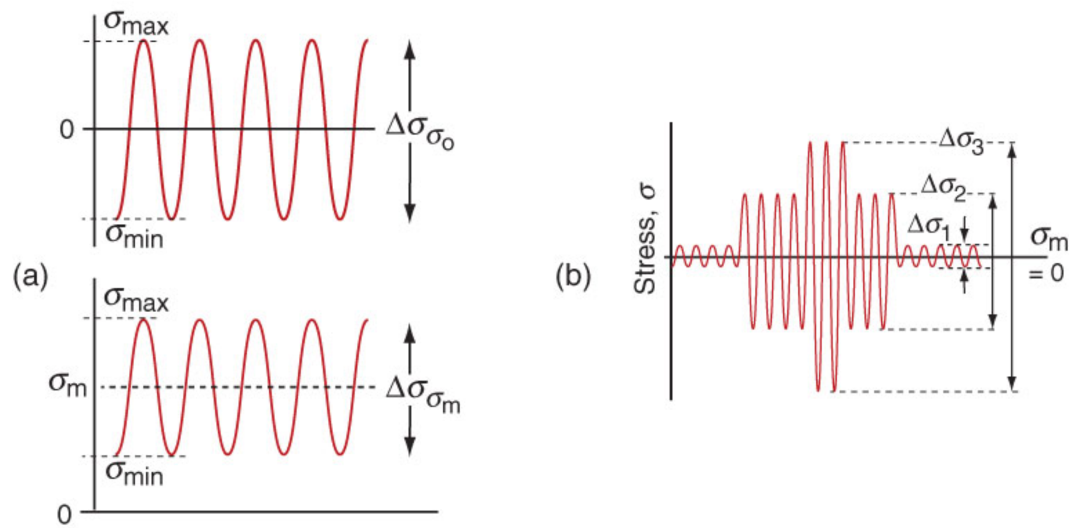


Figure 9.5 (a) The endurance limit refers to a zero mean stress. Goodman's law scales of the stress range to a mean stress σ_m. (b) When the cyclic stress amplitude changes, the life is calculated using Miner's cumulative damage rule.

Example

Example 9.3

The component in Examples 9.1 and 9.2 is loaded for N_1 = 5 000 cycles with a mean stress of 50 MPa and a stress amplitude of 100 MPa (as per Example 9.2). It is then cycled about zero mean for N_2 cycles with the same stress amplitude (as per Example 9.1) until it breaks. Use Miner's rule, equation (9.8), to determine N_2.

NDT = nondestructive testing

Fatigue loading of cracked components

- For many situations (welding, casting) cracks cannot be avoided and NDT methods cannot insure a crack-free sample, just one with cracks below the resolution limit. For critical applications we must design assuming an initial crack.
- We test using pre-cracked specimens.
- Eq. 8.3: Tensile stress intensity factor $K_1 = Y\sigma(\pi c)^{1/2}$ Here: define cyclic stress intensity range $\Delta K = K_{max} - K_{min} = \Delta\sigma(\pi c)^{1/2}$

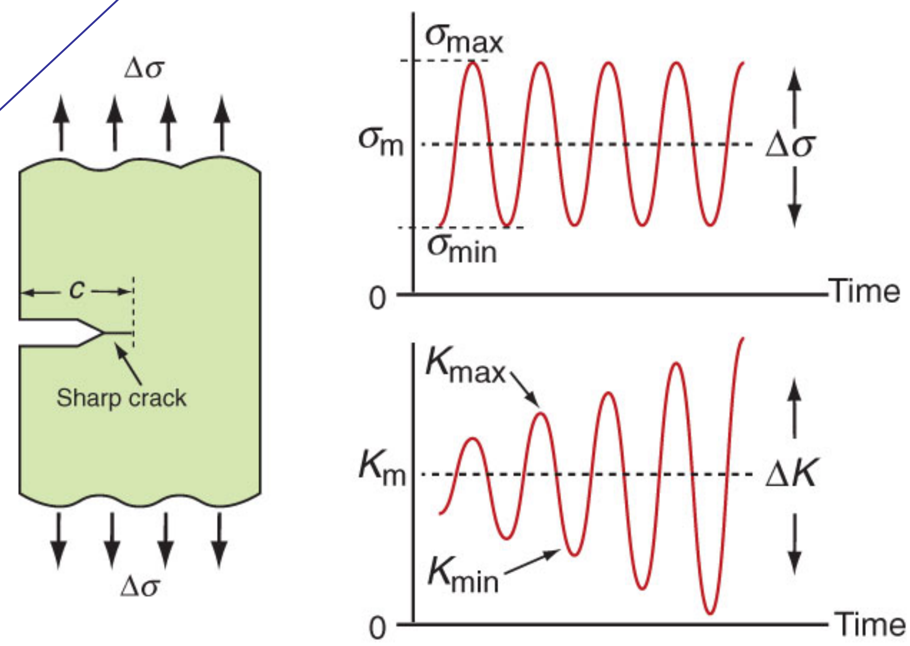


Figure 9.6 Cyclic loading of a cracked component. A constant stress amplitude $\Delta\sigma$ gives an increasing amplitude of stress intensity, $\Delta K = \Delta\sigma\sqrt{\pi c}$ as the crack grows in length.

- ΔK increases with time because the crack grows, $\Delta K = \Delta\sigma(\pi c)^{1/2}$
- The growth per cycle, dc/dN is zero below a threshold ΔK_{th}.
- Above threshold we get steady state (**Paris law**): $dc/dN = A\ \Delta K^m$ where A and m are constants.
- At high ΔK the growth rate accelerates and at $K_{max} = \sigma_{max}(\pi c)^{1/2} = K_{1c}$ the sample fractures in one cycle. (This is Eq. 9.11, not in the book)

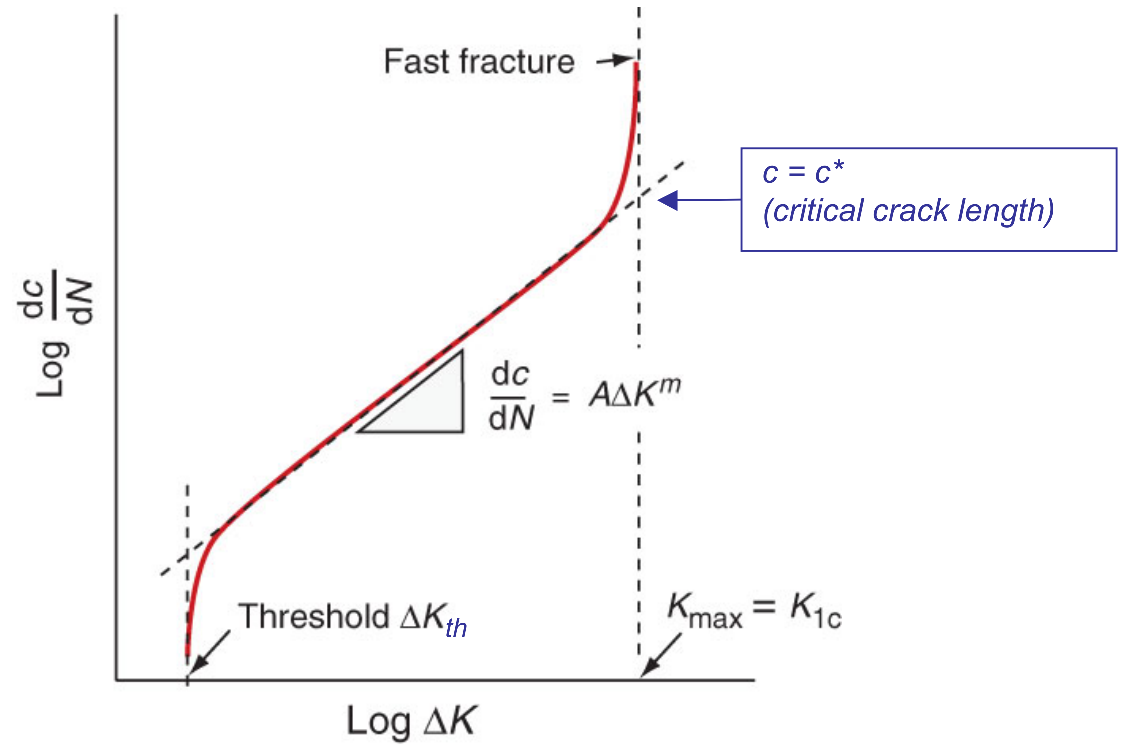
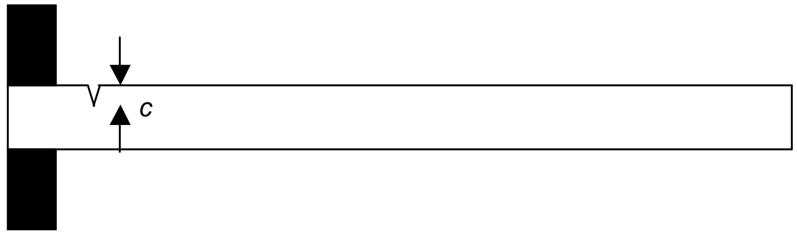


Figure 9.7 Crack growth during cyclic loading.

Example



L = 250 mm

w = 25 mm

t = 4.6 mm

Y = 1.1

From Example 5.2, bottom

Example 9.4

The polystyrene ruler in Example 8.1 is to be loaded as a cantilever with a cyclic force that varies from 0 to 10 N. (a) What depth of transverse crack is needed at the base of the cantilever for fast fracture to occur when the end force is 10 N? (b) If the ruler has an initial transverse scratch of depth c_i = 0.1 mm, how many cycles of the force will it take before the ruler breaks? The Paris law constants in equation (9.10) are $m = 4$ and $A = 5 \times 10^{-6}$ when $\Delta\sigma$ is in MPa. (Assume that the scratch is shallow so that the stress at the crack tip is the same as the stress at the extreme fibre of the beam.)

…before this critical crack depth is reached and the ruler breaks.

And *c* is in m

$dc/dN = A\,\Delta K^m$, Eq. (9.10)

Example

Eq. (8.4):
$K_{1c} = Y\sigma^*(\pi c)^{1/2}$,
or
$K_{1c} = Y\sigma(\pi c^*)^{1/2}$,
depending on which one calls the 'critical' parameter

Answer. (a) The stress in the extreme fibre of the bending beam is given by equation (7.1)

$$\sigma = \frac{Mt/2}{I}, \text{ with } M = FL \text{ and } I = \frac{wt^3}{12}$$

Thus

$$\sigma = \frac{6FL}{wt^2} = \frac{6 \times 10 \times 0.25}{0.025 \times 0.0046^2} = 28.4 \text{ MPa}$$

From equation (8.4), the critical crack length (depth) at fast fracture is

$$c^* = \frac{K_{1c}^2}{\pi Y^2 \sigma^2} = \frac{(10^6)^2}{\pi 1.1^2 (28.4 \times 10^6)^2} = 0.33 \text{ mm}$$

(9.9)

(b) From equation (9.10), the range of stress intensity factor is $\Delta K = Y\Delta\sigma\sqrt{\pi c}$, where $\Delta\sigma = \sigma_{max} - \sigma_{min}$. In this case, $\Delta\sigma = \sigma_{max}$ because $\sigma_{min} = 0$. The crack starts at length c_i and grows steadily according to the Paris law (9.11) $dc/dN = A\Delta K^m$, until it reaches the critical length c^*, when the ruler breaks.

(9.10)

Combining equations (9.10) and (9.11) gives:

$$\frac{dc}{dN} = A\left(Y\sigma_{max}\sqrt{\pi c}\right)^m$$

In this case $\Delta K = K_{max}$, because $K_{min} = 0$

from which

$$\int_0^{N_f} dN = \frac{1}{AY^m \sigma_{max}^m \pi^{m/2}} \int_{c_i}^{c^*} \frac{dc}{c^{m/2}}.$$

where N_f is the number of cycles to failure. Integrating this gives:

$$N_f = \frac{1}{AY^m \sigma_{max}^m \pi^{m/2}(1-m/2)}\left[(c^*)^{1-m/2} - (c_i)^{1-m/2}\right]$$

$$= \frac{1}{5 \times 10^{-6} \times 1.1^4 \times 28.4^4 \pi^2 (-1)}\left[\frac{1}{0.00033} - \frac{1}{0.0001}\right] = 330000$$

9.4 Charts for endurance limit

- Endurance limit and strength are related.
- For metals and polymers $\sigma_e \sim 0.33\ \sigma_{ts}$.
- For ceramics and glasses $\sigma_e \sim 0.9\ \sigma_{ts}$.

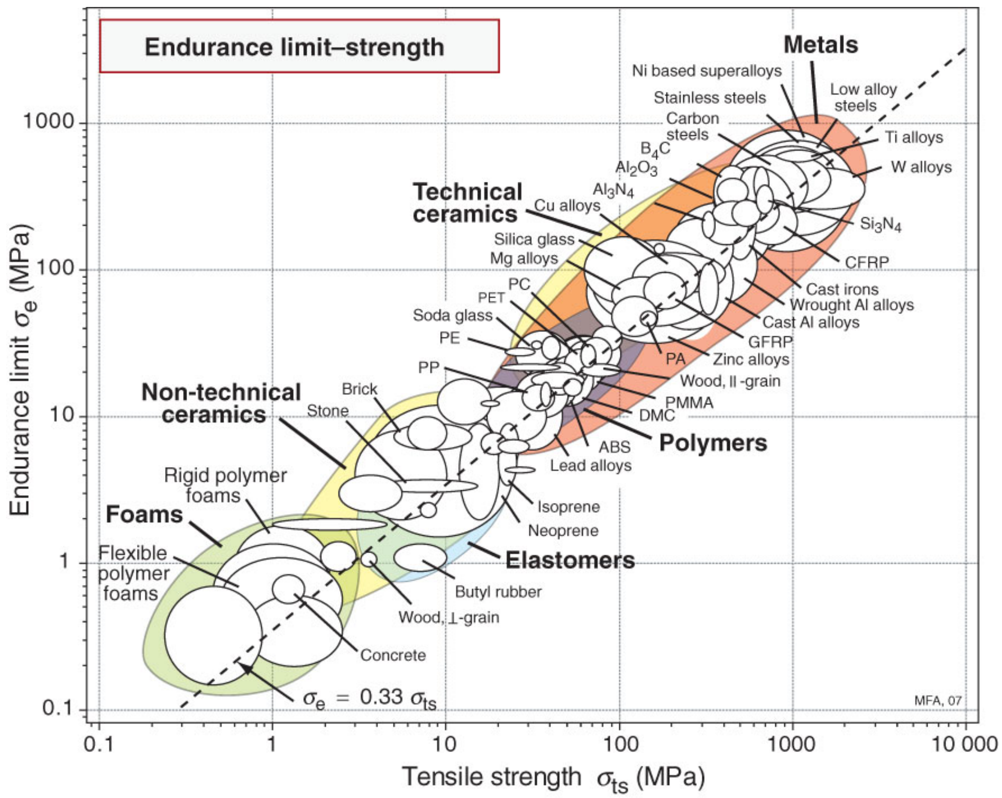


Figure 9.8 The endurance limit plotted against the tensile strength. Almost all materials fail in fatigue at stresses well below the tensile strength.

9.5 Drilling down: the origins of damping and fatigue

Material damping: the mechanical loss coefficient

- Many mechanisms. If process has a specific time constant the loss is centered around a characteristic frequency. Others are frequency independent.
- In metals, largely due to small-scale dislocation motion (damping is high in soft metals, low in heavily alloyed metals where solutes pin dislocations).
- Engineering ceramics – low; porous ceramics high (cracks rub); polymers (chain segments slide against each other) – high above T_g, low below.

Fatigue damage and cracking

- Blemishes (**defecten**) of many types concentrate stress and locally yield eventually creating a crack.
- Striations (**groeven**) on fracture surface are characteristic of fatigue failure and are useful in a ‘forensic’ sense.

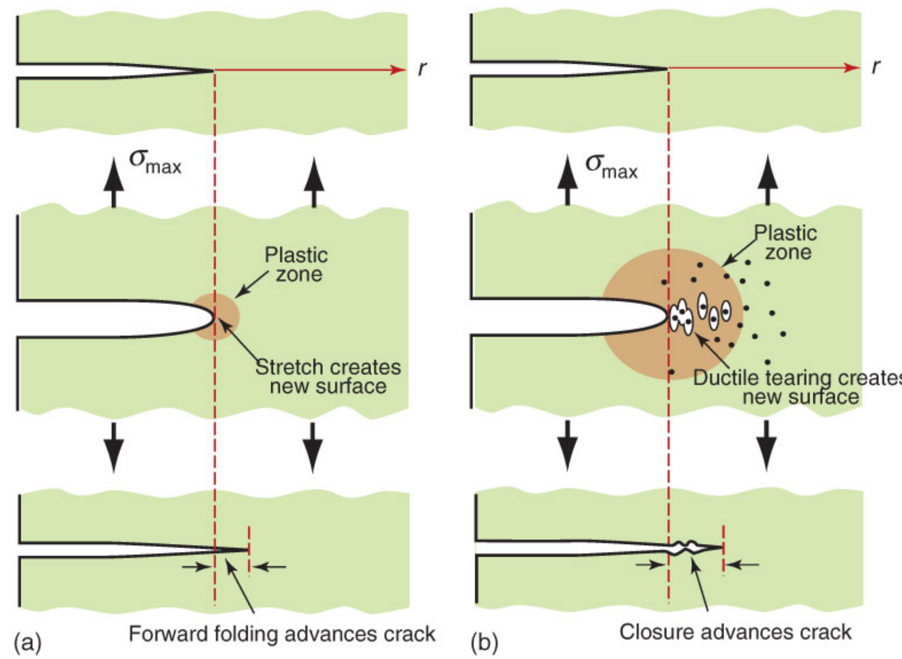


Figure 9.9 (a) In high-cycle fatigue a tiny zone of plasticity forms at the crack tip on each tension cycle; on compression the newly formed surface (blunted tip) folds forwards, generating a striation in each cycle.

(b) In low-cycle (high stress amplitude) fatigue the plastic zone is large enough for voids to nucleate and grow within it. Their coalescence further advances the crack.

9.6 Manipulating resistance to fatigue

- Fatigue life is enhanced by strength, minimum defects, and a surface in compression.

Choosing materials that are strong

- Fatigue ratio, $F_r = \sigma_e/\sigma_y$

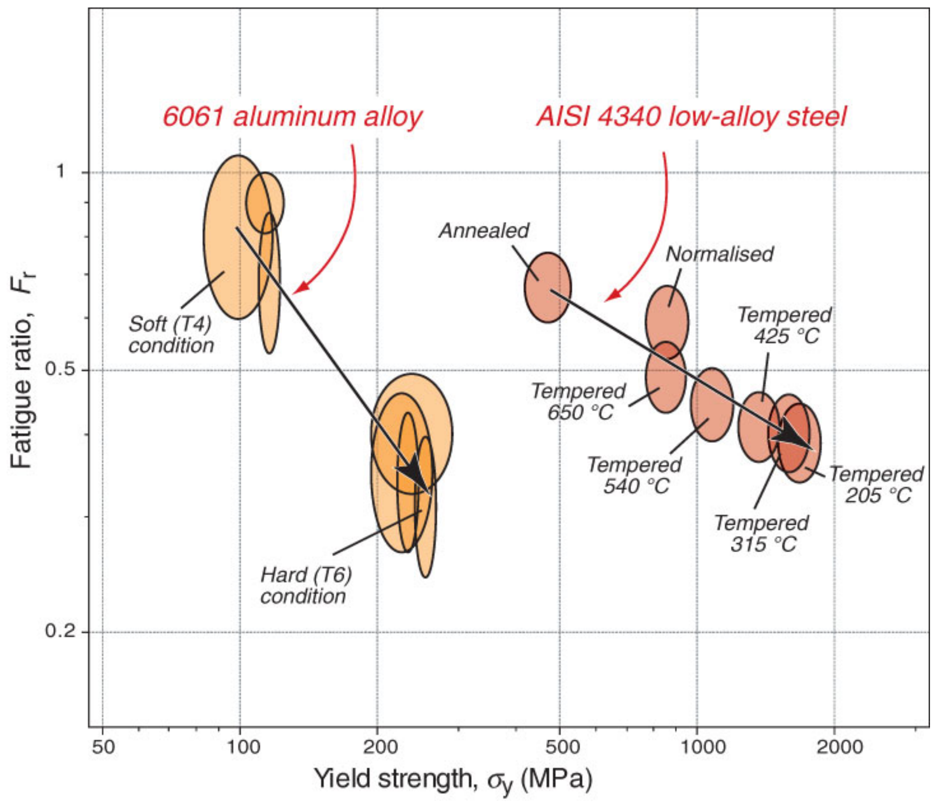


Figure 9.10 The drop in fatigue ratio with increase in yield strength for an aluminum alloy and a steel, both of which can be treated to give a range of strengths.

Gain in endurance limit is less than gain in strength.

Minimizing defects

- Clean alloys, Hot Isostatic Pressing ("HIPing") to seal cracks and collapse porosity.

Compressive surface stress

- Cracks often start at the surface – a compressive stress tends to close cracks; shot-peening **(hameren met kogeltjes),** ion-implantation, diffusion.

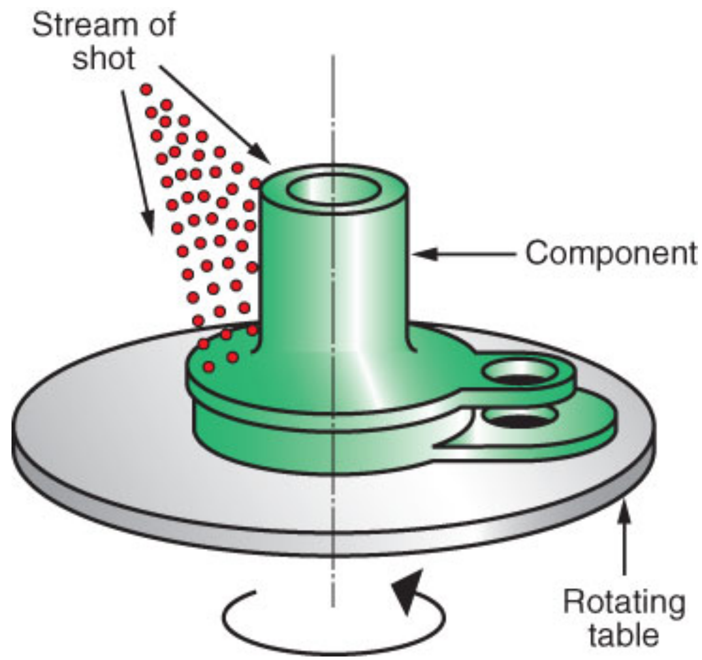
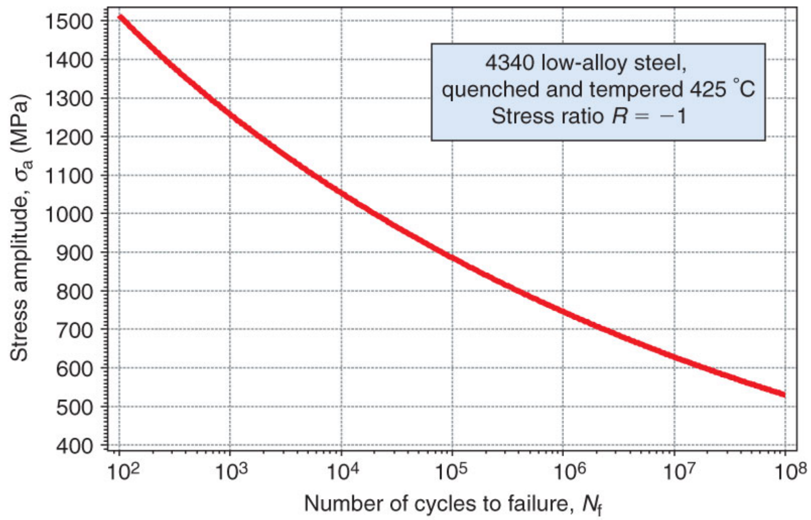


Figure 9.11 Shot peening, one of several ways of creating compressive surface stresses.

Exercise



E 9.4. The figure shows an S–N curve for AISI 4340 steel, hardened to a tensile stress of 1800 MPa.

- What is the endurance limit?
- If cycled for 100 cycles at an amplitude of 1200 MPa and a zero mean stress, will it fail?
- If cycled for 100,000 cycles at an amplitude of 900 MPa and zero mean stress, will it fail?
- If cycled for 100,000 cycles at an amplitude of 800 MPa and a mean stress of 300 MPa, will it fail?