RESIT MEASUREMENT SCIENCE 2012 (EE1320)

(translation of the Dutch exam
of Thursday, Aug. 30, 2012, 9:00h – 12:00h)

This exam consists of three questions, with 5 sub questions each.
All sub questions have the same weight.

At this exam, you may consult the book - only in original form, no printout or copy:
P.P.L Regtien, "Electronic Instrumentation" and / or R.F. Wolffenbuttel,
"Measurement of electrical and non-electrical quantities".

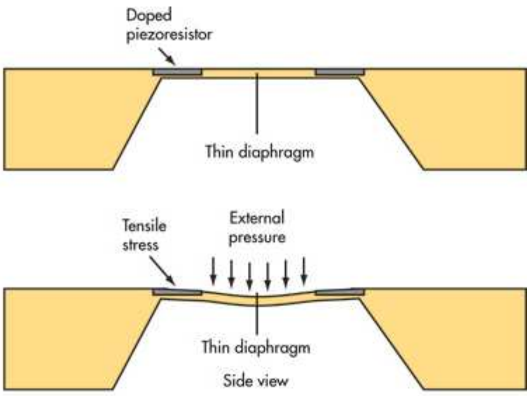
Use of a calculator is permitted.

For every question, give a short motivation for your answer
Start every question on a new answer sheet.
*Write your **name** and **student number** on every sheet.*

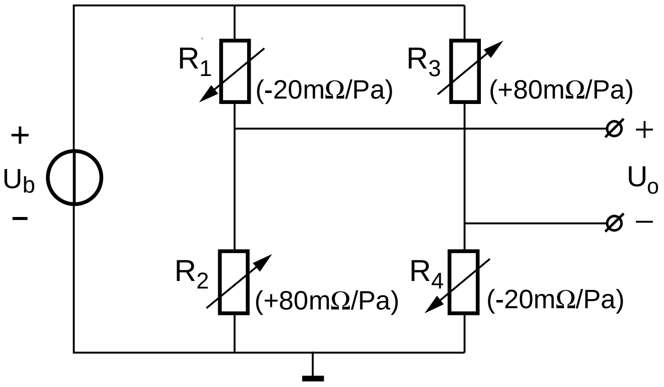
GOOD LUCK!

1. Pressure sensor

A pressure sensor is based on a silicon membrane containing four piezoresistive strain gauges. A cross-section of the sensor is shown on the right (where 2 of the four resistors are visible). When the pressure changes, the membrane will bend, causing the gauges to deform and their resistance to change. We're assuming this change is linearly proportional to the pressure (measured in Pa).



The four resistors are incorporated in a Wheatstone bridge:



At a pressure of 100 kPa, the bridge is balanced and the following holds:
$R_1 = R_2 = R_3 = R_4 = 5.0\ \text{k}\Omega$.

The resistance of R_1 and R_4 decreases at increasing pressure, with 20 mΩ/Pa.
The resistance of R_2 en R_3 increases at increasing pressure, with 80 mΩ/Pa.

a) The sensor is driven by a voltage $U_b = 5.0$ V. Determine the sensitivity of the output voltage U_o (in V/Pa).

For a given pressure difference Δp with respect to 100 kPa, we can write the resistances as: $R_1 = R_4 = 5.0\ \text{k}\Omega - 20\ \text{m}\Omega/\text{Pa} \cdot \Delta p$,
$R_2 = R_3 = 5.0\ \text{k}\Omega + 80\ \text{m}\Omega/\text{Pa} \cdot \Delta p$.

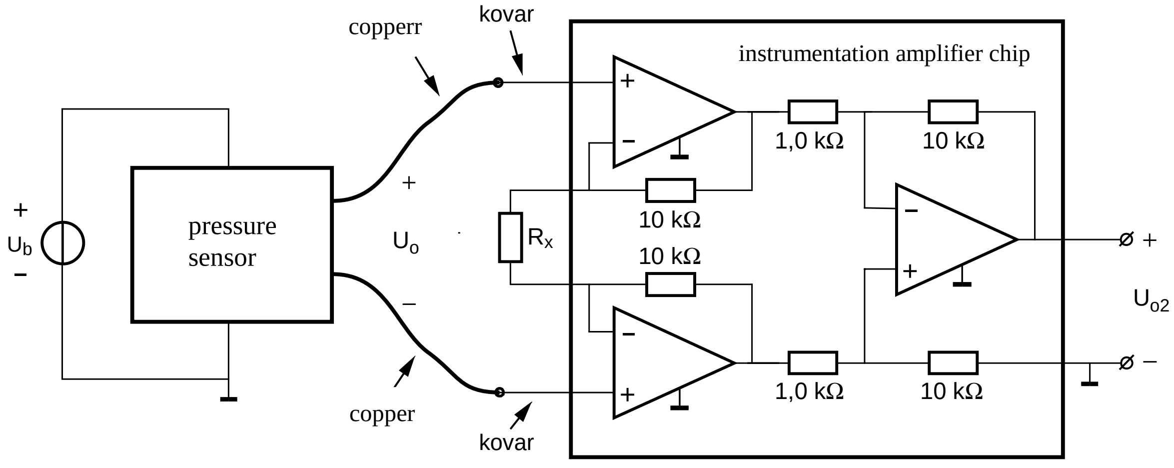
The output voltage is then equal to:

$$U_o = U_b \cdot \{ R_2 / (R_1 + R_2) - R_4 / (R_3 + R_4)$$
$$= U_b \cdot 100\ \text{m}\Omega/\text{Pa} \cdot \Delta p / (10\ \text{k}\Omega + 60\ \text{m}\Omega/\text{Pa} \cdot \Delta p)$$

We can find the sensitivity by taking the derivative of U_o to Δp (around $\Delta p = 0$):

$$S = dU_o / d\Delta p = U_b \cdot 100\ \text{m}\Omega/\text{Pa} / 10\ \text{k}\Omega = \underline{50\ \mu\text{V/Pa}}$$

As shown below, the sensor is connected to an instrumentation amplifier using copper wire. This amplifier is a chip of which the pins are made of the material kovar. It is given that the Seebeck coefficient of a copper-kovar connection equals 50µV/°C. In the following questions, assume that the bridge is driven by U_b = 5.0 V (as before), that the bridge is in balance at 100 kPa (as before), and that the sensitivity of U_o is equal to **2.0µV/Pa** (not the correct answer to question a).



b) Determine the measurement error (in Pa) when there is a temperature difference of 0.1°C between the connector pins of the instrumentation amplifier.

The copper-kovar transitions are thermocouple junctions. When there is no temperature difference between these junctions, the same voltage is across them, which makes the differential voltage at the input of the instrumentation amplifier equal to U_o. When there is a temperature difference, however, there the voltages across the two junctions will differ. Therefore, the differential voltage at the input of the amplifier deviates from U_o, with a measurement error as result.
This error is 0.1°C · 50 µV/°C = 5.0 µV, which corresponds to a measurement error of 5.0 µV / 2.0 µV/Pa = 2.5 Pa.

c) The amplification of the instrumentation amplifier can be set with a resistor R_x. The resistors above may be presumed ideal. Determine the value of R_x at which a pressure of 110 kPa will yield an output voltage U_{o2} of 2.0 V.

At a pressure of 110 kPa the output voltage of the bridge is

$$U_o = (110\text{ kPa} - 100\text{ kPa}) \cdot 2.0\,\mu\text{V/Pa} = 20\text{ mV}$$

Therefore, to obtain an output voltage of 2.0 V, a 100x amplification is needed. The amplification of the 3-opamp instrumentation amplifier is

$$U_{o2} / U_o = (20\text{ k}\Omega + R_x) / R_x \cdot 10\text{ k}\Omega / 1{,}0\text{ k}\Omega$$

Solving gives R_x = 2,2 kΩ.

d) Determine the minimal common-mode rejection ratio (CMRR) of the instrumentation amplifier which is needed to make sure the measurement error due to the common-mode voltage at the output of the bridge is smaller than 10 Pa.

The common-mode voltage at the output of the bridge is equal to half of the source voltage: 2.5 V. At the output, this will give a voltage of $G_c \cdot 2.5$ V, where G_c is the common-mode amplification.

A measurement error of 10 Pa corresponds to a differential voltage $U_o = 10$ Pa $\cdot$ 2.0 µV/Pa = 20 µV. At the output, this gives a voltage $G_d \cdot 20$ µV, where G_d is the differential amplification.

Required is that $G_c \cdot 2.5 \text{ V} < G_d \cdot 20\,\mu\text{V}$, or, the common-mode rejection ratio $H = G_d / G_c > 2.5 \text{ V} / 20\,\mu\text{V} = 1.25 \cdot 10^5$ (102 dB).

e) An ADC will be added to the output signal of the instrumentation amplifier to digitalize the signal once per second, in order to obtain a digital representation of the pressure with a resolution of 1.0 Pa in a range of 10 kPa. If you have the choice between a flash, ADC, a dual-slope ADC and a successively-approximating ADC, which type would you prefer? Concisely motivate your answer.

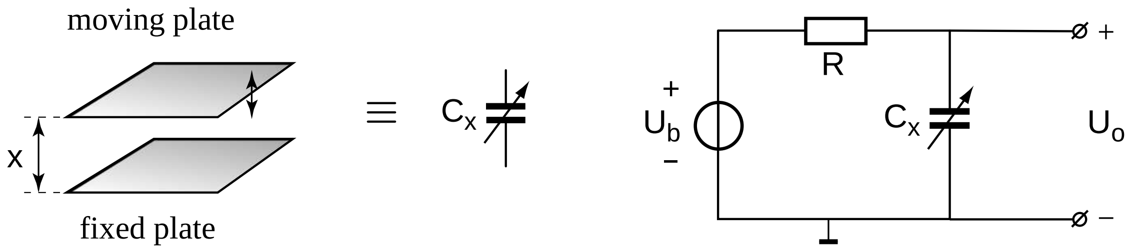
A resolution of 1.0 Pa in a range of 10 kPa is equivalent to $^2\log$ (10 kPa / 1.0 Pa) = 13.3 bits. Therefore a 14-bit ADC is required. Considering that a relatively long measurement time is available, and the required resolution is relatively high, a dual-slope ADC is the most logical choice, with the added advantage of averaging out interference.

A successively-approximating ADC could also be used, however, without the advantage of interference suppression.

With a flash ADC, the number of components (resistors, comparators) depends exponentially on the number of bits, which at 14 bits leads to an impractical complexity. Moreover, flash ADCS are especially known for their quick processing, while this is not required here.

2. Capacitive displacement sensor

The displacement sensor below is based on a parallel plate capacitor of which one of the plates is fixed, and the other is movable. At a distance of x = 1.0 mm the capacitance of the sensor is 100pF. Together with a 10 kΩ resistor R, the sensor forms an RC-filter that is driven by a sinusoidal voltage U_b with an amplitude of 1.0 V and a frequency of 10 MHz. A change in the sensor capacitance C_x changes the corner frequency of this filter. The position of the moving plate can thus be determined from the amplitude of the output signal U_o.



a) Determine the sensitivity of the amplitude of the output signal U_o (in V/mm) for small displacements around x = 1.0 mm.

The sensor capacitance C_x is inversely proportional to the distance x between the plates: $C_x = \varepsilon A/x$, where ε is the dielectric constant of the material between the plates, and A is the surface area of the plates.

The corner frequency of the filter $f_{-3dB} = 1/2\pi RC_x$ is inversely proportional to C_x and hence proportional to x. At x = 1.0 mm this corner frequency is 159 kHz. The source frequency of 10 MHz is much larger, so the output amplitude is in good approximation $U_b \cdot f_{-3dB} / f$. The amplitude at x = 1,0 mm is $U_o \approx$ 1,0 V · 159 kHz / 10 MHz = 16 mV.

This amplitude is proportional to f_{-3dB} and thus to x, so the sensitivity is <u>16 mV/mm</u>.

<u>More formal without approximations</u>: The sensor capacity $C_x = \varepsilon A/x$. The sensitivity of C_x to changes in x is:

$$dC_x / dx = -\varepsilon A/x^2 = -C_x/x$$

The output voltage can be written as $U_o = U_b / (1 + j\omega RC_x)$.
the amplitude of which is $|U_o| = U_b / \sqrt{(1 + (\omega RC_x)^2)}$.
The sensitivity of $|U_o|$ for changes in C_x is:

$$d|U_o| / dC_x = -U_b (\omega R)^2 / (1 + (\omega RC_x)^2)^{1,5}$$
$$= -|U_o| (\omega R)^2 C_x / (1 + (\omega RC_x)^2)$$

The sensitivity of $|U_o|$ for changes is x is therefore:

$$d|U_o| / dx = (d|U_o| / dC_x) \cdot (dC_x / dx)$$
$$= (|U_o|/x) \cdot (\omega RC_x)^2 / (1 + (\omega RC_x)^2)$$

At x = 1.0 mm, $|U_o|$ = 16 mV.
Substituting the numbers: $| dU_o / dx |$ = <u>16 mV/mm</u>.

b) Using a coax cable, the sensor is connected to an oscilloscope. The cable has a capacitance of 50pF. The oscilloscope has an input impedance of 1 MΩ // 20 pF. For x = 1.0 mm, determine the relative error (in %) in the amplitude due to the load on the sensor by the cable and the scope.

The cable capacitance and the input impedance of the oscilloscope are parallel to C_x. At 10 MHz the effect of the 1 MΩ resistor is negligible, so due to the load, the capacitance is effectively enlarged from 100pF to 170 pF. As a consequence, the cross-over point is lowered by a factor 170/100 = 1.7, and thus the amplitude of the output signal is also lowered by a factor 1.7. Therefore, the amplitude is 100% / 1,7 = 59% of the unloaded value, or, an error of – 41%.

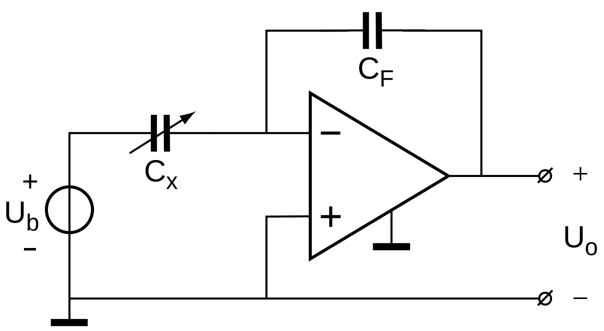
More formal: Unloaded, transfer is $U_o = U_b / (1 + j\omega RC_x)$. The output amplitude at x = 1,0 mm (C_x = 100 pF) is 16 mV.

Loaded, transfer is $U_o = U_b R_s / (R_s + R (1 + j\omega R_s C_{tot}))$, where R_s = 1 MΩ is the resistance of the scope and C_{tot} = 170 pF the total capacity. Substituting the numbers and determining the modulus gives an amplitude of 9.4 mV, or, 41% less.

c) The resistor R has a temperature coefficient of $2.0 \cdot 10^{-5}$ K^{-1}. All other components are temperature independent. The sensor is calibrated at 25°C by measuring the amplitude that corresponds to x = 1.0 mm. Determine the maximum measurement error (in μm) when the sensor is consequently used in a temperature range of -50°C to 125°C.

The largest error will occur at 125°C, because there the temperature difference with respect to the calibration temperature is largest, namely 100°C. The resistance is then $100°C \cdot 2.0 \cdot 10^{5}$ K^{-1} = 0.2% larger. The output amplitude is 0.2% lower. Given that the amplitude is proportional to x, the measurement error will also be 0.2%, or 2,0μm.

Next, the circuit below is used for the sensor readout. The source voltage U_b still has an amplitude of 1.0 V and a frequency of 10 MHz. The opamp in this circuit can be assumed ideal for now. Also, you can assume that the sensor capacitance has a nominal value of 100pF, with a sensitivity of **20 pF/mm**.



d) Determine the value of C_F at which the sensitivity of the amplitude of the output voltage is equal to 10 mV/mm.

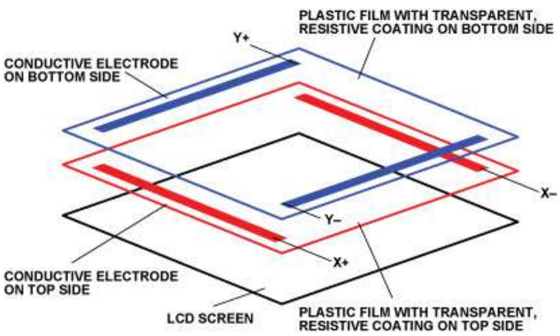
The amplitude of the output voltage U_o is $U_b \cdot C_x / C_F$.
The sensitivity is $\mathrm{d}U_o / \mathrm{d}x = (\mathrm{d}U_o / \mathrm{d}C_x) \cdot (\mathrm{d}C_x / \mathrm{d}x)$
$= (U_b / C_F) \cdot 20$ pF/mm = 10 mV/mm.
Solving yields $\underline{C_F = 2.0\ \text{nF}}$.

e) In practice, the input current of an opamp is not precisely zero: there is a bias current. Explain to what problem this leads in this measurement circuit, and indicate how you can modify the circuit to solve this problem..

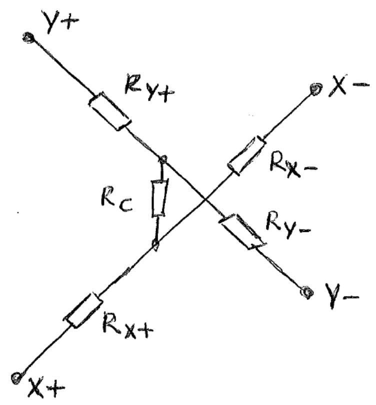
The bias current is integrated on C_F and therefore causes the output voltage to linearly increase with time (or decrease, depending on the polarity of the bias current) until the output of the opamp saturates (due to the limited output range of a practical opamp). Then, a signal can no longer be measured. This problem can be solved by adding a resistor in parallel to C_F. The value of this resistor must be taken large in comparison to the impedance of C_F, so it does not significantly influence the transfer signal, but small enough to prevent the output from being saturated due to the bias current.

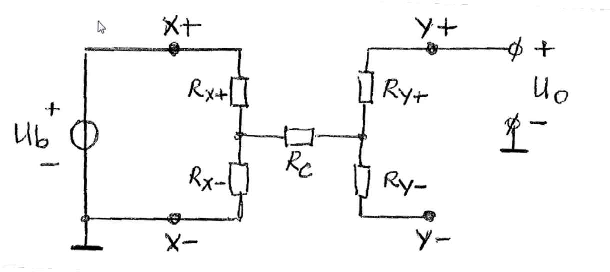
3. Resistive touchscreen

A resistive touchscreem, as used in certain mobile phones, is built up as displayed on the right. It consists of two transparent plastic films, each with a homogeneous resistive layer on top. These layers are electrically connected with conducting electrodes X+, X-, Y+ and Y-. When a pressure is exerted on the foils (for example by pressing on it with a finger), the resistive layers contact each other, creating a resistive divider.



The horizontal position of the contact-point can be determined by applying a voltage U_B across X+ and X- and measuring the resulting voltage on the contact point using one of the remaining electrodes (Y+) , as is displayed below. In this figure, R_{X+}, R_{X-}, R_{Y+}, and R_{Y-} represent the resistance from the different electrodes up to the contact point, and R_C the resistance of the contact point itself.





a) It is given that the distance between the electrodes is 10 cm, and that the total resistance of both resistive layers equals $R_{X+} + R_{X-} = R_{Y+} + R_{Y-} = 2.0$ kΩ. The contact resistance R_C can be neglected. A voltage $U_B = 5.0$ V is applied. How high is the sensitivity of the output voltage U_o for the x-position of the contact point (in V/mm)?

> The output voltage U_o is equal to the voltage on the contact point. If the contact point is at electrode X-, the output voltage is 0 V, if it's at X+, then U_o is equal to U_b. Since the resistance is homogenous between these contact points, the voltage will change linearly with the position between the electrodes. The sensitivity is therefore 5.0 V / 10 cm = 50 mV/mm.

b) The contact resistance R_C is a measure for the pressure which is applied to the touchscreen. Describe how you can measure the resistance R_C through the 4 electrodes of the touch screen, using a current source and a voltmeter.

> For instance, this can be done by connecting the current source to electrodes X+ and Y+, so the current passes through R_{X+}, R_C en R_{Y+}, and connecting the voltage meter to X- and Y-. Since there is no current passing through R_{X-} and R_{Y-} , the voltage meter measures the voltage over R_C. By dividing the measured voltage by the induced current, R_C can be determined.

c) A SAR ADC is used to digitize the measured position of the contact point. The SAR ADC consists of a DAC, a comparator and SAR logic. How should the reference voltage of the DAC be chosen such that the resolution of the ADC is maximized? Concisely motivate your answer.

> In a SAR ADC, the input voltage is compared to the output voltage of the DAC, and a digital code is determined in a number of steps, in which the output voltage of the DAC is matched to the input voltage as closely as possible. The output range of the DAC must therefore equal the range of U_o, that is, 0 V to $U_b = 5.0$ V. To this end, a reference voltage of 5.0V must be chosen.

d) Determine the minimal clock-frequency of the SAR ADC, given that the position must be determined with a resolution of minimally 0.10 mm, in a measurement time of maximally 1.0 ms. You can assume the resolution of the ADC is fully utilized.

> A resolution of 0.10 mm on a range of 10 cm corresponds to
> $^{2}\log$ (10 cm / 0,10 mm) = 10 bits. A10-bit SAR ADC needs 10 clock cycles for a conversion. To realize this within a measurement time of 1.0 ms, a clock frequency of minimally 10 / 1.0 ms = 10 kHz is needed.

e) Say an 8-bit SAR ADC is used, and this is connected such that a contact at position *x*=0 cm corresponds to the minimal output code, and *x*=10 cm to the maximal output code. It is given that the touch screen is touched at position *x*=2.25 cm. During a second comparison of the conversion process, the comparator makes a wrong decision due to a distortion. Determine the measured position.

> In the first step in the SAR algorithm, *x* is compared to 5.0 cm:
>
> 2.25 cm < 5.0 cm ⇒ most significant bit becomes 0.
>
> After this, *x* is compared to 2.5 cm:
>
> 2,25 cm < 2.5 cm ⇒ The next bit should become 0, but (wrongly) becomes 1. As a consequence, all the following comparisons will therefore be between 2.5 and 5.0, causing all the subsequent bits to become 0. The resulting binary code is 0100000, which corresponds to 2.5 cm.

--- END OF EXAM ---