1 Introduction, Level Spacing and Density of States

2a The Schrödinger equation.

b $E_n = \hbar\omega(n + 1/2)$.

c $1/\hbar\omega$. (Note that we cannot normalize this with the length as there is no unique length-scale in this system.)

d The DOS is flat, while for a particle in a box, it decreases like $E^{-1/2}$.

e $L \approx 89.5 \cdot (\hbar/2m\omega)^{1/2}$.

3 For 1D: $k_F = \pi n/2 \approx 1.57/d_{e-e}$.

For 2D: $k_F = (2\pi n)^{1/2} \approx 2.51/d_{e-e}$.

For 3D: $k_F = (3\pi^2 n)^{1/3} \approx 3.09/d_{e-e}$.

4a $k_F = 1.56\,\mathsf{A}^{-1}$ and $E_F = 9.3\,\mathsf{eV}$.

b The bandstructure consists of cones at the edges of the Brillioun zone. The effective mass cannot be calculated using the free-electron model. When the bandstructure is compared to the result for relativistic particles, it follows that they have zero mass.

c $D(E) = 2\frac{|E-E_F|}{v_F^2\hbar^2}$.

5a A metallic material.

b For the 1 nm case: $\Delta E = 7.6\,\mathsf{eV}$ and $E_C = 1.4\,\mathsf{eV}$.
For the 10 nm case: $\Delta E = 0.58\,\mathsf{eV}$ and $E_C = 0.14\,\mathsf{eV}$.

c $k_F = 0.6\,\mathsf{A}^{-1}, E_F = 5.5\,\mathsf{eV}$ and $D(E_F) = 16\ \mathsf{eV}^{-1}\ \mathsf{nm}^{-3}$.

2 Length scales, transport regimes and classical conduction

1a Due to the higher electron concentration.

3 Phase coherent transport

1a $\Phi_0 = h/e = 4.17 \cdot 10^{-15}\,\mathsf{Wb}$

b $B_P = \Phi_0/\frac{1}{4}\pi d^2$.
$B_P = 5.3\,\mathsf{mT}$ for $d = 1\,\mu\mathsf{m}$,
$B_P = 10.4\,\mathsf{T}$ for $d = 20\,\mathsf{nm}$,
$B_P = 1.04 \cdot 10^3\,\mathsf{T}$ for $d = 2\,\mathsf{nm}$.
In the last case only a small part of the period can be observed.

c $B_P = \Phi_0/\frac{1}{4}\pi d^2$.

2a $B_P = \Phi_0/\frac{1}{4}\pi d^2$
for $d_{in} = 0.48\,\mathsf{nm} \rightarrow B_P = 23.0\,\mathsf{mT}$,
for $d_{out} = 0.52\,\mathsf{nm} \rightarrow B_P = 19.6\,\mathsf{mT}$.
The AB effect is larger at low magnetic fields

b The peaks become broadened due to windowing.

c λ_F.

d The two arms are not identical on the order of λ_F.

e Always a maximum.

3a The magnitude of the conductance oscillations are always e^2/h.

b $R_\square = \rho/H$.

c $R = \frac{\rho L}{HW} = R_\square \frac{L}{W} = R_\square N_\square$.

d $\Delta R \approx \left|\frac{\partial R}{\partial G}\right| \Delta G = 0.39\,\Omega = 0.39\%$.

4 Ballistic transport

1a $\lambda_F \ll L \ll l_\phi, l_e$.

b $R_\square = \rho/H$.

c $R = \frac{\rho L}{HW} = R_\square \frac{L}{W} = R_\square N_\square$.

d $\Delta R \approx \left|\frac{\partial R}{\partial G}\right| \Delta G = 0.39\,\Omega = 0.39\%$.

5 Single electron tunneling and Coulomb blockade

1a $E_C = e^2/2C$.

b $0.72\,\mathsf{eV}$.

2a The RC time $\tau = RC$.

b $\delta E = \frac{\hbar}{2RC}$.

c $\delta E \ll E_C$.

d $R \ll \left(\frac{2e^2}{h}\right)^{-1}$.

3a $E_C = 80\,\mathsf{meV}$,
$C_{tot} = 1.0\,\mathsf{aF}$.

b $C_g = 0.09\,\mathsf{aF}$,
$C_d = 0.18\,\mathsf{aF}$,
$C_s = 0.72\,\mathsf{aF}$.

c $\Delta V_g = 0.29\,\mathsf{V}$,
$\Delta Q_g = 2.9 \cdot 10^{-20}\,\mathsf{C} = 0.18 \cdot e$.

d $C_{coupling} = 0.18\,\mathsf{aF}$.

4a The difference is: $U(R) - U(\infty) = e^2/2C$, is positive. This means that is costs energy to place the electron onto the sphere. When calculating the potential energy at every location of the electron, you find that the first electron is actually attracted by the sphere: $U(a) - U(\infty) = -\frac{e^2}{2C}\frac{R^2}{a^2-R^2}$.

b $C = 4\pi\epsilon_0/R$ and $V = -e/C = -eR/4\pi\epsilon_0$.

c $U(R) - U(\infty) = 3 \cdot e^2/2C$.

d $U(R) - U(\infty) = e^2/2C$ for each electron.