Lecture

OFFSHORE HYDROMECHANICS

OE 4620-d

MODULE 4

ch. 12 Wave Forces on Slender Cylinders
ch. 13 Survival Loads on Tower Structures
ch. 14 Sea Bed Boundary Effects

TU Delft

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Chapter 12

Wave Forces on Slender Cylinders

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Basic assumptions & definitions

- Slender cylinder : Cylinder Diameter (D) small relative to Wavelength (λ)

  $D / \lambda < 0.1 - 0.2$

- Unit length of cylinder : Forces / meter [N/m]

- Ambient water motions in immediate vicinity are the same at any instant time

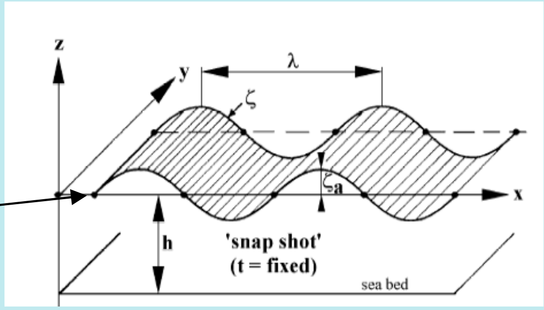
  ➡ Spatial variation near cylinder Neglected

- Flow around cylinder segment is 2-Dimensional

- Flow components and resulting forces parallel to the cylinder axis are Neglected

- Co-ordinate system convention (ch.5) :

still water



Concerning kinematics

Resulting water motions (ch.5) : Based on Potential Flow

$$u = \frac{\partial \Phi_w}{\partial x} = \frac{dx}{dt} = \zeta_a \omega \cdot \frac{\cosh k(h+z)}{\sinh kh} \cdot \cos(kx - \omega t)$$ x-direction (horizontal)

$$w = \frac{\partial \Phi_w}{\partial z} = \frac{dz}{dt} = \zeta_a \omega \cdot \frac{\sinh k(h+z)}{\sinh kh} \cdot \sin(kx - \omega t)$$ z-direction (vertical)

Simplifications :
- location "x" is fixed : kx = 0
- consider vertical cylinder : formula yields desired flow velocity

Undisturbed horizontal flow velocity :

$$u(z,t) = \zeta_a \omega \cdot \frac{\cosh k(h+z)}{\sinh kh} \cdot \cos(-\omega t)$$

.... at chosen elevation z :

Horizontal flow velocity :

$$u(t) = u_a \cos(\omega t)$$

$$\begin{aligned} u_a &= \text{amplitude the wave-generated horizontal} \\ &\quad\ \text{water velocity at elevation } z \text{ (m/s)} \\ \omega &= \text{wave frequency (rad/s)} \end{aligned}$$

Horizontal flow acceleration :

$$\dot{u}(t) = -\omega \; u_a \sin(\omega t)$$

accel. amplitude

$$\dot{u}_a = \omega \; u_a$$

See the phase difference between velocity (*cos*) and Acceleration (*sin*)

Derivations hold for any Undisturbed flow, even if viscosity is involved.

Force Components in Oscillating Flows

Inertia Forces

- D'Alembert's Paradox :
  NO resultant drag force in Time-INdependent potential flow
- Consider : Undisturbed ambient flow
  Time Dependency !!

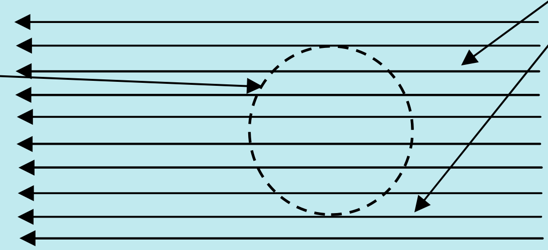
accel.

- Newton's Second Law : Force = mass x acceleration.
- Force in the fluid which causes the horizontal accel. of the flow
- Force itself caused by horizontal Pressure Gradient
- Pressure Gradient for a 'block' of fluid :

$$\frac{dp}{dx} = \rho \frac{du}{dt} = \rho \cdot \dot{u}$$

Pressure Gradient Force

- Virtual Cylinder in the flow
  Flow still **undisturbed**



- Forces from undisturbed pressure field acting on the perimeter
  Integration along the perimeter yields :

$$F_{x1}(t) = 2 \cdot \int_0^{\pi} p(R,\theta,t)\ R\ \cos\theta \cdot 1 \cdot d\theta$$

Undisturbed Pressure [N/m2]

- Simplifications and substitutions lead to :

$$F_{x1}(t) = \rho\ \pi\ R^2 \cdot \dot{u}(t)$$

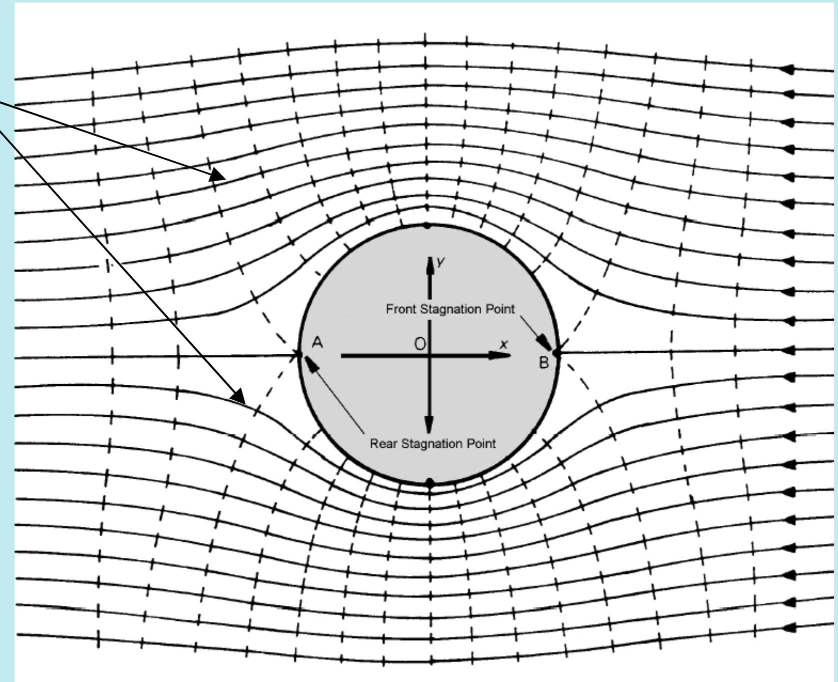
; force = M1 x accel. (*per unit length*)

M1 is the mass within the perimeter.

Force component = Froude Krylov force. *(ch.6)*

Disturbance Force

stream lines



Flow **disturbed** by a (real) cylinder.

Cylinder is 'forcing' the flow to go around the geometry.

Force from cylinder acts on the fluid, and local **velocities** and **accelerations** occur.

To evaluate **this** extra force causing disturbance ->

Difference in Kinetic Energy (E) of the **disturbed and undisturbed** flow field :

$$E = \underbrace{\iint_{cyl.\ wall}^{\infty} \frac{1}{2}\rho \cdot \left[u\left(x,y,t\right)\right]^2 dx \cdot dy}_{\textbf{Disturbed}} - \underbrace{\iint_{cyl.\ wall}^{\infty} \frac{1}{2}\rho \cdot u_{\infty}^2(t) \cdot dx \cdot dy}_{\textbf{Undisturbed}}$$

For convenience, associate to equivalent mass M2, moving with ambient, undisturbed flow velocity :

-> $$E = \frac{1}{2} M_2 \, u_\infty^2$$

M2 is the mass of fluid displaced by the cylinder, *just like M1 !.*

$$M_2 = \pi \ R^2 \ \rho$$

Force due to disturbance :

$$F_{x2} = \pi \ R^2 \ \rho \cdot \dot{u}(t)$$

The F_{x2} force is the result of time dependent flow.

Resultant Inertia Force

Undisturbed Disturbed

$$
\begin{aligned}
F_I(t) &= F_{x1}(t) + F_{x2}(t) \\
&= 2 \cdot \pi \; R^2 \; \rho \cdot \dot{u}(t)
\end{aligned}
$$

In theory !!

Cm : Inertia coefficient

Inertia coefficient : Cm = 1 + Ca.

Disturbed field : value most uncertain due to vortices in the wake !

Undisturbed field : value acceptable.

Ca : Coefficient of added mass ; Ca < 1.

Summarized :

Force Component	Force Term	Experimental Coefficient	Theoretical Value	Experimental Value
Froude-Krylov	F_{x1}	1	1	1
Disturbance	F_{x2}	C_a	1	Usually < 1
Inertia	F_I	C_M	2	Usually 1 to 2

Concerning 'ADDED MASS' phrase

Misleading interpretation : Ca interpreted as a **physical 'Hydrodynamic' mass** of the surrounding fluid.

The right interpretation :

Ca is FORCE per UNIT ACCELERATION, or

Ca ~ Force/UnitAccel.

See 'Keel Clearance' experiment in reader.

Case : Fixed Cylinder in Waves

Regarding the Inertia Force per unit length :

$$
\begin{aligned}
F_I(t) &= F_{x1}(t) + F_{x2}(t) \\
&= \rho \, \frac{\pi}{4} C_M \, D^2 \cdot \dot{u}(t)
\end{aligned}
$$

in which:

$F_I(t)$	=	inertia force per unit cylinder length (N/m)
ρ	=	mass density of the fluid (kg/m^3)
C_M	=	dimensionless inertia coefficient (-)
$\dot{u}(t)$	=	time dependent undisturbed flow acceleration (m/s^2)

Case: Oscillating Cylinder in Still Water

Flow around an oscillating cylinder in still water

is **NOT EQUAL** to

to oscillating flow passing a fixed cylinder

In still water : NO ambient dynamic pressure field.
Therefore the Froude Krylov (F_{x1}) force is zero.

Suppose oscillation movement of cylinder is : $\dot{X}(t) = a \cos(\omega t)$

The resultant hydrodynamic force on the cylinder is :

$$F_I(t) = -F_{x2}(t) = -C_a \cdot \pi R^2 \rho \cdot \ddot{X}(t)$$

Minus sign : Force is opposite to direction of acceleration.

Force Components in Oscillating Flows

Discussed Inertia Forces, now …. **Drag Forces**

For a time-dependent flow :

$$F_D(t) = \frac{1}{2}\,\rho\; C_D\; D\; u_a^2 \cdot \cos(\omega t)\,|\cos(\omega t)|$$

$F_D(t)$	=	drag force per unit length of cylinder (N/m)
C_D	=	dimensionless drag coefficient (-)
D	=	cylinder diameter (m)
u_a	=	water velocity amplitude (m/s)
ω	=	circular water oscillation frequency (rad/s)
t	=	time (s)

Values of C_D for constant flow expected to be different than for time-dependent flow.

Morison Equation

Superimposing the Inertia Force and the Drag Force leads to Resultant force :

(Per Unit Length)

$$F(t) = F_{inertia}(t) + F_{drag}(t)$$

MORISON equation:

$$\boxed{F(t) = \frac{\pi}{4}\,\rho\, C_M D^2 \cdot \dot{u}(t) + \frac{1}{2}\,\rho\, C_D D \cdot u(t)\,|u(t)|}$$

In an oscillatory motion the velocity and acceleration phase are 90 degrees shifted to each other. So do the inertia and drag forces.

Morison Equation Coefficient Determination

Assume a vertical cylinder fixed in a horizontal sinusoidal oscillatory flow.

The force per unit length can be predicted using Morison :

$$\boxed{F(t) = \frac{\pi}{4}\,\rho\, C_M D^2 \cdot \dot{u}(t) + \frac{1}{2}\,\rho\, C_D D \cdot u(t)\,|u(t)|}$$

The two empirical dimensionless force coefficients C_D and C_M can be determined by performing experiments.

To be recorded :

1- the time, t.
2- the force as a function of time, F(t).
3- the flow characteristic: e.g. wave height (h(t))
or velocity (u(t))

Experimental Set Up

Wave Tank at Ship Hydromechanics (WbMt)

Methods to determine Cd and Cm, or Ca :

- Morison's Method
- Fourier Series Approach
- Least Squares Method
- Weighted Least Squares Method
- Alternative Approach

Morison’s Method

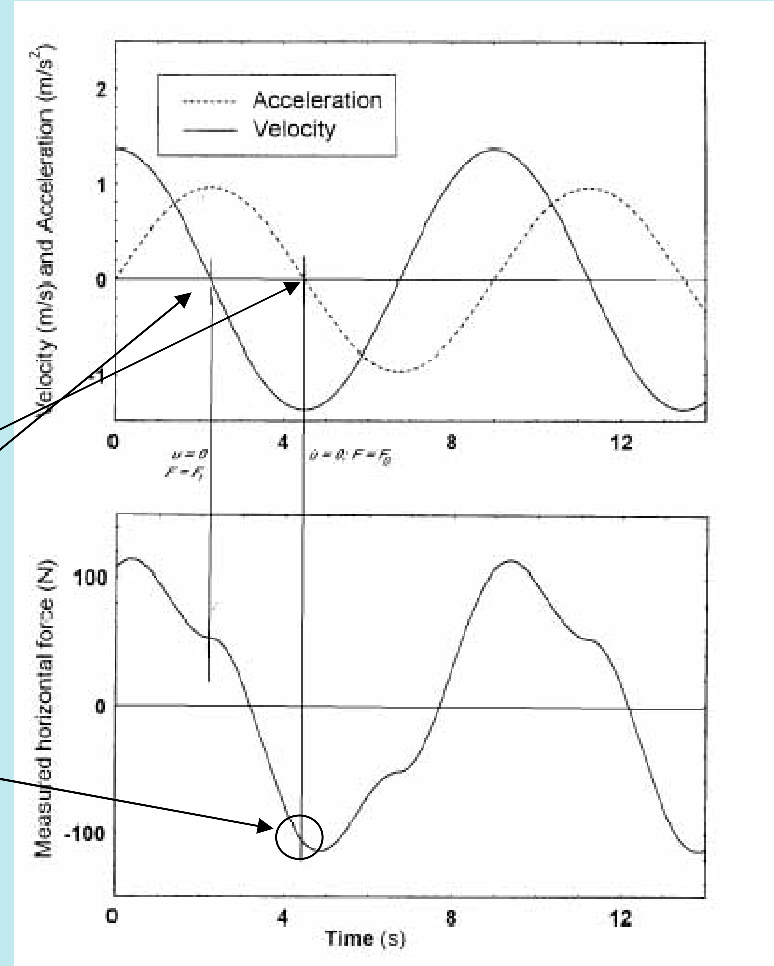
Approach :
- when u(t1) is Max, then d/dt u(t1) is Zero,
  -> F(t1) = F_D
- when d/dt u(t2) is Max, then u(t2) is Zero,
  -> F(t2) = F_I
- This yields :

$$C_D = \frac{2F}{\rho D \cdot u_a |u_a|} \quad \text{at an instant } t_1 \text{ when } \dot{u} = 0$$

$$C_M = \frac{4F}{\pi\, \rho\, D^2 \cdot \omega\, u_a} \quad \text{at an instant } t_2 \text{ when } u = 0$$

IF Small error in velocity record
-> large effect on F_D (t) due to steepness

Errors can be reduced by performing large amount of measurements and take the average value.



Fourier Series Approach

Any Time-dependent, periodic (T) signal ***F(t)***
can be expressed as :

$$F(t) = a_0 + \sum_{n=1}^{\infty} \left[a_n \cos(n\omega t) + b_n \sin(n\omega t) \right]$$

With :

$$\begin{array}{lcl}
F(t) & = & \text{Arbitray periodic function} \\
a_n & = & \text{Coefficients; } n = 0, 1, 2, \ldots \\
b_n & = & \text{Coefficients; } n = \quad 1, 2, \ldots \\
n & = & \text{An integer} \\
t & = & \text{Time} \\
\omega = 2\pi/T & = & \text{Frequency} \\
T & = & \text{Period of the function}
\end{array}$$

Fourier Series Approach

From Chapter 5,
and change in u_a over the altitude neglected:

Velocity : $u(t) = u_a \cdot \cos(\omega t)$

Acceleration : $\frac{d}{dt} u(t) = -\omega\, u_a \sin(\omega t)$

Linearization of periodic signal (n=1) is sufficient.

Series development of the quadratic DRAG term requires function of the form :

$$F(t) = A\cos(\omega t) \cdot |\cos(\omega t)|$$

Dominance

5 different methods to determine Cd, Cm coefficients could give different values !

Exact value of Cd and Cm impossible to determine. Tolerance of a few Percent is at best.

Widely varying values can be obtained due to dominance of the inertia or drag force.

E.g. Inertia dominance of F(t) :

- Drag force relatively unimportant
- Information to calculate F_D is relatively small

Presentation Parameters

C_D and C_M are the (flow) ***dependent*** variables,
what are the ***INdependent*** variables to represent the flow condition ?

Reynolds number, for unsteady flow :

$$Rn = \frac{u_a \cdot D}{\nu}$$

Flow velocity amplitude

Kinem. Viscosity [m2/s]

Keulegan Carpenter number, oscillating flow :

(Most realistic and useful)

$$\boxed{KC = \frac{u_a \cdot T}{D}}$$

Oscillating Flow period [s]

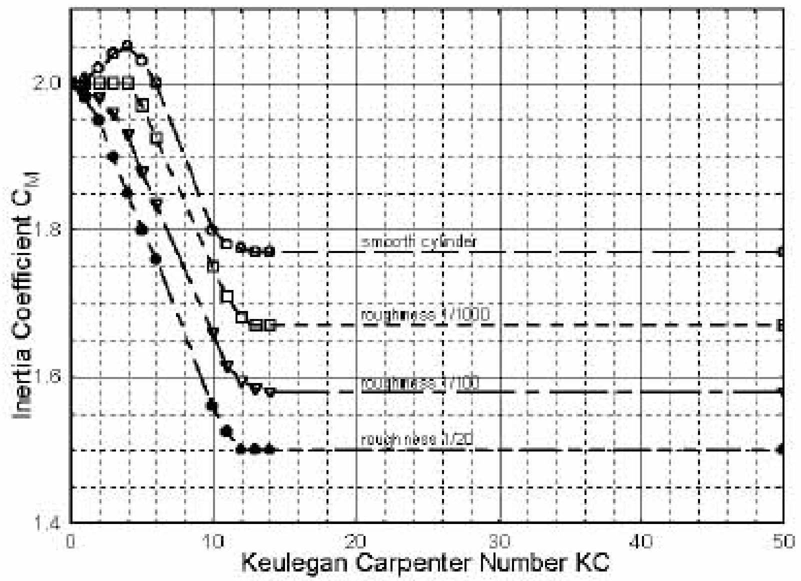
For sinusoidal wave :

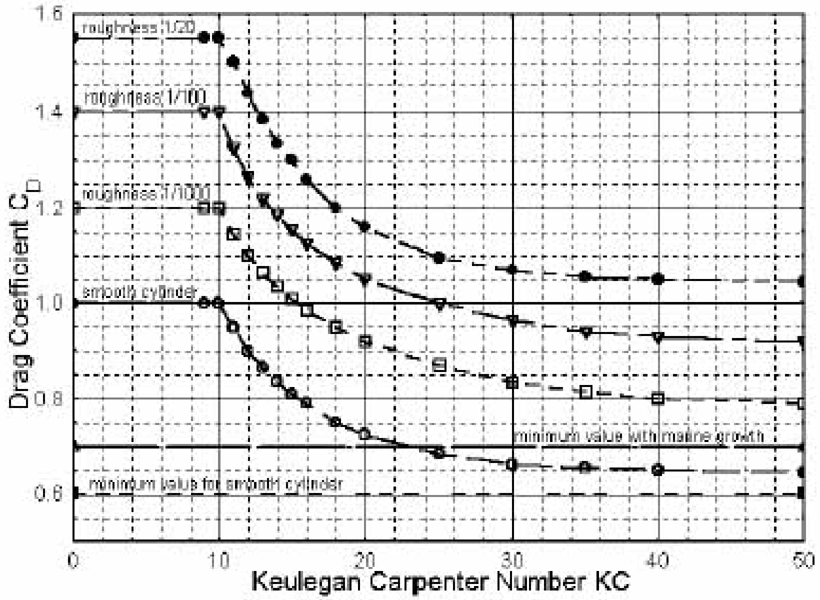
$$KC = 2\pi \cdot \frac{\text{water displacement amplitude}}{\text{cylinder diameter}} = 2\pi \frac{x_a}{D}$$

For deep water :

$$KC = \pi \cdot \frac{H}{D} = 2\pi \cdot \frac{\zeta_a}{D} \quad \text{(deep water only)}$$

Typical Coefficient values, suggested by DNV for design purposes.





Roughness (dim.less.) : $\frac{\varepsilon}{D} = \frac{\text{roughness height}}{\text{cylinder diameter}}$

Cd and Cm Values from various design codes differs up to 30-40% .
Less for extreme wave condition which is significant for Survival Load calculations.

Inertia or Drag dominance

The KC-number can be utilized as an indication of the relative importance of **Drag** versus **Inertia** forces in a particular situation.

Comparing the force component Amplitudes of Fd and Fi :

$$\frac{F_{drag_a}}{F_{inertia_a}} = \frac{1}{\pi^2} \cdot \frac{C_D}{C_M} \cdot \frac{u_a \cdot T}{D} = \frac{1}{\pi^2} \cdot \frac{C_D}{C_M} \cdot KC$$

KC < 3 : Inertia Dominance , Drag neglected.
3 < KC < 15 : Linearize the Drag.
15 < KC < 45 : full Morison Equation (non linear Drag !).
KC > 45 : Drag force is Dominant, Inertia neglected, near uniform flow.
KC -> infin. : constant current.

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Forces on A Fixed Cylinder in Various Flows

Current Alone

Current force acting perpendicular on the cylinder :

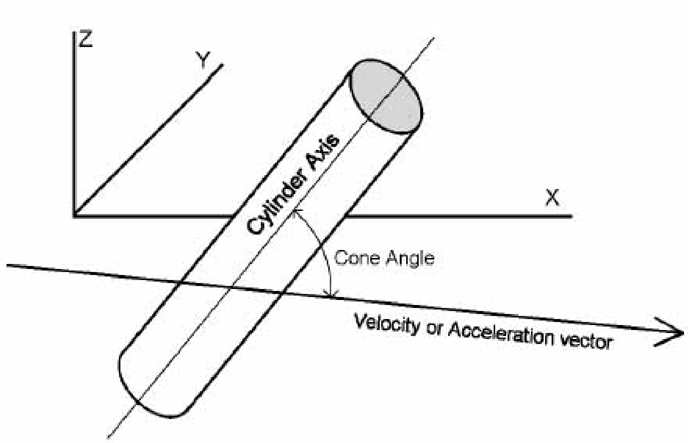
$$F_c = \frac{1}{2}\,\rho\,U^2 D\,C_D \sin^2\kappa$$

(only the Quadratic Drag Force)

U_p	=	Perpendicular velocity component (m/s)
C_D	=	Drag coefficient for constant current (-)
κ	=	Cone angle between the velocity vector, U, and the cylinder axis.
F_c	=	Current force per unit cylinder length (N/m)

Cylinder can be horizontal, vertical …. etc.

Sign convention :



Waves Alone

Basic Morison eq.

$$F = \overbrace{\frac{1}{4}\,\pi\,\rho\,D^2\,C_M\,\dot{u}(t)}^{\text{Inertia Force}} + \overbrace{\frac{1}{2}\,\rho\,C_D\,D\,u(t)\,|u(t)|}^{\text{Drag Force}}$$

F	=	Force per unit length of cylinder (N/m)
D	=	Cylinder diameter (m)
$u(t)$	=	Horizontal velocity component (m/s)
$\dot{u}(t)$	=	Horizontal acceleration component (m/s^2)

To determine the force on a cylinder or structure's member the water acceleration and velocity have to be known at any time.

Especially when the wave is irregular.

Recipe for Non-Vertical position of cylinder :

- Determ. instantaneous **acceleration and velocity** in a fixed co-ordin. syst.
- Determ. instantaneous **cone angles** for the accel. (κ_I) and veloc. (κ_D)
- Determ. instantaneous **perpendicular** component of accel. (d/dt Up) and veloc. (Up). Not generally colinear. In the plane perpendicular to cylind. axis.
- Determ. Inertia (F_I) and Drag (F_D) forces at each instant.
- Integrate Inertia and Drag force components separate over the member's (beam) length.
- Resulting Force magnitude and direction: vector addition of F_I and F_D.

Careful 'bookkeeping' is essential, especially for complex structures and irregular waves.

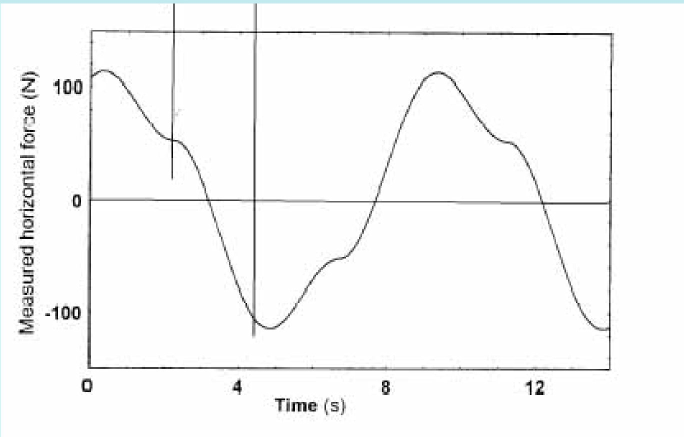
Special Orientations

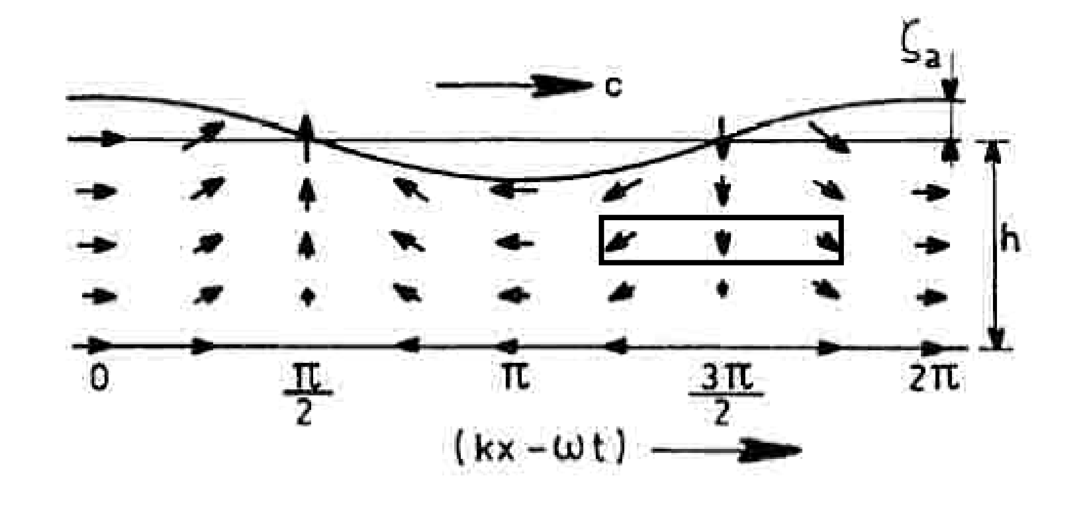
Cylinder : - horizontal

- parallel to wave propagation (perpendicular to wave crests)

Force :

- trace is equal as vertical cylinder case.
- phase is 90 degrees shifted !
- relative phases of Fd and Fi of consecutive segments corresp. to wave profile.



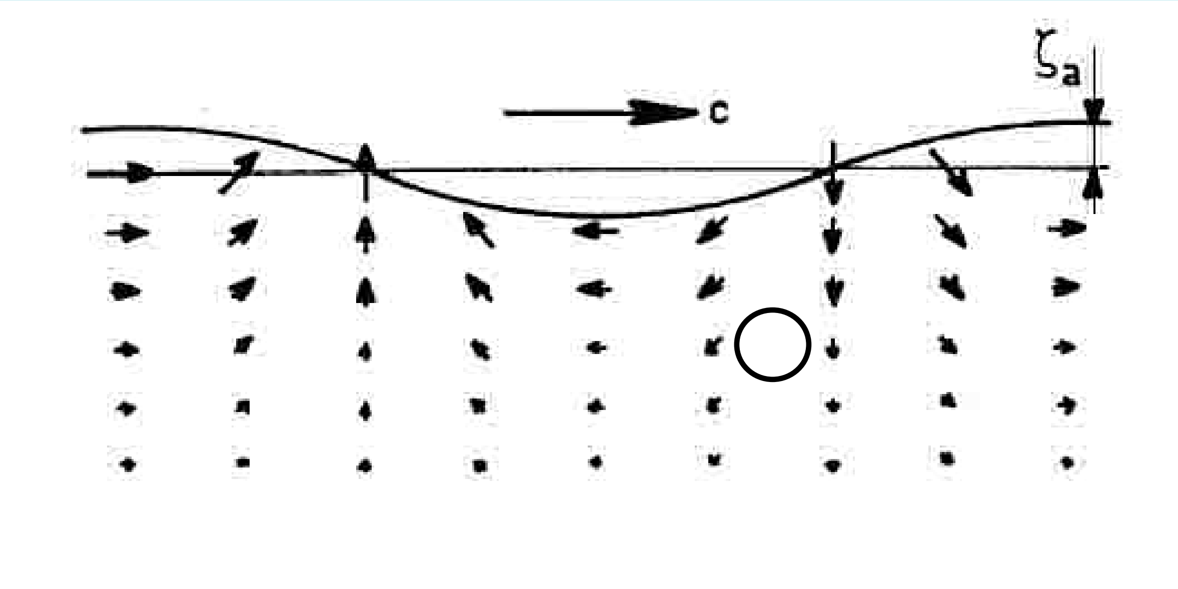


Cylinder : - horizontal

- parallel to wave crests (perpendicular to wave propagation)
- in Deep water

Force :

- Horiz. and Vertic force components have the same magnitude ! (ch5.)
- Resultant force sweeps around the cylinder once per wave period
- Horiz. force component has sinusoidal form, despite the quadratic drag !



Current plus Waves

DO NOT compute the Wave and Current forces separately !!

OTHERWISE : quadratic Drag Force will be underestimated, due to the following :

$$U_p^2 + u_p^2 < (U_p + u_p)^2$$

Calculate the Drag Force after vectorially superpose the current and wave velocity.

Current does not contribute to Inertia Force.

Forces on An Oscillating Cylinder in Various Flows

"Moving Cylinder in various Flows"

Distinction :

External Force : Exerted by the cylinder on the surrounding water.
Internal Force : Needed to oscillate the cylinder.

Internal/Structural force often measured in lab tests -> measures the hydrodynamic force, but ALSO the force to accelerate the cylinder itself !

Still Water

No ambient pressure gradient in still water -> No Froude Krylov force

-> $C_M = C_A$

Inertia force associated with C_A ;
Drag force associated with C_D

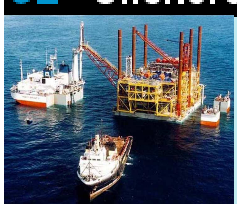
Current Alone

Vortex-induced cylinder vibration, due to the lift force, usually has it's largest component perpendicular to the current direction.

No ambient time-dependent pressure gradient : Inertia force, association to Ca.

Ocillating cylinders are rather slender (e.g. cable to ROC) ->
KC numebr is large -> Inertia forces small.

In many situations only the drag force to be considered.



Waves Alone

Inertia Forces

Waves contribute to :

- Froude Krylov force term (1)
- Disturbance force term (Ca)

Cylinder oscillations contribute to :

- Only disturbance term (Ca)

Simplification : motion cylinder is small wrt. the wave length

-> No phase change due to this movement.

The Equation of Motion :

NOT a true equality : only selected terms are included.

$$M\,\ddot{X}(t) \boxminus \underbrace{C_M\,M_D\,\dot{u}_p(t)}_{\text{Wave Inertia Force}} - \underbrace{C_a\,M_D\,\ddot{X}(t)}_{\text{Cylinder Inertia Force}}$$

$$\begin{array}{lcl}
M & = & \text{Mass of the cylinder segment (kg/m)} \\
M_D & = & \text{Displaced water mass } = \frac{\pi}{4} D^2 \rho \text{ (kg/m)} \\
\dot{u}_p(t) & = & \text{Perpendicular acceleration component from the waves (m/s}^2\text{)} \\
\ddot{X}(t) & = & \text{Cylinder acceleration (m/s}^2\text{)}
\end{array}$$

In general form :

$$(M + C_a\,M_D)\,\ddot{X}(t) \boxminus C_M\,M_D\,\dot{u}_p(t)$$

In case both the cylinder and wave accelerations have the same magnitude and direction :

$$M\ \ddot{X}(t) \boxminus 1\ M_D\ \dot{u}_p(t)$$

No disturbance at all, only Froude Krylov force remains.

Waves Alone

Drag Forces

Drag Forces result from :

- Flow disturbance
- Wake near the cylinder

Two different approaches to describe and calculate the drag forces :

- Relative Velocity Approach
- Absolute Velocity Approach

Relative Velocity Approach

The relative velocity of the water to the moving cylinder : $u - \dot{X}$

The drag force is proportional to the square of this relative velocity.
The velocity-dependent terms in the equation of movement is :

$$c\,\dot{X}(t) = \frac{1}{2}\rho\, C_D\, D\, \left(u_p(t) - \dot{X}(t)\right) \left|u_p(t) - \dot{X}(t)\right|$$

Structural Damping of Cylinder.

c	$=$	Material damping coefficient $(\mathrm{N \cdot s/m})$
$u_p(t)$	$=$	Time-dependent perpendicular water velocity $(\mathrm{m/s})$
$\dot{X}(t)$	$=$	Time-dependent cylinder velocity $(\mathrm{m/s})$

This differential equation has to be simultaneously stepwise solved in time domain.
In each time-step an iterative loop is necessary to solve X.

Absolute Velocity Approach

Resultant Force is summation of :

- Force caused by waves plus current on stationary cylinder, proportional to $Up.|Up|$
- Force exerted on a cylinder oscillating in still water, proportional to $(d/dt.X)^2$.

This approach does not count the cross product ($-2\ Up\ d/dt\ X$) !

Due to the fact that $d/dt\ X << Up$, the largest contribution comes from the term $Up.|Up|$.

The $(d/dt.X)^2$ term can be linearized and put on the left hand of the eq. of motion, resulting in a "Linear Damper" behavior.

An even more pragmatic approach consists of the linearization of the cross product (-2 Up d/dt X) and the (d/dt.X)^2 term which is associated to cylinder velocity d/dt X, and put this on the left hand side.
This will modify the structural damping of the cylinder.

-> fully decouples the motion of the cylinder (d/dt X) and the wave motion forces.
-> each motion is treated as if it is in a fixed system
-> Absolute Motion (velocity) Approach

Comparing Both Approaches :

Conservatism in Design leads to utilization of the Absolute Velocity Approach, but NOT modifying the linearized damping.

Advantages :

- Lower damping results in larger dynamic response, and leads to conservative proposed design.
- Simple computational procedure.

...... continuing with Chapter 12

Point wise summary of last part previous lecture

Forces on An Oscillating Cylinder In Various Flows

- **Still Water**
- **Current Alone**
- **Waves Alone**
- *... Continue...*

Current Plus Waves

All hydrodynamic velocity components have to be **superposed before** force computations. Just as the fixed cylinder case.

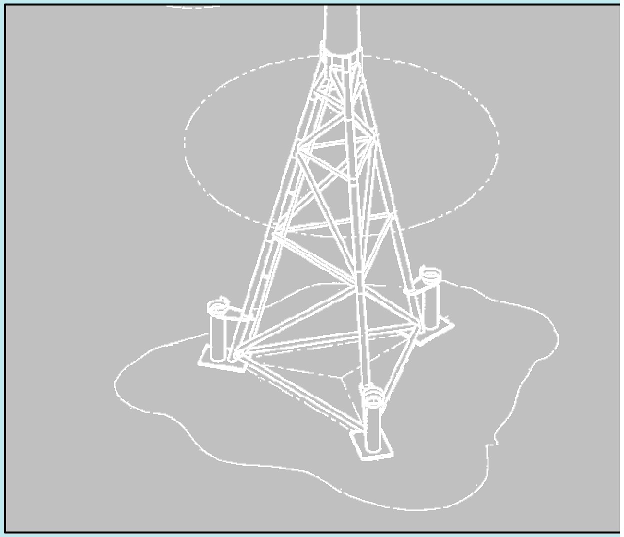
Further treatment is identical to oscillating cylinder in waves alone.

Force Integration over A Structure

So far : Forces calculated on a unit length of a cylinder.

Integration over the length of this cylinder "element"
-> obtain the loads over one element
-> loads on the end nodes of an element known

Space truss structures consists of a finite number of elements (e.g. cylinder, pipe, beam, ...).



The structure's geometry is defined by the positions of the node points, and elements located between these points.
-> **Finite Element model**

Computer programs for FE Analysis, with time-dependent (environment) loadings :

Load sequence divided in discrete time steps.

Performs for each discrete time-step :

From element loads -> to loads on node points

Generate time-dependent loadings for structural analysis (dynamic, fatigue, ...)