Exam OE4630D1 Offshore Hydromechanics Module 1
Friday January 29th 2010
09:00-12:00u
Answer sheet

1 Hydrostatics

1.1 Rectangular floating beam

a Determine for which (range(s) of) values of ρ_b the beam will be floating in a stable upright condition.

Beam will float upright when:

1. Condition 1: $t \geqslant 0$ and $t \leqslant h$
2. Condition 2: $GM \geqslant 0$

Condition 1:

$t \geqslant 0$ and $t \leqslant h$ => $t \leqslant 0.5$

Condition 2:

$GM \geqslant 0$ => $KB + BM - KG \geqslant 0$

With:

$$KB = \frac{t}{2} \qquad BM = \frac{I_t}{\nabla} = \frac{\frac{1}{12} l b^3}{l b t} = \frac{b^2}{12t} \qquad KG = \frac{h}{2}$$

Results in:

$$\frac{t}{2} + \frac{b^2}{12t} - \frac{h}{2} \geqslant 0$$

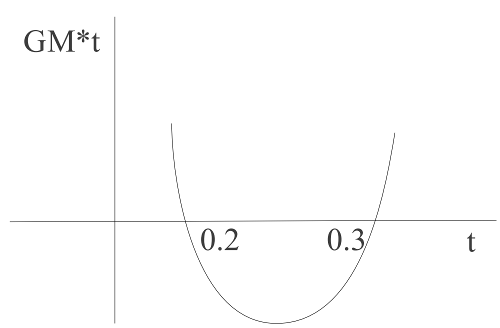
To solve this:

$$\frac{t}{2} + \frac{b^2}{12t} - \frac{h}{2} = 0 \quad |\cdot t| \quad \text{resulting in:} \quad \frac{t^2}{2} + \frac{b^2}{12} - \frac{ht}{2} = 0$$

Abc-rule to solve quadratic equality:

$$t_1 = \frac{\frac{h}{2} - \sqrt{\frac{h^2}{2^2} - 4 \cdot \frac{1}{2} \cdot \frac{b^2}{12}}}{2 \cdot \frac{1}{2}} = \frac{h}{2} - \sqrt{\frac{h^2}{4} - \frac{b^2}{6}} = \frac{0.5}{2} - \sqrt{\frac{0.5^2}{4} - \frac{0.6^2}{6}} = 0.25 - 0.05 = 0.2\, m$$

$$t_2 = \frac{\frac{h}{2} + \sqrt{\frac{h^2}{2^2} - 4 \cdot \frac{1}{2} \cdot \frac{b^2}{12}}}{2 \cdot \frac{1}{2}} = \ldots = 0.25 + 0.05 = 0.3\, m$$



The parabole $\frac{t^2}{2} + \frac{b^2}{12} - \frac{ht}{2} = 0$ is shaped like the figure:

Meaning than GM is positive when: $t \leqslant 0.2 \quad \vee \quad t \geqslant 0.3$

Both conditions combined:

Beam is floating stable and upright when: $0 \leqslant t \leqslant 0.2\,m \quad \vee \quad 0.3\,m \leqslant t \leqslant 0.5\,m$.

The weight of the beam should equal the weight of the displaced water:

$$F_{z\,beam} = F_{z\,displacement} \quad => \quad \rho_b\, g\, l\, b\, h = \rho\, g\, l\, b\, t \quad => \quad \rho_b\, h = \rho\, t \quad => \quad t = \frac{\rho_b}{\rho} h = \frac{\rho_b}{2000}$$

Thus, in terms of the density of the beam the condition becomes:

$$0 \leqslant \frac{\rho_b}{2000} \leqslant 0.2 \quad \vee \quad 0.3 \leqslant \frac{\rho_b}{2000} \leqslant 0.5$$

$$\boxed{0 \leqslant \rho_b \leqslant 400\,kg/m^3 \quad \vee \quad 600\,kg/m^3 \leqslant \rho_b \leqslant 1000\,kg/m^3}$$

b Explain under which conditions the formula of Scribanti for the righting arm is *exact*.

This is true under 2 conditions:

1. Wall sided structures (vertical sides).
2. Corner of side and deck is not immersed into the water, corner of side and bottom (bilge) remains immersed in the water (i.e. the water surface cuts through the sides).

c Is the beam floating upright in a stable condition (for the rotation about the longitudinal axis) when the density of the material of the beam is $\rho_b = 550\ kg/m^3$**? If not, what at heeling angle will the beam float?**

From question a) can be concluded that GM is negative. In any case lets first compute GM:

$$GM = KB + BM - KG$$

with:

$$t = \frac{\rho_b}{2000} = \frac{550}{2000} = 0.275\,m$$

$$KB = \frac{t}{2} = \frac{0.275}{2} = 0.1375\,m$$

$$BM = \frac{I_t}{\nabla} = \frac{\frac{1}{12} l\, b^3}{l\, b\, t} = \frac{b^2}{12t} = \frac{0.6^2}{12 \cdot 0.275} = 0.1091\,m$$

$$KG = \frac{h}{2} = \frac{0.5}{2} = 0.25\,m$$

thus:

$$GM = 0.1375 + 0.1091 - 0.25 = -0.003\,m$$

Thus: for this beam material density the beam will not float upright! It will start to turn until the stability moment becomes zero again.

Now the Scribanti equations gives the arm of the stability moment for slightly larger angles. It contains with respect to the inital stability using just GM an additional term that improves the stability.

Stability moment according to the initial stability:

$$M_{st} = \rho\, g\, \nabla\, GM \sin\phi$$

Stability moment using Scribanti:

$$M_{st} = \rho\, g\, \nabla \left(GM + \frac{1}{2} \tan^2 \phi\, BM \right) \sin\phi$$

Using Scribanti we may find the new heeling angle:

$$M_{st}=\rho\, g\, \nabla \left(GM+\frac{1}{2}\tan^2\phi\, BM \right)\sin\phi=0$$

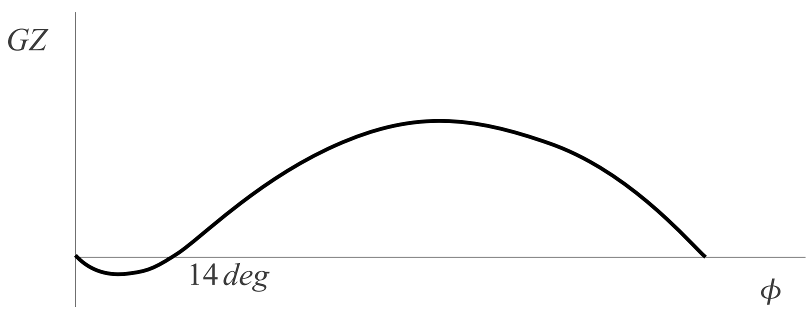
$$\left(GM+\frac{1}{2}\tan^2\phi\; BM \right)=0 \quad \vee \quad \sin\phi=0$$

The second result yields the obvious: $\phi=0,90,180,\ldots$. We know already that at 0 degrees the beam is not stable (GM < 0), and at 90 degrees and higher probably the same will happen, so lets try the other solution:

$$\left(GM+\frac{1}{2}\tan^2\phi\; BM \right)=0 \quad \Rightarrow \quad \boxed{\phi=\arctan\left(\sqrt{\frac{-BM}{2\,GM}}\right)=14\,deg}$$

This is less than 90 degrees, so we do not even need to check what happens there, as the beam will stop turning already at 14 degrees.

A rough indication of the resulting GZ curve (arm of stability moment) is depicted below. We know that at 0 degrees heeling angle GZ becomes smaller than 0 (GM < 0) so automatically at the next intersection (at 14 degrees) GZ should become again positive; the beam floats with positive stability at 14 degrees.



1.2 A rectangular barge loaded with blocks of concrete in a dock

This you may try to solve yourself.

2 *Potential Flow (10 points)*

2.1 The velocity components of a steady three-dimensional flow are given as:

$$u = A\,x\,;\quad v = -2\,A\,y\,;\quad w = A\,z$$

where A > 0 is a constant. The flow is assumed to be incompressible and inviscid.

a Show whether the flow field satisfies the conditions necessary for potential flow.

Check two things:

1. Whether the flow field satisfies conservation of mass.
2. Whether the flow field is irrotational.

Condition 1 Continuity of mass:

For potential flow conservation of mass is enforced by the Laplace equation:

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2} = 0 \quad \text{which equals:} \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

In this case:

$$\frac{\partial u}{\partial x} = A \qquad \frac{\partial u}{\partial y} = -2\text{A} \qquad \frac{\partial w}{\partial z} = A$$

Substitution in the Laplace equation indeed yields zero:

$$A - 2\text{A} + A = 0$$

Condition 2 irrotationality:

Irrotationality is given by:

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0 \qquad \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} = 0 \qquad \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} = 0$$

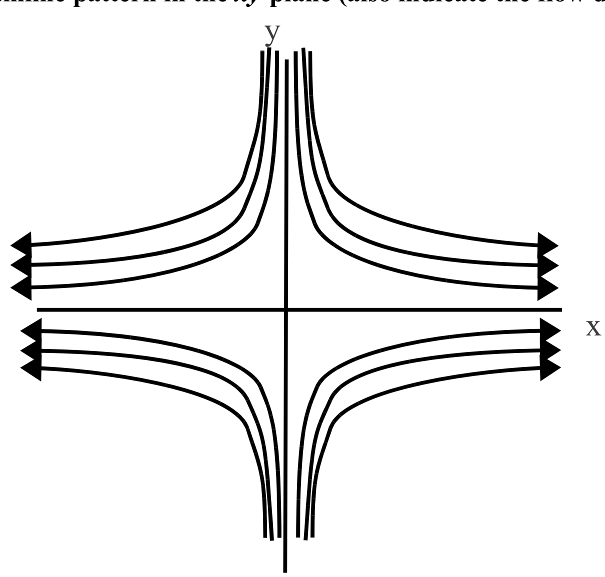
In fact all cross derivatives of u, v and w are 0, so all three conditions are satisfied.

Conclusion:

The flow field describes a potential flow field.

b Determine the streamline pattern in the *xy*-plane (also indicate the flow direction).

A sketch:



2.2 Consider a given by the velocity potential and the stream function given below:

$$\Phi=\frac{Q}{2\pi}\cdot\ln\sqrt{x^2+y^2}+U_\infty\cdot x \qquad \Psi=\frac{Q}{2\pi}\cdot\arctan\left(\frac{y}{x}\right)+U_\infty\cdot y$$

a The flow is build up by two components, which are these and what are their flow directions?
(Positive) source flow (outward) plus uniform flow in positive *x*-direction.

b Derive the location of the stagnation point.
Conditions for finding location stagnation point:

$$u(x_{stag}, y_{stag})=0$$
$$v(x_{stag}, y_{stag})=0$$

This question may be solved in two different ways, either using the stream function to obtain expressions for *u* and *v*, or using the velocity potential to obtain expressions for *u* and *v*. Both lead to the same result.

Using the velocity potential:

$$u=\frac{\partial\Phi}{\partial x}=\frac{Q}{2\pi}\cdot\frac{1}{\sqrt{x^2+y^2}}\cdot\frac{0.5}{\sqrt{x^2+y^2}}\cdot 2\mathrm{x}+U_\infty=\frac{Q}{2\pi}\cdot\frac{x}{x^2+y^2}+U_\infty$$
$$v=\frac{\partial\Phi}{\partial y}=\frac{Q}{2\pi}\cdot\frac{1}{\sqrt{x^2+y^2}}\cdot\frac{0.5}{\sqrt{x^2+y^2}}\cdot 2\mathrm{y}=\frac{Q}{2\pi}\cdot\frac{y}{x^2+y^2}$$

Setting:

$$v=\frac{\partial\Phi}{\partial y}=\frac{Q}{2\pi}\cdot\frac{y}{x^2+y^2}=0$$

This is only true when either:

$$y=0$$

or:

$$x=\pm\infty$$

The first option seems to be the most obvious one, the second refers to the situation where far from the source the influence of the source diminishes and only the uniform flow in *x*-direction remains.

So:

$$y=0$$

Substitution in *u* and equaling to zero yields:

$$u=\frac{\partial\Phi}{\partial x}=\frac{Q}{2\pi}\cdot\frac{x}{x^2+0^2}+U_\infty \quad => \quad \frac{Q}{2\pi}=-U_\infty x \quad => \quad x=-\frac{Q}{2\pi U_\infty}$$

Thus:

$$\boxed{(x_{stag}, y_{stag})=\left(-\frac{Q}{2\pi U_\infty}, 0\right)}$$

3 Real Flows (8 points)

3.1 In order to calculate the resistance of a ship advancing with forward speed through calm water a method based on potential flow is used.

a Name the three main resistance components this ship is experiencing.

Wave making resistance } -> residual resistance (Froude)
Form resistance \ /
Frictional resistance } -> viscous resistance (Hughes)

b Explain which of these components can be calculated by applying the potential flow method.

Both frictional resistance and form resistance are related to viscosity of the water. Frictional resistance is related to tangential stresses between water particles and the ship's surface and the formation of the boundary layer. Form resistance is related to the flow separation region behind the ship. Due to viscosity the water particles are slowed down along the hull and are not anymore able to 'climb' towards the high pressure region at the stern of the ship (the stagnation point). Due to this the particles start to separated and form a separation region around the stern; this causes the stagnation pressure to lower at the stern as the pressure is not anymore reacing the stagnation pressure, leading to an increase in resistance: the form resistance.

Potential flow does not include viscosity and both resistance components cannot be calculated using potential flow. The wave making resistance on the other hand can be computed using potential flow, as potential flow thoery can dequately describe waves and their associated velocities and pressures.

3.2 Explain why an increase in blade area ratio of a ship propeller reduces the occurrence of cavitation. Why does one still try to keep the blade area ratio as low as possible?

Increase in blade area ratio means that the same amount of force that needs to be generated by the propellor to overcome the ship's resistance is spread over an larger propeller area. This means that the pressure difference per unit area over the propeller may be lower for the same thrust force. So also the low pressure regions will contain less lower pressures and the change that the pressure dips below the vapour pressure becomes smaller: the risk on cavitation becomes smaller.

On the other hand may the increase in blade area lead to larger skin friction force on the propeller area and thus to larger viscous losses and thus a less efficient propulsion system.

3.3 We are performing a extrapolation of the resistance of a model towed in waves. The dimensions of the experiment are chosen such that the Froude number during the experiments equals the full scale Froude number. Given that the scaling factor for length is α_L and the scaling factor for density is α_ρ, derive scaling factors for the following quantities:

- **velocity (m/s)**
- **acceleration (m/s^2)**
- **time (s)**
- **displacement (m^3)**
- **force (N)**
- **wave frequency (rad/s)**

expressing these only in α_L and α_ρ.

Velocity

The Froude numbers of model and full scale ship are equal, thus:

$$Fn=\frac{V_m}{\sqrt{g L_m}}=\frac{V_s}{\sqrt{g L_s}} \quad => \quad Fn=\frac{V_m}{V_s}=\frac{\sqrt{g L_m}}{\sqrt{g L_s}}=\frac{\sqrt{L_m}}{\sqrt{L_s}} \quad => \quad \boxed{\alpha_V=\sqrt{\alpha_L}}$$

Time

The length traveled equals the velocity times the time traveled:

$$x = Vt \quad => \quad \alpha_L = \alpha_V \cdot \alpha_t \quad => \quad \boxed{\alpha_t = \frac{\alpha_L}{\alpha_V} = \frac{\alpha_L}{\sqrt{\alpha_L}} = \sqrt{\alpha_L}}$$

Acceleration

Velocity equals time times acceleration:

$$V = at \quad => \quad \alpha_V = \alpha_a \cdot \alpha_t \quad => \quad \boxed{\alpha_a = \frac{\alpha_V}{\alpha_t} = \frac{\sqrt{\alpha_L}}{\sqrt{\alpha_L}} = 1}$$

Of course this is 1: also the acceleration of gravity *g* is kept constant for model and full scale!

Displacement

Displacement has the unit of length to the 3rd power:

$$\boxed{\alpha_\nabla = \alpha_L^3}$$

Force

Second law of Newton:

$$F = ma \quad => \quad F \propto \rho \nabla a \quad => \quad \boxed{\alpha_F = \alpha_\rho \cdot \alpha_\nabla \cdot \alpha_a = \alpha_\rho \cdot \alpha_L^3 \cdot 1 = \alpha_\rho \alpha_L^3}$$

Wave frequency

Unit is *rad/s*, and angles on model scale and full scale are identical, thus:

$$\boxed{\alpha_\omega = \frac{1}{\alpha_t} = \frac{1}{\sqrt{\alpha_L}}}$$

4 *Waves (8 points)*

4.1 We consider an irregular sea state in a storm with a mean wave period of $T_1 = 6.3\ s$ and a significant wave height of $H_{1/3} = 3.4\ m$. You may assume that the wave amplitudes are Rayleigh distributed.

a In order to assume Rayleigh distributed wave amplitudes, what two conditions must the wave elevation spectrum fulfill?

Wave elevations are Gaussian and narrow banded distributed.

b Calculate the maximum expected wave height when the duration of the storm is 6 hours.

The probabilty that the maximum wave height is exceeded equals 1 in the number of waves that pass during the 6 hours.

The mean wave period is 6.3 seconds so during $6 \cdot 3600 = 21600\ s$ $21600/6.3 = 3429$ waves will pass.

The probability of the highest wave is then:

$$P(H > H_{max}) = 1/3429 = 0.00029$$

The probability of exceedance for a Rayleigh distributed signal is given by:

$$P(H > H_{max}) = \exp\left(\frac{-2H_{max}^2}{H_{1/3}^2}\right)$$

Thus:

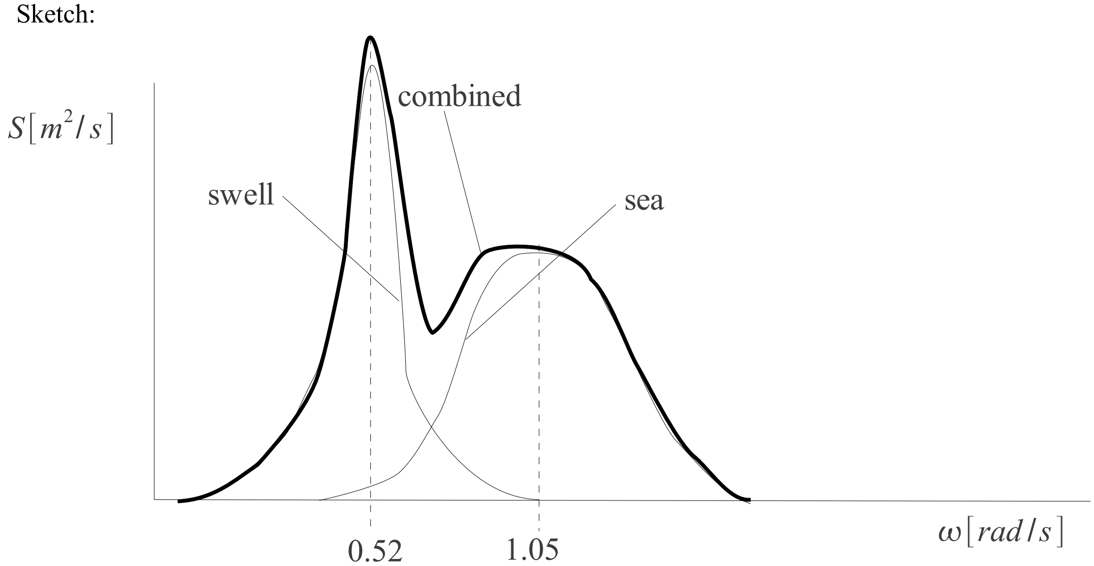
$$\exp\left(\frac{-2H_{max}^2}{H_{1/3}^2}\right) = 0.00029 \quad => \quad \boxed{H_{max} = 2.02 H_{1/3} = 2.02 \cdot 3.4 = 6.86\ m}$$

4.2 A fully developed sea and swell are defined by:

$$H_{1/3\,sea} = 3\ m \quad \textbf{and} \quad H_{1/3\,swell} = 4\ m$$
$$T_{1\,sea} = 6\ s \qquad\qquad T_{1\,swell} = 12\ s$$

a Give a realistic sketch (including dimensions) of the energy spectra of this sea and swell and of the combined sea and swell.

Sketch:



Important points:

- Location mean wave frequency sea and swell $\omega = 2\pi / T$
- Make sure area under combined curve equals the sum of the areas under the sea and swell curves (energy needs to be conserved at each frequency)
- Swell spectrum is narrow banded, sea spectrum is wider banded (energy is spread over more frequencies).
- Maximum energy spectrum swell higher than for sea (narrow banded plus higher significant wave height).

b Calculate the characteristics $H_{1/3}$ and T_1 of the combined sea and swell.

This can be computed using the spectral moments:

$$H_{1/3} = 4\sqrt{m_0} \quad => \quad m_0 = \left(\frac{H_{1/3}}{4}\right)^2$$

$$T_1 = 2\pi \frac{m_0}{m_1} \quad => \quad m_1 = 2\pi \frac{m_0}{T_1}$$

Using this:

$$m_{0\,sea} = 0.56\, m^2 \qquad m_{0\,swell} = 1.00\, m^2$$

$$m_{1\,sea} = 0.59\, m^2/s \qquad m_{1\,swell} = 0.52\, m^2/s$$

Combined:

$$m_0 = 0.56 + 1.00 = 1.56\, m^2$$

$$m_1 = 0.59 + 0.52 = 1.11\, m^2/s$$

This results in:

$$H_{1/3} = 4\sqrt{1.56} = 5.00\, m$$

$$T_1 = 2\pi \frac{1.56}{1.11} = 8.82\, s$$