Formula sheet Offshore Hydromechanics Module 1
OE4630d1

Chapter 2 Hydrostatics

$$p = \rho g h$$

$$F_\nabla = \rho g \nabla$$

$$m = \rho \nabla$$

$$\rho g \nabla = g m$$

$$\Delta T = \frac{p}{\rho A_{\mathrm{WL}}}$$

$$\overline{G_0 G_1} = \frac{p \cdot c}{m}$$

$$M_S = \rho g \nabla \cdot \overline{GN}_\phi \cdot \sin\phi$$

$$\overline{GZ} = \overline{GM} \cdot \sin\phi$$

$$\overline{BM} = \frac{I_t}{\nabla}$$

$$\overline{GM} = \overline{KB} + \overline{BM} - \overline{KG}$$

$$\overline{BN}_\phi = \overline{BM} \cdot \left(1 + \frac{1}{2}\tan^2\phi\right)$$

$$\overline{GN}_\phi = \overline{KB} + \overline{BN}_\phi - \overline{KG}$$

$$\overline{GZ}_\phi = \left(\overline{GM} + \overline{BM}\,\frac{1}{2}\tan^2\phi\right)\sin\phi$$

$$\phi = \arccos\left\{\frac{\rho\nabla \cdot \overline{GN}_\phi \cdot \sin\phi}{p \cdot c}\right\}$$

$$\phi = \arctan\left\{\frac{p \cdot c}{\rho\nabla \cdot \overline{GM}}\right\}$$

$$\overline{GM} = \frac{p \cdot c}{\rho\nabla \cdot \tan(\phi_1 - \phi_0)}$$

$$\overline{GG''} = \frac{\sum\{\rho' i\}}{\rho\nabla} \cdot \left(1 + \frac{1}{2}\tan^2\phi\right)$$

$$I_T = \int_{x_1}^{x_2}\int_{y_1}^{y_2} y^2 dy\, dx$$

Chapter 3 Constant potential flow phenomena

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\frac{v_r}{r} + \frac{\partial v_r}{\partial r} + \frac{1}{r} \cdot \frac{\partial v_\theta}{\partial \theta} = 0$$

$$\frac{\dot\alpha + \dot\beta}{2} = \frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)$$

$$\dot\varphi = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$u = \frac{\partial \Phi}{\partial x} \qquad v = \frac{\partial \Phi}{\partial y} \qquad w = \frac{\partial \Phi}{\partial z}$$

$$v_r = \frac{\partial \Phi}{\partial r} \qquad v_\theta = \frac{1}{r} \cdot \frac{\partial \Phi}{\partial \theta}$$

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2} = \nabla^2 \Phi = 0$$

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0 \qquad \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} = 0 \qquad \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} = 0$$

$$\frac{v_\theta}{r} + \frac{\partial v_\theta}{\partial r} - \frac{1}{r} \cdot \frac{\partial v_r}{\partial \theta} = 0$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho} \cdot \frac{\partial p}{\partial x}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{1}{\rho} \cdot \frac{\partial p}{\partial y}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho} \cdot \frac{\partial p}{\partial z}$$

$$\frac{\partial \Phi}{\partial t} + \frac{1}{2}V^2 + \frac{p}{\rho} + gz = C(t)$$

$$\frac{\partial \Phi}{\partial t} + \frac{1}{2}\left(u^2 + v^2 + w^2\right) + \frac{p}{\rho} + gz = 0$$

$$\frac{\partial \Phi}{\partial t} + \frac{p}{\rho} + gz = 0\,(\text{linearized})$$

$$u = \frac{\partial \Psi}{\partial y} \qquad v = -\frac{\partial \Psi}{\partial x}$$

$$v_r = \frac{1}{r} \cdot \frac{\partial \Psi}{\partial \theta} \qquad v_\theta = -\frac{\partial \Psi}{\partial r}$$

Chapter 4 Constant real flow phenomena

$$\tau = \eta \cdot \frac{\partial V}{\partial y}$$

$$\nu = \frac{\eta}{\rho}$$

$$Rn = \frac{V \cdot D}{\nu}$$

$$Fn = \frac{V}{\sqrt{gL}}$$

$$F_D = \frac{1}{2}\rho U^2 C_D D$$

$$F_l = \frac{1}{2}\rho U^2 \cdot D \cdot C_L \cdot \sin(2\pi f_v t + \varepsilon_{Ft})$$

$$St = \frac{f_v \cdot D}{U}$$

$$U_r = \frac{U}{f_n \cdot D}$$

$$C_f = \frac{R_f}{\frac{1}{2}\rho V^2 S}$$

$$C_f = 1.328 \cdot \sqrt{Rn} \text{ (Blasius)}$$

$$\frac{0.242}{\sqrt{C_f}} = \log_{10}(Rn \cdot C_f) \text{ (Schoenherr)}$$

$$C_f = \frac{0.075}{(\log_{10}(Rn) - 2)^2} \text{ (ITTC} - 1957)$$

$$C_t = \frac{R_t}{\frac{1}{2}\rho V^2 S}$$

$$C_w = C_t - (1 + k) \cdot C_f$$

$$C_{t\,\text{ship}} = (1 + k) \cdot C_{f\,\text{ship}} + C_w + C_a$$

$$X_w = \frac{1}{2}\rho_{\text{air}} V_{rw}^2 \cdot C_{Xw}(\alpha_{rw}) \cdot A_T$$

$$Y_w = \frac{1}{2}\rho_{\text{air}} V_{rw}^2 \cdot C_{Yw}(\alpha_{rw}) \cdot A_L$$

$$N_w = \frac{1}{2}\rho_{\text{air}} V_{rw}^2 \cdot C_{Nw}(\alpha_{rw}) \cdot A_L \cdot L$$

$$V_{rw} = \sqrt{V_s^2 + V_{tw}^2 + 2V_s \cdot V_{tw} \cdot \cos\alpha_{tw}}$$

$$\alpha_{rw} = \arctan\left(\frac{V_{tw} \sin\alpha_{tw}}{V_s + V_{tw} \cdot \cos\alpha_{tw}}\right)$$

$$X_c = \frac{1}{2}\rho_{\text{air}} V_c^2 \cdot C_{Xc}(\alpha_c) \cdot A_{TS}$$

$$Y_c = \frac{1}{2}\rho_{\text{air}} V_c^2 \cdot C_{Yc}(\alpha_c) \cdot A_{LS}$$

$$N_c = \frac{1}{2}\rho_{\text{air}} V_c^2 \cdot C_{Nc}(\alpha_c) \cdot A_{LS} \cdot L$$

$$\eta = \frac{P_{\text{in}}}{P_{\text{out}}} = \frac{P_E}{P_D} = \frac{T \cdot V_e}{Q \cdot 2\pi n}$$

$$\sigma = \frac{p - p_v}{\frac{1}{2}\rho V^2}$$

$$\beta_{0.7R} = \arctan\left(\frac{V_e}{0.7\pi \cdot nD}\right)$$

$$J = \frac{V_e}{nD}$$

$$K_T = \frac{T}{\rho D^4 n^2}$$

$$K_Q = \frac{Q}{\rho D^5 n^2}$$

$$P_D = Q \cdot 2\pi n$$

$$P_E = T \cdot V_e$$

$$\eta_O = \frac{P_E}{P_D}$$

$$w_n = \frac{V_s - V_e}{V_s}$$

$$t = \frac{T - R}{T}$$

$$\eta_T = \frac{R \cdot V}{Q \cdot 2\pi n} = \eta_O \cdot \eta_H \cdot \eta_R$$

$$\eta_R = \frac{Q_O}{Q}$$

$$F_L = 4\pi R \rho V^2 C$$

$$C = -\frac{\Gamma}{4\pi R V}$$

Chapter 5 Ocean surface waves

$$H = 2\zeta_a$$

$$k\lambda = 2\pi$$

$$\omega T = 2\pi$$

$$c = \frac{\lambda}{T}$$

$$\zeta = \zeta_a \cos(kx - \omega t)$$

$$\Phi_w(x,z,t) = \frac{\zeta_a g}{\omega} \cdot \frac{\cosh k(h+z)}{\cosh kh} \cdot \sin(kx - \omega t)$$

$$\Phi_w(x,z,t) = \frac{\zeta_a g}{\omega} \cdot e^{kz} \cdot \sin(kx - \omega t)$$

$$\frac{\partial \Phi_w}{\partial t} + g\zeta = 0 \quad \text{for: } z = 0$$

$$\frac{\partial \Phi_w}{\partial z} = \frac{\partial \zeta}{\partial t} \quad \text{for: } z = 0$$

$$\omega^2 = kg \cdot \tanh\ kh$$

$$c = \sqrt{\frac{g}{k} \cdot \tanh kh}$$

$$K = \int_{\text{vol}} \frac{1}{2}\left(u^2 + w^2\right) dm$$

$$P = \frac{1}{2} \int_0^{\lambda} \rho g \zeta^2 dx$$

$$E = \frac{1}{2} \rho g \zeta_a^2$$

$$\bar{W} = \frac{1}{T} \int_t^{t+T} \int_{-h}^{0} p \cdot u \cdot dz \cdot dt$$

$$\bar{W} = \frac{1}{2} \rho g \zeta_a^2 \cdot \frac{c}{2} \cdot \left(1 + \frac{2kh}{\sinh 2kh}\right)$$

$$c_g = \frac{c}{2} \cdot \left(1 + \frac{2kh}{\sinh 2kh}\right)$$

$$K_{sh} = \frac{H_h}{H_\infty} = \sqrt{\frac{1}{\tanh kh \cdot \left(1 + \frac{2kh}{\sinh 2kh}\right)}}$$

$$P\left\{\tilde{H}_w > a\right\} = \int_a^{\infty} f(x) dx$$

$$\sigma = \sqrt{\frac{1}{N-1} \sum_{n=1}^{N} \zeta_n^2}$$

$$\zeta_{a_{1/3}} = 2 \cdot \sigma$$

$$H_{1/3} = 4 \cdot \sigma$$

Gaussian/Normal distribution:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot \exp\left\{-\left(\frac{x}{\sigma\sqrt{2}}\right)^2\right\}$$

$$P\{\zeta > a\} = \frac{1}{\sigma\sqrt{2\pi}} \int_a^{\infty} \exp\left\{-\left(\frac{x}{\sigma\sqrt{2}}\right)^2\right\} \text{dx}$$

Rayleigh distribution:

$$f(x) = \frac{x}{\sigma^2} \cdot \exp\left\{-\left(\frac{x}{\sigma\sqrt{2}}\right)^2\right\}$$

$$P\{\zeta > a\} = \exp\left\{-\frac{a^2}{2\sigma^2}\right\}$$

$$\zeta(t) = \sum_{n=1}^{N} \zeta_{a_n} \cos(k_n x - \omega_n t + \varepsilon_n)$$

$$S_\zeta(\omega_n) \cdot \Delta\omega = \sum_{\omega_n}^{\omega_n + \Delta\omega} \frac{1}{2} \zeta_{a_n}^2(\omega)$$

$$S_\zeta(\omega_n) \cdot d\omega = \frac{1}{2} \zeta_{a_n}^2$$

$$\sigma_\zeta^2 = \int_0^{\infty} S_\zeta(\omega) \cdot d\omega$$

$$S_\zeta(\omega) \cdot d\omega = S_\zeta(f) \cdot df$$

$$m_{n\zeta} = \int_o^\infty \omega^n \cdot S_\zeta(\omega) \cdot d\omega$$

$$\sigma_\zeta = \text{RMS} = \sqrt{m_{0\zeta}}$$

$$\zeta_{a_{1/3}} = 2 \cdot \sqrt{m_{0\zeta}}$$

$$T_1 = 2\pi \cdot \frac{m_{0\zeta}}{m_{1\zeta}}$$

$$T_2 = 2\pi \cdot \sqrt{\frac{m_{0\zeta}}{m_{2\zeta}}}$$

Bretschneider wave spectrum:

$$S_\zeta(\omega) = \frac{173 \cdot H_{1/3}^2}{T_1^4} \cdot \omega^{-5} \cdot \exp\left\{\frac{-692}{T_1^4} \cdot \omega^{-4}\right\}$$

$$T_1 = 1.086 \cdot T_2 = 0.772 \cdot T_p$$

JONSWAP wave spectrum:

$$S_\zeta(\omega) = \frac{320 \cdot H_{1/3}^2}{T_1^4} \cdot \omega^{-5} \cdot \exp\left\{\frac{-1950}{T_1^4} \cdot \omega^{-4}\right\} \cdot \gamma^A$$

$$\gamma = 3.3 \qquad A = \exp\left\{-\left(\frac{\frac{\omega}{\omega_p} - 1}{\sigma\sqrt{2}}\right)^2\right\}$$

$$\omega_p = \frac{T_p}{2\pi} \qquad \sigma = \begin{cases} 0.07 & \text{if } \omega < \omega_p \\ 0.09 & \text{if } \omega > \omega_p \end{cases}$$

$$T_1 = 1.073 \cdot T_2 = 0.834 \cdot T_p$$

$$\zeta_{a_n} = 2\sqrt{S_\zeta(\omega) \cdot \Delta\omega}$$

Weibull distribution:

$$P(H) = \exp\left\{-\left(\frac{H - c}{a}\right)^b\right\}$$

Some useful derivatives:

$$\frac{d}{dx}\ln x = \frac{1}{x}$$

$$\frac{d}{\mathrm{dx}}\arctan x = \frac{1}{1 + x^2}$$