Exercises on Offshore Fluid Flows

(Lecture Code OT4620)

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A number of exercises on fluid flow problems in offshore activities are given here. As far as possible, they are given here in the same order as the underlying theory has been treated in the textbook of the lecture OT4620:

Offshore Hydromechanics
(First Edition)
by
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Contents:

A: Constant Potential Flow Phenomena

A-1. Potential Flows

Exercise:

a) Give a definition (in words, no formulas) of each of the following potential flow expressions:

- incompressible fluid
- potential function
- stream line
- stream function
- irrotational flow
- stagnation point
- stagnation pressure
- potential flow
- uniform flow
- source and sink
- doublet or dipole
- Rankine ship forms
- D'Alembert's paradox
- circulation and lift

b) Give the physical fluid and flow requirements which have to be fulfilled when describing a flow by the potential theory.

c) Explain why in the potential theory the sum of the individual velocity potentials of different flows provides the velocity potential of the new flow.

d) Determine the Continuity equation for an incompressible flow.
Present the result as a function of the velocity potential, Φ, and give the general name of this equation.

e) Determine the Bernoulli equation for a constant potential flow.

f) Show that a flow described by:

$$\Phi = 2x^3 - x^2 + 2xy - 6y^2x$$

does not represent a constant potential flow.

g) How should the term $+2xy$ be changed to let Φ (above) describe a potential flow?

Solution:

a) -

b) Fluid must be non-viscous, homogeneous and incompressible.
Flow must be irrotational.

c) Individual potentials are linear - doubled velocity results in doubled accelerations, pressures, etc. Therefore they may be superimposed which yields a new linear potential flow.

d) -

e) -

f) $\frac{\partial\Phi}{\partial x} = 6x^2 - 2x + 2y - 6y^2 \quad \rightarrow \quad \frac{\partial^2\Phi}{\partial x^2} = 12x - 2$
$\frac{\partial\Phi}{\partial y} = 2x - 12xy \quad \rightarrow \quad \frac{\partial^2\Phi}{\partial y^2} = -12x$
So: $\frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} = -2 \neq 0$

g) $+2xy$ must be $+y^2$, which yields: $\frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} = 0$

Solution:

A-2. Potential Body

Exercise:
Suppose a velocity potential of a 2-D flow described by:

$$\Phi = U_\infty \cdot (r\cos\theta + a\ln r)$$

where:

$$\begin{array}{lcl} U_\infty & = & \text{velocity at infinity} \\ a & = & \text{a constant} \\ r,\theta & = & \text{polar coordinates} \end{array}$$

a) Show that this is a constant potential flow.

b) Explain the two components of this flow.

c) Determine the equation of the potential body - the shape that may be replaced by a solid body.

d) Determine the stagnation point.

Solution:

a) $\frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} = 0$

b) Flow = uniform flow + source.

c) -

d) -

A-3. Stagnation Point Flow

Exercise:
Suppose the superposition of a velocity potential of a stagnation point flow:

$$\Phi_1 = \frac{a}{2} \cdot \left(x^2 - y^2\right)$$

and a uniform flow, where:

$$\begin{array}{lcl} U & = & \text{velocity of the uniform flow} \\ a & = & \text{a constant} \\ x, y & = & \text{Cartesian coordinates of an axis system} \end{array}$$

a) Give the resulting velocity potential.

b) Show that this is a potential function.

c) Give the general equation of the stream lines.

d) Give the resulting stream function.

e) Is it possible that $y = 0$ is the equation of an impervious wall? Explain your answer.

f) Give the coordinates of the stagnation point.

g) What is the pressure in the stagnation point, as the pressure in the origin is p_0.

h) Sketch the equipotential lines and the stream lines, such that their mutual relation will be clearly shown.

<u>Solution:</u>

a) -

b) $\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} = 0$

c) -

d) -

e) -

f) -

g) -

h) -

A-4. Flow around a Cylinder

<u>Exercise:</u>
The flow around a cylinder can be described by the superposition of a uniform flow and a doublet:

$$\Psi = -U_\infty \cdot y + \frac{U_\infty \cdot R^2 \cdot y}{x^2 + y^2}$$

where:

U_∞	$=$	velocity in the undisturbed fluid
R	$=$	radius of the cylinder
x, y	$=$	Cartesian coordinates of an axis system with the origin in the center of the cylinder

a) Show that this is a constant potential flow.

b) The cylinder is located in a uniform flow without any boundaries; see figure 0.1-a. What is the velocity along the cylinder surface at $x = 0$ and $y = \pm R$?

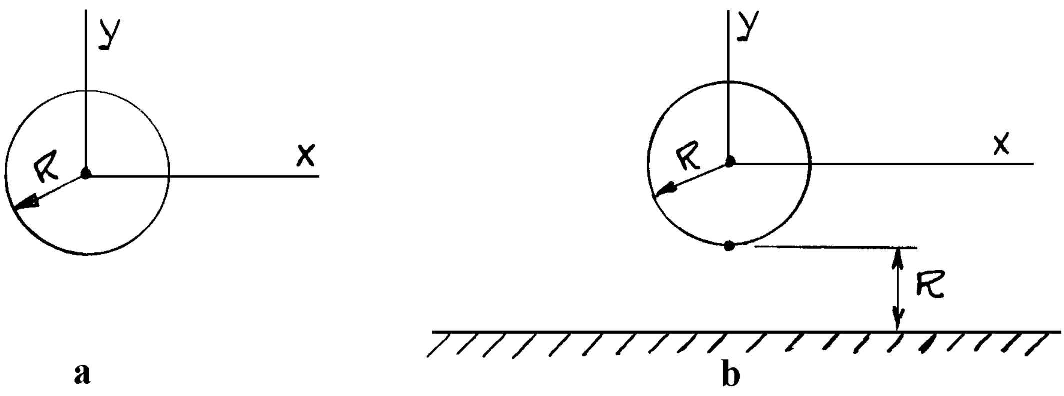

Figure 0.1: Horizontal Cylinder

c) The cylinder is located on a constant distance R above the sea bed; see figure 0.1-b. What is the velocity along the cylinder surface at $x = 0$ and $y = \pm R$?

Solution:

a) $\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} = 0$

b) -

c) -

A-5. Source in a Uniform Flow

Exercise:
Suppose a stream function of a flow defined by:

$$\Psi = U_\infty \cdot \left(y - c + \frac{c}{\pi} \cdot \arctan \frac{y}{x} \right)$$

where:

$$\begin{array}{lcl} U_\infty & = & \text{velocity at infinity} \\ c & = & \text{a constant} \\ x,y & = & \text{Cartesian coordinates} \end{array}$$

a) Show that this is a constant potential flow.

b) Show that this stream function can be obtained by a superposition of a uniform flow and a source.

c) Determine the coordinates of the stagnation point.

d) Write this stream function in polar coordinates (r, θ).

Solution:

a) $\frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} = 0$

b) -

c) -

d) -

A-6. Rankine Oval

Exercise:
A "Rankine oval" is a potential-theoretical body found by the superposition of three two-dimensional flow elements: a source, a sink with the same strength and a uniform flow parallel to a line through this source and sink.
The uniform flow has a velocity U in the positive x-direction:

$$\Phi_1 = U \cdot x \qquad \text{and} \qquad \Psi_1 = U \cdot y$$

The source $+Q$ lies in $x = -a$ and $y = 0$:

$$\Phi_2 = \frac{+Q}{4\pi} \cdot \ln\left[(x+a)^2 + y^2\right] \qquad \text{and} \qquad \Psi_2 = \frac{-Q}{2\pi} \cdot \arctan\left[\frac{y}{x+a}\right]$$

The sink $-Q$ lies in $x = +a$ and $y = 0$:

$$\Phi_3 = \frac{-Q}{4\pi} \cdot \ln\left[(x-a)^2 + y^2\right] \qquad \text{and} \qquad \Psi_3 = \frac{+Q}{2\pi} \cdot \arctan\left[\frac{y}{x-a}\right]$$

a) Show that each component is a constant potential flow.

b) Determine the velocities u and v of the water particles in the x- and y-directions of the new flow.

c) Determine the strength of the source and sink, $\pm Q$, such that the breadth of the body (measured in its middle, so at $x = 0$) is equal to $2b$.

d) Locate the stagnation points in the flow.

e) Determine the length, L, of the body.

f) Determine the magnitude of the stagnation pressure coefficient in $(0,b)$.

Solution:

a) $\frac{\partial^2\Phi_1}{\partial x^2} + \frac{\partial^2\Phi_1}{\partial y^2} = 0$ and $\frac{\partial^2\Phi_2}{\partial x^2} + \frac{\partial^2\Phi_2}{\partial y^2} = 0$

b) Superpose stream functions, $u = +\partial\Psi/\partial y = ...$ and $v = -\partial\Psi/\partial x = ...$

c) $Q = [2\pi U b] \,/\, [2 \arctan(b/a)]$

d) $x = \pm a\sqrt{1 - [b/a] \,/\, [\arctan(b/a)]}$ and $y = 0$

e) $L = 2a\sqrt{1 - [b/a] \,/\, [\arctan(b/a)]}$

f) $C_p = 1 - \{1 - [ab] \,/\, [(a^2 + b^2) \cdot \arctan(b/a)]\}^2$

B: Constant Real Flow Phenomena

B-1. Cycling against the Wind

Exercise:
When cycling against the wind, one experiences an increased wind resistance.
If one assumes that:

- the aerodynamic resistance is the dominant resisting force,
- this resistance is directly proportional to the square of the relative velocity, and
- one gets tired by delivering impulse (equal to force multiplied by time),

then one can compute that the optimum speed over the ground (that makes one the least tired when going from A to B) to pedal the bicycle (against the wind) is numerical identical to the given wind speed.

a) Evaluate the reasoning above.

b) Why does this advise fail in practise?

Solution:

a) -

b) -

B-2. Torpedo

Exercise:
Consider a submarine, launching a torpedo from the forebody of the ship, see figure 0.2.

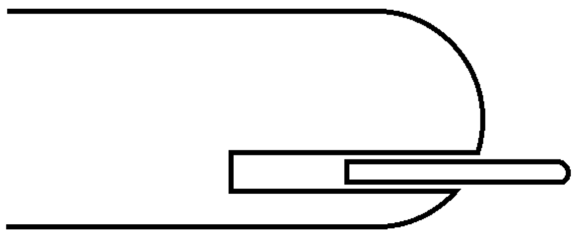

Figure 0.2: Launching a Torpedo by a Submarine

This launching is caused by an impulse of air under high pressure in the launching tube. The expanding air presses the torpedo out of the tube. The torpedo leaves the tube with a velocity, V_t, relative to the submarine which has a forward speed, V.
With increasing speeds of newly build submarines, the problem arises that the torpedo leans in the tube and even jammed.
Consider the bow of the submarine as a vertical cylinder instead of a sphere and explain this with carrying out calculations:

a) Give (qualitatively) the flow pattern around the bow of the submarine without the torpedo.

b) How will this flow pattern change as the forward speed of the submarine will be increased?

c) Which hydromechanical forces act on the torpedo to counteract its launching? Where upon are they depending?

d) Which hydromechanical forces act on the torpedo to let it lean in the tube. Where upon are they depending?

e) Is there any reason to expect secondary flows along the torpedo? Why?

Solution:

a) -

b) Two answers are possible:
- if potential flow, then all velocities increase proportionally; the pattern does not change.
- if boundary layer, then earlier separation and turbulence.

c) Torpedo experiences a resistance force to its forward motion at nose end:

$$F_R = \frac{1}{2}\rho(V + V_t)^2 \cdot \frac{\pi}{4}D^2 \cdot C_D \qquad \text{(not } C_D \text{ for cylinder!)}$$

plus, possibly a skin friction caused by the relative flow along the sides parallel to the motion direction plus an inertia force (if it is still accelerating).

d) Transverse forces are caused by flow components perpendicular to projecting torpedo. These forces cause a moment, moving the nose away from the ship's center line and causing it to jam.

e) Yes, possibly. The velocity component perpendicular to the torpedo axis is not constant along its length.

B-3. Aerodynamic Drag on a Windmill

Exercise:
This question is designed to let you apply the principles of your knowledge about hydrodynamic forces to a different situation: the forces on the rotor of a modern, operating windmill having two blades on its rotor.
Large windmills are being considered for installation at offshore locations in the Dutch part of the North Sea. We are talking now of a windmill with a rotor which is 80 meters in diameter with a horizontal axis of rotation which is 60 meters above the sea.
The wind speed varies as a function of height above the sea.

a) Why does the wind above the sea exhibit such a velocity profile? Explain this in a few words.

For the purposes of this discussion let us assume that this velocity profile varies linearly from a speed, V, 20 meters above the sea (at the lower part of the rotor circle) to $2V$ at an elevation of 100 meters (the top of the rotor circle). We want to examine the cyclic loadings which can occur in the rotor blades and in the support tower as the rotor turns. Consider for this only the wind which acts perpendicular to the plane of the rotor's rotation and the drag force that it causes coincident with the wind direction. For convenience, let us define the X-axis as coinciding with the wind direction and the Z-axis vertically upward. The rotor axis is therefore parallel to the X-axis; the rotor is in the Y-Z plane.
In all of the following, the drag force on a rotor blade segment of length dL can be expressed as:

$$dF_d = C \cdot [U(z)]^2 \cdot dL$$

where: C = a constant
$U(z)$ = wind speed at height z

b) How does the bending moment at the axis-end of each rotor blade vary as it follows a full circle? Examine the situation when the blade is vertically upward, vertically downward, and horizontal on each side. Express each of these bending moments as an equation.

c) Examine the forces on each of the blades when the rotor is 45 degrees from the vertical. Describe the torsional moment in the supporting tower (about the Z-axis) at this moment, when the rotor is vertical, and when it is turned 45 degrees to the other side. Again, express the torsion in each position as an equation.

d) How (explain in a few words) would this situation in the last two questions change if the wind velocity is independent of the elevation, thus constant over the entire area swept by the rotor blades?

Solution:

a) -

b) -

c) -

d) -

B-4. Remote Controlled Vehicle

Exercise:
A remote controlled vehicle is moving forward on the sea bed with constant speed u along the x-axis of a fixed coordinate system. There are no waves in the ocean, but there can be a constant current with speed V. In general, the direction of this current forms an angle θ (in plan - as seen from above) with respect to the x-axis. Our task is to evaluate the hydrodynamic drag forces on a slender vertical cylindrical antenna (of diameter D and length L) extending upward from our vehicle. Assume that the current velocity is the same at all elevations.

a) At first there is no current; $V = 0$. Write an expression for the drag force per unit length of the antenna. Explain the symbols used.

b) There is now a constant current of magnitude V. If the vehicle were moving with a speed $u = V$ (in magnitude) in the same direction as the current, what would then be the drag force on the antenna? How does this force relate (how many times larger or smaller) to that found in the first question.

c) If the vehicle in the second question were now moving in the opposite direction (directly against the current) but with the same speed, how would the drag force now be related to that found in the first question?

d) The vehicle has turned 90 degrees so that the current is now crosswise to its own direction of motion. The two velocity magnitudes are still identical. How does the magnitude of the resulting drag force now relate to that found in the first question?

e) Write a general relationship relating the magnitude of the resultant drag force to the three independent parameters, u, V and θ. (Hint: start with the x and y components of u and V.)

f) Under what conditions of u, V and θ will the magnitude of the drag force be identical to that found for the vehicle moving with speed u in still water?

Solution:

a) Drag force (in this case) is opposite to the velocity of motion of the antenna (through the still water):

$$F_D = -\frac{1}{2}\rho U^2 \cdot D \cdot 1 \cdot C_D$$

with:

$$\begin{array}{lcl} U & = & \text{velocity } (\omega/r \text{ antenna}) \text{ m/s} \\ \rho & = & \text{density of fluid} \\ D & = & \text{antenna diameter} \\ C_D & = & \text{dimensionless coefficient} \end{array}$$

b) Vehicle moves with the current; relative velocity becomes zero, so:

$$F_D \to 0$$

c) Vehicle moves against current (also of speed u):

$$\begin{aligned} F_D &= -\frac{1}{2}\rho\,(U+u)^2 \cdot D \cdot 1 \cdot C_D \\ &= 4 \text{ times } F_D \text{ in first question} \end{aligned}$$

d) -

e) -

f) -

B-5. Fixed Platform

Exercise:
Mention and explain the hydromechanical forces acting on a offshore platform - fixed to the sea bed - in a tidal current.

Solution:
See textbook.

B-6. Scaling Laws of Froude and Reynolds

Exercise:
A supply vessel is scaled by $1 : \alpha$ to a model ($\alpha > 1$).

a) Determine the scaling factors - at the same time fulfilling the scaling law of R.E. Froude - for each of the following magnitudes:

- water plane area
- wetted surface of the hull
- volume of displacement
- mass of displacement
- mass moment of inertia
- pressures in the fluid
- fluid forces on the body
- fluid moments on the body
- forward speed
- propeller diameter
- propeller pitch
- wave length
- wave period
- frequencies of oscillation
- vertical displacements
- angular displacements
- vertical velocities
- angular velocities
- vertical accelerations
- angular accelerations
- propeller rpm
- propulsive power

b) Determine now the scaling factors - at the same time fulfilling the scaling law of Osborn Reynolds - for each of the previous magnitudes.

Solution:

Phenomenon	Froude	Reynolds	Phenomenon	Froude	Reynolds
water plane area	$1 : \alpha^2$	$1 : \alpha$	wave length	$1 : \alpha$	$1 : \alpha$
wetted hull surface	$1 : \alpha^2$	$1 : \alpha^2$	wave period	$1 : \alpha^{\frac{1}{2}}$	
volume of displacement	$1 : \alpha^3$	$1 : \alpha^3$	oscillation frequency	$1 : \alpha^{-\frac{1}{2}}$	
mass of displacement	$1 : \alpha^3$	$1 : \alpha^3$	vertical displacements	$1 : \alpha$	$1 : \alpha$
mass moment of inertia	$1 : \alpha^5$	$1 : \alpha^5$	angular displacements	$1 : 1$	$1 : 1$
pressures in the fluid	$1 : \alpha^2$	$1 : \alpha^{-2}$	vertical velocities	$1 : \alpha^{\frac{1}{2}}$	$1 : \alpha^{-1}$
fluid forces	$1 : \alpha^3$	$1 : 1$	angular velocities	$1 : \alpha^{-\frac{1}{2}}$	
fluid moments	$1 : \alpha^4$	$1 : \alpha$	vertical accelerations	$1 : 1$	$1 : 1$
forward speed	$1 : \alpha^{\frac{1}{2}}$	$1 : \alpha^{-1}$	angular accelerations	$1 : \alpha^{-1}$	
propeller diameter	$1 : \alpha$	$1 : \alpha$	propeller rpm	$1 : \alpha^{\frac{1}{2}}$	
propeller pitch (deg)	$1 : 1$	$1 : 1$	propulsive power	$1 : \alpha^{4\frac{1}{2}}$	

B-7. Still Water Resistance

Exercise:

a) Discuss the different components of the resistance of a ship (or a model). Pay hereby attention to the scaling laws which should be applied for each of these components and give a realistic non-dimensional presentation in one figure of the individual and total components.

b) Explain the resistance extrapolation method of William Froude.
Explain also the method of Hughes for extrapolating the form resistance component. Why is this form resistance component not fully independent of the Reynolds number?

Solution:

a) -

b) -

B-8. Propulsors

Exercise:

a) Discuss the different types of propulsion systems used in offshore activities and give their mutual "pros and cons".

b) Discuss the most important parameters used when dimensioning a propeller.

Solution:

a) -

b) -

B-9. Open Water Propellers

Exercise:

a) Sketch a realistic presentation of the propulsive characteristics of a fixed pitch propeller of a crude oil carrier.

b) Determine the relation between the propulsive efficiency, η_O, of an open water propeller on one hand and the thrust coefficient, K_T, the torque coefficient, K_Q, and the speed ratio, J, on the other hand.

c) Consider the K_T-K_Q-J characteristics. Explain why the K_T-curve intersects its zero-axis at a smaller J-value than the K_Q-curve; in other words, why lies the K_T-curve there beneath the K_Q-curve? Give a physical explanation of what happens in the fluid when $K_T = 0$.

d) The results of open water tests on a 0.30 meter diameter propeller model - run at 500 rpm - are shown below.

J	0.0	0.2	0.4	0.6	0.8	1.0	1.2
K_T	0.715	0.620	0.500	0.370	0.250	0.140	0.030
K_Q	0.130	0.114	0.094	0.072	0.050	0.032	0.014

What will be the maximum efficiency, η_O, obtainable for this propeller, and the appropriate speed of advance, V_e?

e) What is the reason of the presentation of so-called "Four Quadrant Measurements" of propellers?

<u>Solution:</u>

a) -

b) -

c) -

d) $\eta_O = 0.70$ and $V_e = 2.45$ m/s.

e) -

B-10. Cavitation

<u>Exercise:</u>

a) Explain the possible occurrence of cavitation on propeller blades. How can this be avoided when designing a propeller?

b) Explain why experiments on models to determine the effects of cavitation cannot be carried out in an open model tank.

<u>Solution:</u>

a) -

b) -

B-11. Wake and Thrust Deduction

<u>Exercise:</u>

a) Give a physical explanation of the wake and the thrust deduction of a sailing ship.

b) Define the wake fraction and the thrust deduction fraction. Describe shortly how these two phenomena can be obtained by model tests.

c) Contrarily to the thrust deduction fraction, the wake fraction is very sensitive for scale effect. Explain this.

Solution:

a) -

b) -

c) -

B-12. Propulsion versus Resistance

Exercise:

a) Determine the three components of the total propulsive efficiency of a sailing ship and explain these components.

b) Explain why a propeller **behind** a sailing ship results in a higher total propulsive efficiency than a propeller connected just **before** the ship.

c) The forward speed of a free running (self-propelled and radio controlled) model of a ship has been scaled by Froude's law. Explain the generally remarkable good manoeuvering characteristics of this free running model when compared to the full scale ship.

d) Consider a free running tug and the same tug when towing a barge. Discuss the resistance and propulsion characteristics in both conditions. Show for both cases in one sketch how one can determine the forward speed of the tug (such that resistance and propulsion forces are in equilibrium).

e) A ship of 50,000 ton displacement is driven at a speed of 12 knots. A ship of 65,000 ton of similar form is being designed. At what speed of the larger ship should its performance be compared with the 50,000 ton ship?

f) A 50,000 m^3 displacement ship, length 120 m, is to be represented by a model of 3.00 meter long. What is the displacement, ∇_m, of the model? At what speed, V_m, must it be run to represent a speed of 20 knots in the ship and what is the ratio, α_P, of the ship to model propulsion power at this speed?

Solution:

a) -

b) -

c) -

d) -

e) $V = 12.54$ knots.

f) $\nabla_m = 0.781$ m^3, $V_m = 1.63$ m/s and $\alpha_P = 4.05 \cdot 10^5$.

C: Wave Forces on Slender Cylinders

Useful or useless formulas and coefficients for this chapter are:

$$
\begin{aligned}
\widehat{F}_d &= 1/2 \cdot \rho \cdot \widehat{u}\,|\widehat{u}| \cdot C_d \cdot D \\
\widehat{F}_i &= \pi/4 \cdot D^2 \cdot \rho \cdot C_m \cdot \widehat{\dot{u}} \\
F &= \widehat{F}_d \cdot \sin^2 \omega t + \widehat{F}_i \cdot \cos \omega t \\
KC &= \widehat{u} \cdot T/D \\
KC &= \pi^2 \cdot (C_m/C_d) \cdot \left(\widehat{F}_d/\widehat{F}_i\right) \\
\widehat{u} &= 1/2 \cdot \omega \cdot H \cdot e^{-kz} \\
C_d &= 1.2 \\
C_m &= 1.5
\end{aligned}
\qquad
\begin{aligned}
k &= 2\pi/\lambda \\
\omega &= 2\pi/T \\
\lambda &= 1.56 \cdot T^2 \\
C_m &= 1 + C_a \\
\widehat{\dot{u}} &= \omega \cdot \widehat{u} \\
Rn &= \widehat{V} \cdot D \cdot \rho/\mu \\
C_l &= 1.0 \\
St &= 0.2
\end{aligned}
$$

C-1. Miscellaneous

Exercise:

a) Explain why the inertia force on a cylinder oscillating in still water is different from that on a fixed cylinder in waves - even though the flow patterns look very much similar when viewed from the cylinder.

b) Define and give two uses (or interpretations) for the Keulegan-Carpenter number.

c) Compute the amplitudes of the drag and inertia forces on a 5 meter long segment of a fixed vertical steel riser located 100 meters below the surface of the sea. The riser is 0.75 meters in diameter. The local flow is caused by a tidal current of 1.3 m/s.

d) Compute the amplitudes of the drag and inertia forces on a 5 meter long segment of a fixed vertical steel riser located 100 meters below the surface of the sea. The riser is 0.75 meters in diameter. The local flow is now caused by a regular wave with a period of 15 seconds and a height of 12 meters. The water is 150 meters deep.

e) Compute the amplitudes of the drag and inertia forces on a 5 meter long segment of a fixed vertical steel riser located 100 meters below the surface of the sea. The riser is 0.75 meters in diameter. The local flow is now caused by a regular wave with a period of 15 seconds and a height of 12 meters superimposed on a (colinear) tidal current of 1.3 m/s. The water is 150 meters deep.

f) What are the distinctions between the absolute and relative velocity approaches for computing drag forces?

g) What are the distinctions between the absolute and relative velocity approaches for computing inertia forces?

h) Explain why the horizontal component of the force on a submerged slender horizontal cylinder placed parallel to the crest of sinusoidal waves will be sinusoidal - even though there is an important drag force component.

i) Given the following:

Wave height	=	10 meters
Wave period	=	12.56 seconds
Water depth	=	150 meters
Cylinder diameter	=	3.50 meters
Cylinder length	=	7 meters
Cylinder location	=	20 meters below the still water level
Cylinder orientation	=	Horizontal
		Parallel to wave propagation direction

Other data: The cylinder has a roughness of 1/100 and is part of a complete offshore tower structure. Graphs of the DNV recommended C_D and C_M values are given in a graph in the lecture notes. Neglect wave phase differences along the length of the cylinder.

Questions:

- Determine the amplitudes of the vertical water velocity and acceleration at the sea surface and at the bottom of the cylinder. Determine the Reynolds and Keulegan-Carpenter number values that should be used to determine the drag and inertia coefficients.
- Determine the amplitudes of the drag and inertia force components acting on the given cylinder.
- Determine the maximum vertical force acting on the given cylinder.

j) Explain in words how one linearizes the quadratic drag term in the Morison equation. What are the motivations for and effects of doing this?

<u>Solution:</u>

a) -

b) -

c) -

d) -

e) -

f) -

g) -

h) -

i) -

C-2. Forces on a Truss Structure

Exercise:
This question is intended to test your insight about hydrodynamic forces on slender cylinders. It involves no numerical computations!
Situation:
You are to work with a computer scientist to make a program to compute hydrodynamic forces on a fixed, rigid, three-dimensional truss structure in currents and waves. The Morison equation is to be used to compute these forces. The word "Morison" reminds the computer scientist only of a specific centerfold picture in his (or her) favorite leisure time magazine! The computer scientist (a real specialist) is only able to translate the mathematical formulas which you prepare into a working computer program; you must develop the necessary mathematical formulations!
Given information:
- Each of the nodes of the truss structure is numbered sequentially, and the coordinates (x, y, z) of each node is known.
- The members are also numbered sequentially. For each member we know its diameter, drag and inertia coefficients, and the number of the node at each of its ends. From this, one can then easily look up the coordinates (x, y, z) of each end of the member.
- Additionally, the water velocity (as a vector) at any location and time van be computed: $\vec{V} = \text{Vel}\ (x, y, z, t)$.
- A similar known expression describes the acceleration: $\vec{A} = \text{Acc}\ (x, y, z, t)$.
- A series of specific time values, t, is specified.

Desired information:
The team - you and your computer science friend - are to produce a computer program to compute the total force (x, y and z components) on the structure at each end of the given times, t.
Task:
Your task is to explain to the computer scientist - via a series of simple steps - the computational procedure he must follow in order to compute the total force on the structure as described above.

Solution:
See textbook.

C-3. Wave Force Tests

Exercise:
A student conducts wave force tests on a segment of a slender horizontal submerged cylinder fixed with its axis parallel to the wave crests in a deep water test tank. Regular waves are generated in the tank. He measures the wave profile as well as the horizontal and vertical components of the force acting on the cylinder segment.
Our student, accustomed to calculating forces on vertical cylinders, is astonished to find that:
- the wave profile,
- the horizontal force component, and
- the vertical force component

all have very nice sinusoidal forms (with different phases, of course).

a) Is our student correct in assuming, therefore, that the Morison equation is only valid for vertical cylinders?

b) What series of tests must he do - without changing the orientation of the cylinder in the tank - to test his assumption?

Solution:

a) -

b) -

C-4. Flow around a Submerged Horizontal Cylinder

Exercise:
Consider a segment of a smooth, horizontal cylinder which is mounted parallel to the y-axis (see below) in a large, open, rectangular tank filled with water. (We will not discuss, here, how this is mounted; let us assume that all of the motions suggested below can be carried out.) The cylinder itself has a diameter D and its mass is M kg/m. The water in the tank has a density ρ. The cylinder segment we are considering is well below the water surface and also far from the ends of the cylinder, the walls and tank bottom, and the mounting of the cylinder.
Use the following axis convention: the x-axis is along the longest tank axis; the y-axis is also horizontal and perpendicular to x; the z-axis is vertical. Hydrodynamic interaction forces will have x and z components, therefore.
Use sketches and explanations as well as formulas to describe the lift, drag and inertia hydrodynamic interaction forces for this cylinder segment under each of the conditions and at the time instants listed below. Use the absolute velocity and acceleration approach.

a) The cylinder moves with constant velocity along the positive x-axis of the tank. The water in the tank is at rest.
Be sure to sketch the force situation at two instants in time:
- when the lift force is maximum, and
- when the lift force is zero.
What is different about the wake near the cylinder at each of these two instants?

b) The cylinder is at rest. The water moves with constant velocity along the positive x-axis of the tank.
What is the difference between this force situation and that resulting from the first question, above?

c) The cylinder is oscillated horizontally and sinusoidally (along the x-axis) while the water in the tank is at rest. The cylinder has a motion amplitude of W and period T.
Describe the force components at the following time instants:
- when the horizontal cylinder velocity is zero, and
- when the horizontal cylinder velocity is maximum.

d) There are sinusoidal waves (propagating along the positive x-axis) such that the water at the cylinder segment of interest is oscillating both horizontally and vertically with an amplitude W and period T. There is no current and the cylinder remains at rest. Sketch the force situation at the following instants in time:
- when the horizontal water velocity is zero, and
- when the horizontal water velocity is maximum.

e) Now, the water in the tank is again at rest. The cylinder, instead, now translates along a circular path (in the x-z plane). This path has a radius W and the cylinder passes the uppermost point if its path every T seconds.
What is the difference between this force situation and that resulting from the previous question, above?

Solution:

a) -

b) -

c) -

d) -

e) -

C-5. Flow around a Vertical Cylinder

Exercise:
Consider a segment of a smooth, vertical cylinder which tends down into a flume below a towing carriage. The cylinder itself has a diameter D and its mass is M kg/m. The water in the flume has a density ρ. The cylinder segment we are considering is well below the water surface but also far from the exposed end of the cylinder.
Use the following axis convention: the X-axis is along the flume axis; the Y-axis is also horizontal and perpendicular to X; the Z-axis is vertical.
Use sketches and explanations to describe the lift, drag and inertia hydrodynamic interaction forces under each of the conditions and at the time instants listed below. Use the relative velocity and acceleration approach.

a) The water in the flume, the towing carriage and the cylinder are completely at rest.

b) There is a flow with a constant velocity (directed along the positive X-axis) in the flume; the towing carriage and cylinder remain fixed.
Be sure to sketch the force situation at two times:
- when the lift force is maximum
- when the lift force is zero.
What is different about the wake near the cylinder at each of these two instants?

c) There are waves (propagating along the positive X-axis) such that the water motion at the cylinder segment of interest is oscillating both horizontally and vertically with an amplitude W and period T. There is now no current in the flume; the cylinder

remains at rest.
Sketch the force situation at the following instants:
- when the horizontal water velocity is zero
- when the horizontal water velocity is maximum.

d) The towing carriage is oscillated with period T and amplitude A back and forth along the flume (X) axis. The water in the flume is again at rest.
Sketch the force situation at the following instants:
- when the cylinder velocity is zero
- when the cylinder velocity is maximum.

e) The towing carriage is oscillated with period T and amplitude A back and forth along the flume (X) axis. There are also waves in the flume causing a water motion of amplitude W and also of period T at the point of interest.
Further, the motions are synchronized so that:
- the maximum of the water velocity and the maximum of the cylinder velocity occur at the same moment and location, but they are in opposite directions
- both the horizontal water motion and the cylinder are instantaneously at rest at the same location.
Describe the force components at each of these moments.

Solution:

a) -

b) -

c) -

d) -

e) -

C-6. Cable Laying Problems

Exercise:
A Dutch-Iceland consortium of electric companies is investigating the possibility of importing electric power from Iceland via a submarine cable to be laid between Iceland and The Netherlands via Scotland. We are to concentrate our attention, here, on the line between Iceland and Scotland.
The following data may be used for the purpose of this exercise:

Water depth	=	800 m	Cable dir. (plan)	=	NW - SE
Water density	=	1025 kg/m^3	Cable diameter	=	0.5 m
			Cable mass	=	800 kg/m
Current speed	=	1.0 m/s	Cable drag coeff.	=	0.7
Current direction	=	toward NE	Cable inertia coeff.	=	1.8
Wave height	=	15.0 m			
Wave period	=	15.0 s			
Wave direction	=	toward NE			

a) The cable is to be laid by reeling it off over a wheel on the stern of a cable laying ship. Assuming, as a first guess that the ship sails fast (more than 10 m/s), compute (approximately) the speed at which the cable being paid out will settle through the water. Neglect the current and note as well that when a ship is sailing at this speed, there is no storm around, either.

b) The laying ship and the cable are now stopped in the water. Calculate the maximum horizontal hydrodynamic force on a one meter long segment of this cable at a depth of 100 meters. There are still no waves, but the design current is now present.

c) Assuming that we have deep water waves as well as the current, compute the maximum and minimum horizontal water velocity to be expected at a water depth of 100 meters.

d) Determine the maximum drag force component and maximum inertia force component on a one-meter section of this cable at this 100 meter depth. The environmental flow conditions are the same as in the previous question Neglect ship motions, but assume further that the cable is vertical and it is horizontal in the Northwest-Southeast direction. How are these force component amplitudes different for the two cable orientations?

e) Explain how (you do not have to carry this out) one would determine the maximum horizontal force on the cable segment in the previous question. Again assume that the cable is vertical and it is horizontal in the Northwest-Southeast direction.

Solution:

a) -

b) -

c) -

d) -

e) -

C-7. Stone Dumping Problem

Exercise:
A contractor must dump stone to cover a pipeline laid across the Strait of Gibraltar between Gibraltar and Morocco. At the location in question, this Strait is 1000 meters deep. He has decided to 'guide' this dumping process by first extending a smooth polyethylene pipe, 2.0 meters in diameter, vertically downward from his ship to just above the sea bottom.
The ship's navigating system indicates that the ship (with its pipe hanging straight down below it) is moving toward the West with a constant speed of 0.5 m/s.
There is a tidal current (with a period of 12.42 hours) along an East-West axis (in either direction) which has an amplitude of 0.8 m/s.
There is also a density current present here: Atlantic Ocean water with an density of 1020 kg/m^3 flows into the Mediterranean Sea (toward the East) with a velocity of 1.0 m/s. in the upper 600 meters of the Straight, while more saline water with a density of 1030 kg/m^3

flows outward (toward the West) with that same velocity in the lower 400 meters of the Strait. (The extra transport of water into the Mediterranean is lost by evaporation.)
For the purpose of this exercise, it is sufficient to consider only a 1.0 meter long segment of that pipe at a depth of 500 meters.

a) Describe briefly in words (supplemented by mathematics if desired) at two distinct approaches to determining the hydrodynamic interaction force between this pipe segment and the surrounding sea.

b) Determine quantitatively the maximum horizontal force acting on a 1.0 meter long segment of this pipe at a depth of 500 meters. Indicate at what instant in the tidal period that this occurs.

Solution:

a) -

b) -

D: Survival Loads on Tower Structures

Miscellaneous

Exercise:

a) A student was using Wheeler stretching when computing the wave forces on a cylinder for which the Keulegan-Carpenter number was equal to 3. Why was this a waste of time on his part?

b) List the assumptions behind the schematization of an entire offshore tower structure as a single vertical stick for the purpose of computing maximum survival loads.

c) What is the philosophy underlying the selection of hydrodynamic (as well as aerodynamic) conditions for the purpose of computing survival loads on an offshore tower structure?

d) Explain why the vertical "stick" - resulting from the schematization of a tower structure - has two effective diameters at each elevation, one for drag and one for inertia forces.

e) Why will the vertical "stick" model - used for computing survival loads on an offshore tower structure - not have a uniform diameter over its entire height?

f) What justifies - for a first approximation - neglecting the inertia force when estimating survival loads on an offshore tower structure

Solution:

a) -

b) -

c) -

d) -

e) -

f) -

E: Sea Bed Boundary Effects

Miscellaneous

Exercise:

a) It is agreed that waves alone do not generally cause large scale sediment transport. Why is it then, that they do contribute to the transport caused by a current?

b) What two physical mechanisms form an equilibrium to maintain sediment transport in suspension?

c) Why is usually acceptable to neglect suspended sediment transport when discussing the morphology of the sea bed near a pipeline or other small object?

d) Most sediment transport formulas will predict a sediment transport whenever there is a current, even though this is not true in practice. Why is this not true?

e) Explain why one finds an erosion pit near the stagnation point on the upstream side of a pile which extends well above and well into the sea bed.

f) Indicate and describe (with the help of a sketch) the forces acting on a horizontal pipeline laying in contact with the sea bed. Assume that there is a current acting perpendicular to the pipe axis.

g) What prevents tunnel erosion from continuing indefinitely under a pipeline?

h) Explain why it is advantageous to allow a limited amount of water to flow downward with the transported solids in a fallpipe.

i) What is the disadvantage of letting too much water flow through the fallpipe?

j) Explain why a downward lift force can act on a segment of a pipeline under which tunnel erosion is taking place?

k) What parameter or parameters must one reproduce in a physical model in which one wants to study the stability of the sea bed near a single exposed wellhead?

Solution:

a) -

b) -

c) -

d) -

e) -

f) -

h) -

i) -

j) -

k) -