P-L-2

Fluid Mechanics

Basic principles and concepts

The Modelling Team
Department of Design Engineering
Faculty of Industrial Design Engineering
Delft University of Technology

Aim

Aim

- To develop a basic understanding of the properties of fluids;
- To show how fluids behave in a real environment
- To demonstrate that those simple behaviours might be modelled so as to provide useful data for designs
- To communicate with experts in their professional languages

Contents

What is fluid?

The Modelling Team
Department of Design Engineering
Faculty of Industrial Design Engineering
Delft University of Technology

Fluid

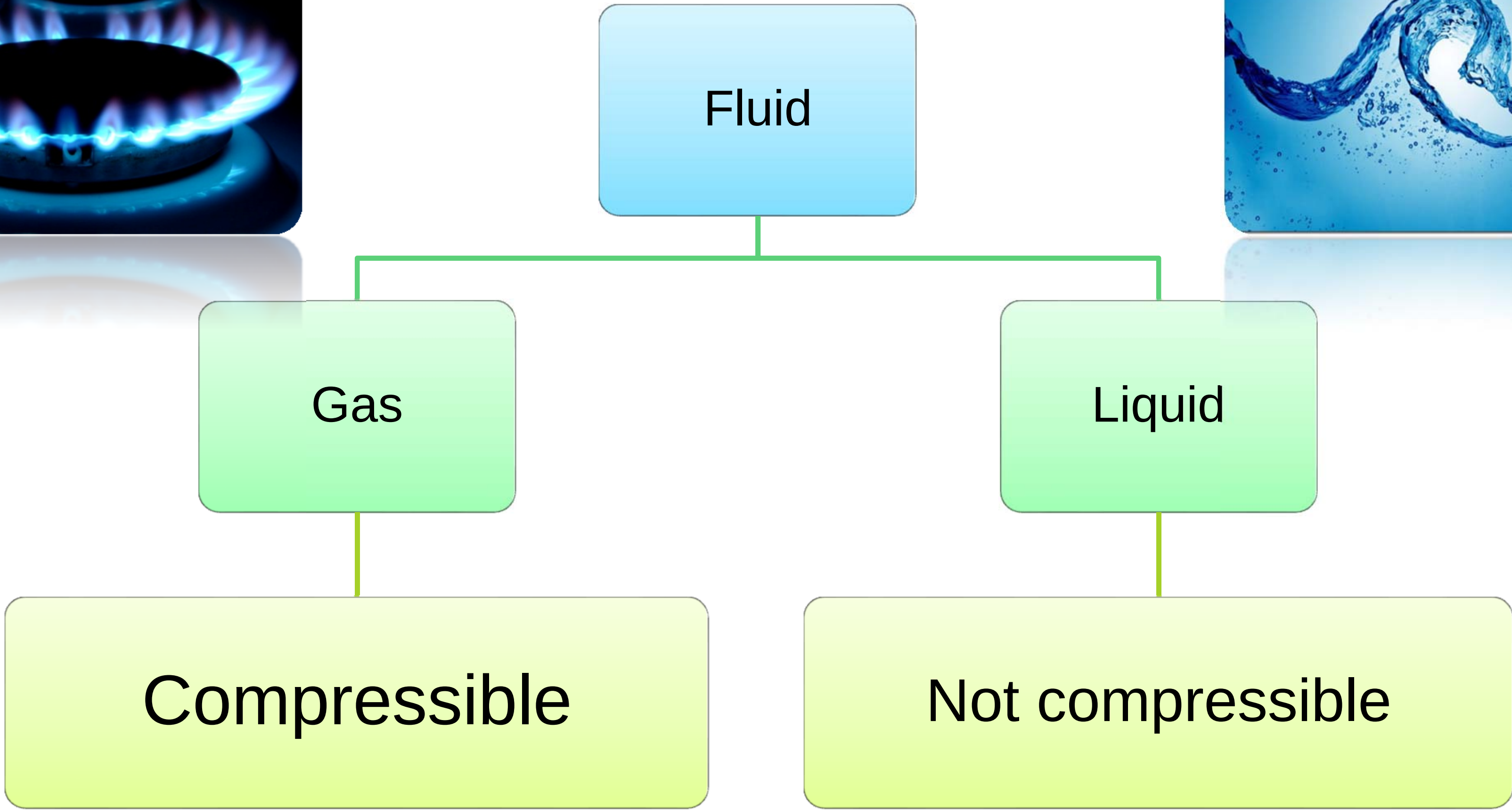
Fluid

Gas | Liquid

Differences

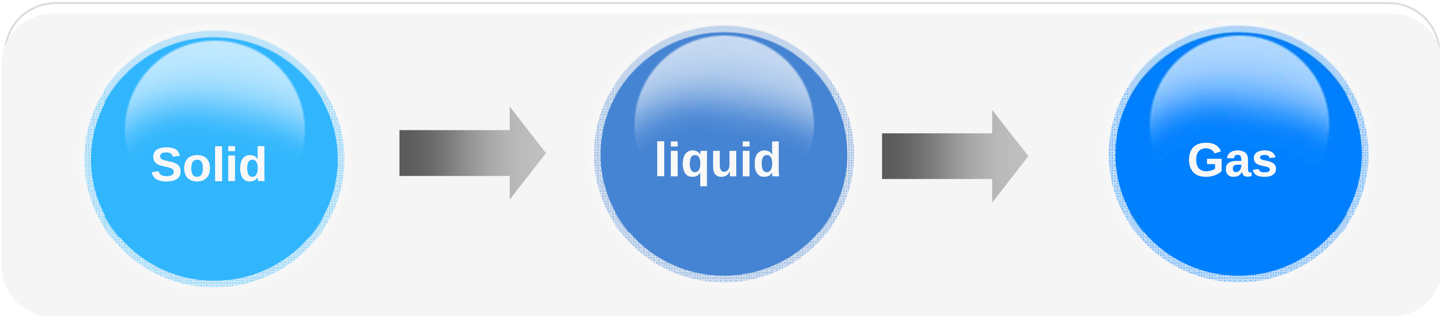
Ref. http://en.wikipedia.org/wiki/Fluid, Courtesy of http://eggfuel.ie/, http://allsizewallpapers.blogspot.com/2009/03/blue-water-widescreen-wallpaper.html

Fluid



Ref. http://en.wikipedia.org/wiki/Fluid, Courtesy of http://eggfuel.ie/, http://allsizewallpapers.blogspot.com/2009/03/blue-water-widescreen-wallpaper.html

States of Matter

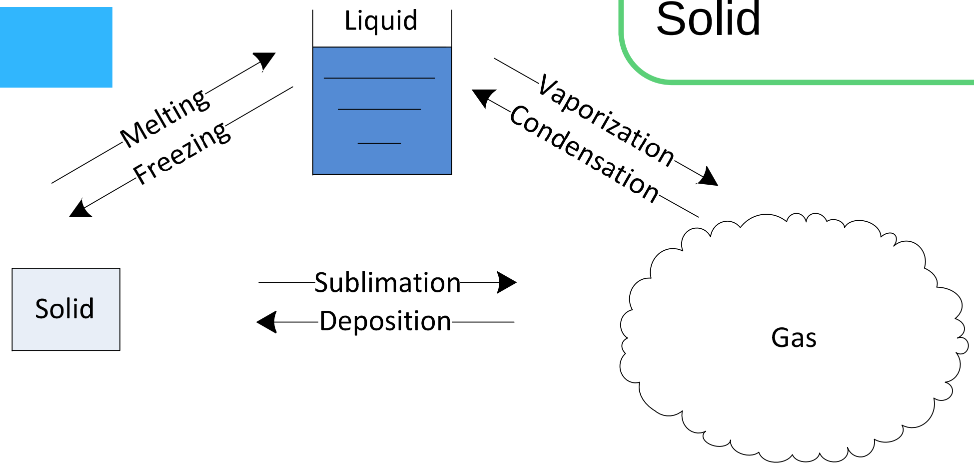


Why is gas compressible?

Liquid

- Shape of the container
- Fixed volume
- Free surface

Distances between molecules in gas are much larger than liquid & Solid



Solid

- Holds shape
- Fixed volume

Gas

- Shape of the container
- Volume of the container

Basic properties of fluids

Density

Density	The density of a fluid is defined as its mass per unit volume. It is denoted by the Greek symbol, ρ.

$$\rho = \frac{m}{V}$$

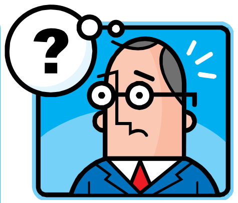
Density. Unit kg/m^3

Mass. Unit kg

Volume. Unit m^3

ρ_{water}= 998 kg/m^3(20°C) How about 4°C?

ρ_{air}= 1.2 kg/m^3(20°C)



Ref. http://en.wikipedia.org/wiki/Properties_of_water, http://en.wikipedia.org/wiki/Density_of_air

Pressure

Pressure	Pressure is the force per unit area, where the force is perpendicular to the area.

Pressure. Unit N/m^2 Or Pascal (Pa)

$$p = \frac{F}{A}$$

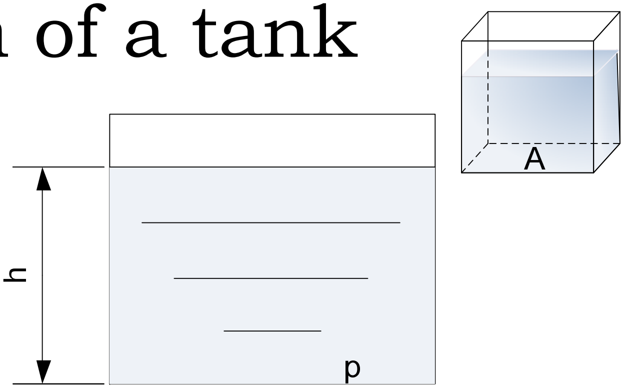
Force. Unit N

Area. Unit m^2

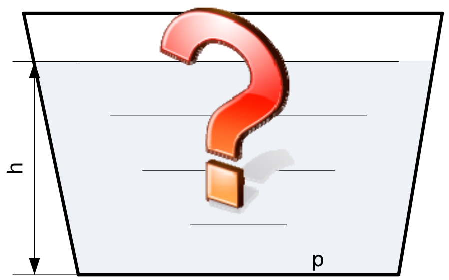
- 1 pa (pascal) = 1 N/m^2(named after Blaise Pascal)
- 1 bar = 100,000 pa ; 1 atm = 101,325 pa

Courtesy of http://en.wikipedia.org/wiki/Blaise_Pascal

Case study:
Pressure on the bottom of a tank



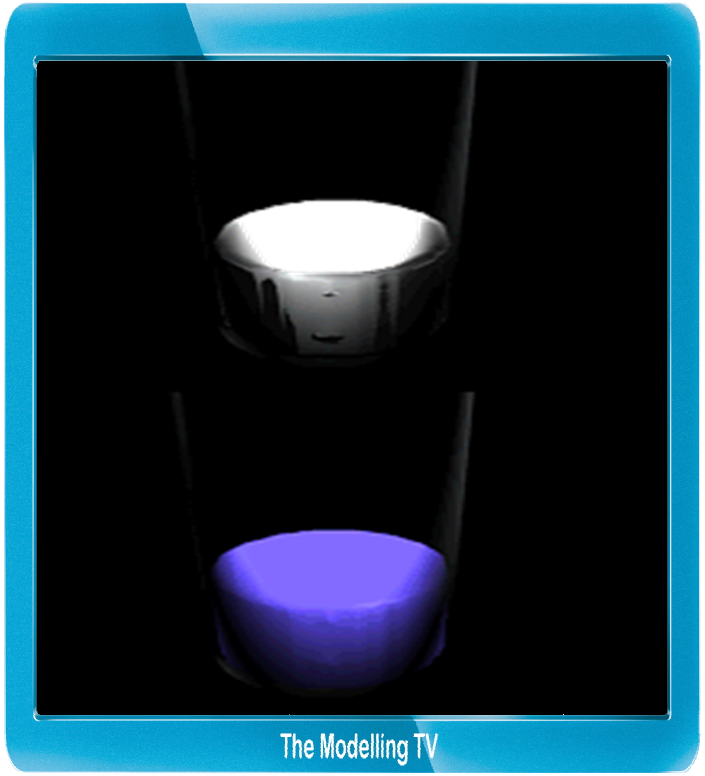
The basic	$p = \dfrac{\text{Weight of the Water}}{\text{Area of the bottom}}$
Mass	$m = \text{Density} \cdot \text{Volume}$ $= \rho \cdot \text{bottom area} \cdot \text{height} = \rho A h$
Pressure	$p = \dfrac{mg}{A} = \dfrac{\rho A g h}{A} = \rho g h$ — Differential form: $dp = \rho g dh$



Viscosity – An example

Water

Honey



- Measures "how thick" the fluid is
- Different types of fluid: Different viscosity
- It is varying, for example, when honey is heated.

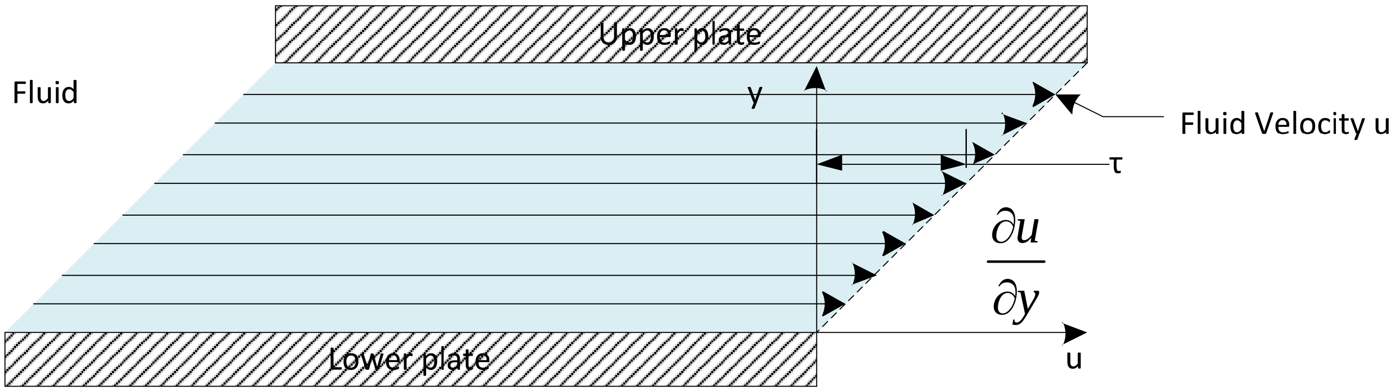
Ref. http://en.wikipedia.org/wiki/Viscosity

Viscosity

Viscosity (Newtonian fluids)	Viscosity is a measure of the resistance of a fluid which is being deformed by either shear stress or tensile stress

$$\tau = \mu \frac{\partial u}{\partial y}$$

- Shear stress . Unit Pa
- Viscosity. Unit Pa.s
- Shear velocity. Unit m/s
- Length. Unit m



Ref. http://en.wikipedia.org/wiki/Viscosity

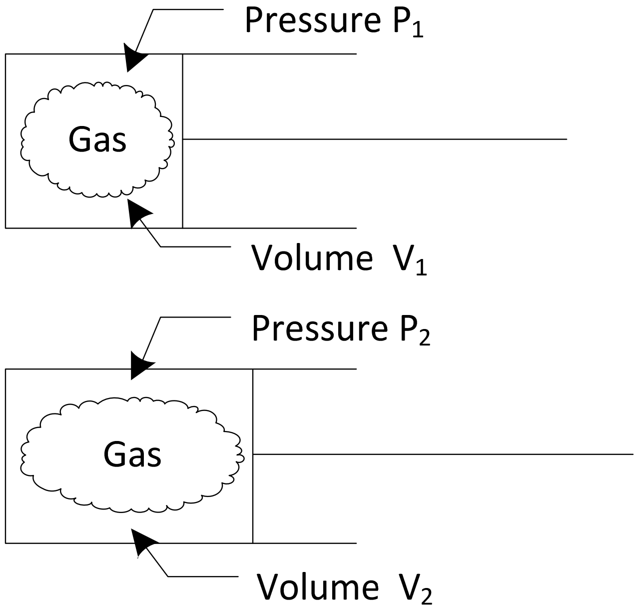
Viscosity – examples

Liquid (25°C)	Viscosity (Pa*s)
Water	8.94e-4
Blood(37°C)	3e-3 to 4e-3
Air	17.4e-6
Olive oil	0.081
Ketchup	50-100
Molten chocolate*	45-130
Honey	2-10
Oil (light)	1.1
Oil (heavy)	6.6
Gasoline	0.006

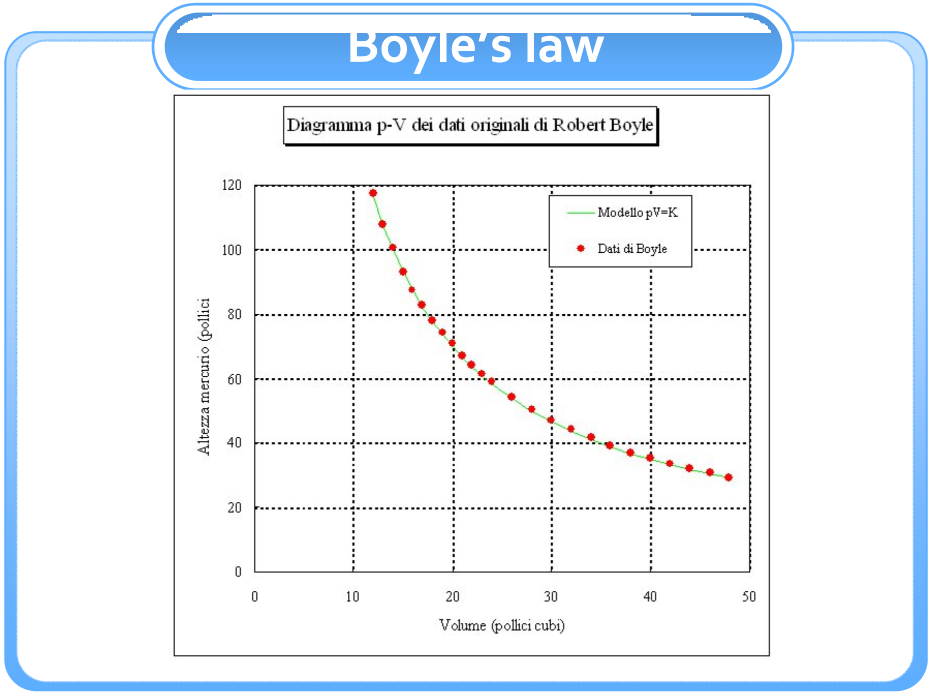
Ref. http://en.wikipedia.org/wiki/Viscosity

Ideal Gas

Physics behind – Boyle's Law



$P_1V_1 = P_2V_2$	Boyle's law



Ref. http://www.antonine-education.co.uk/New_items/STA/Gas%20Laws/Gas_Laws.htm
Courtesy of http://youareaflyonmywall.blogspot.com/2010/04/surviving-and-thriving-in-disaster.html

Case study: The bike pump

The bike pump

```
> restart;
```

Boyles's law

> $equ := p1 \cdot v1 = n \cdot p2 \cdot v2;$

$$equ := p1\,v1 = n p2\,v2 \quad (1.1)$$

Pressure in tyre

> $p1 := (2+1)\cdot 1\cdot 10^5;\ v1 := 2.2\cdot 10^{-3};$

$$p1 := 300000$$
$$v1 := 0.002200000000 \quad (1.2)$$

Pressure in pump & volumne of the pump

> $p2 := 1\cdot 10^5;\ area := 3.14\cdot 0.015^2;\ v2 := 0.5\cdot area;$

$$p2 := 100000$$
$$area := 0.00070650$$
$$v2 := 0.000353250 \quad (1.3)$$

Full strokes

> $fullStrokes := solve(equ, n);$

$$fullStrokes := 18.68365180 \quad (1.4)$$

Max foce needed

> $force := (p1 - p2)\cdot area;$

$$force := 141.3000000 \quad (1.5)$$

```
>
```

The bike pump

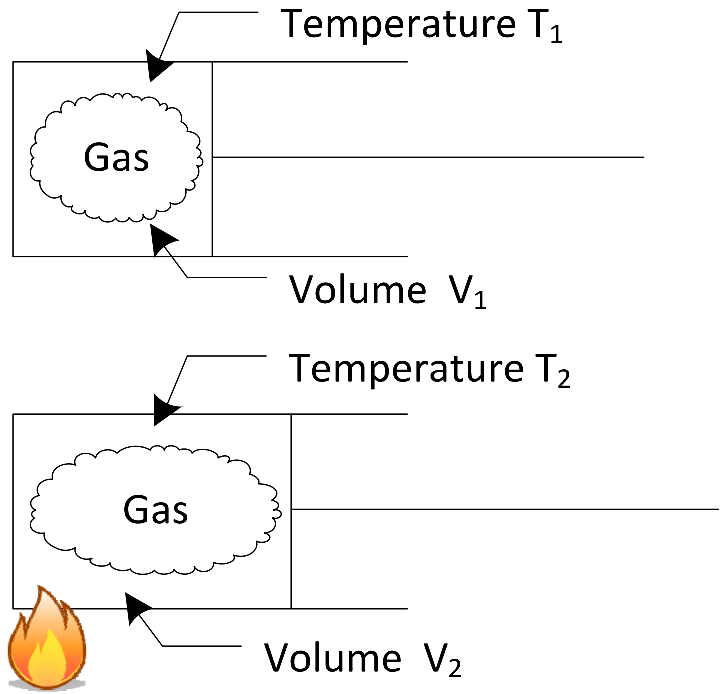
Question:
How many full strokes are needed to pump a flat tyre to **2** bar (relative pressure) using a bike pump?

We choose:
1. the pump has a cylindrical shape; The length is about **50** cm and the inner diameter is **30** mm;
2. the temperature of the pump, air and the tyre are constant during the process; the atmosphere pressure is **1** bar;
3. the tyre is torus-shaped, the volume of the tyre is **2.2** Liter.

Physics behind – Charles' Law

Ref. http://en.wikipedia.org/wiki/Charles's_law

Physics behind – Charles' Law



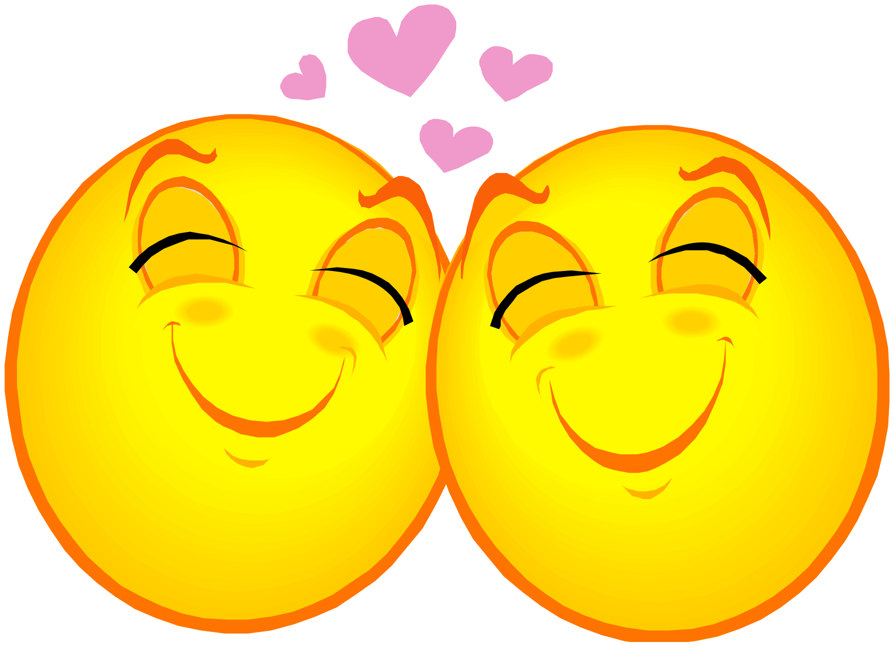
Charles' Law	$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

The Microwave Popcorn

What is the unit of temperature?

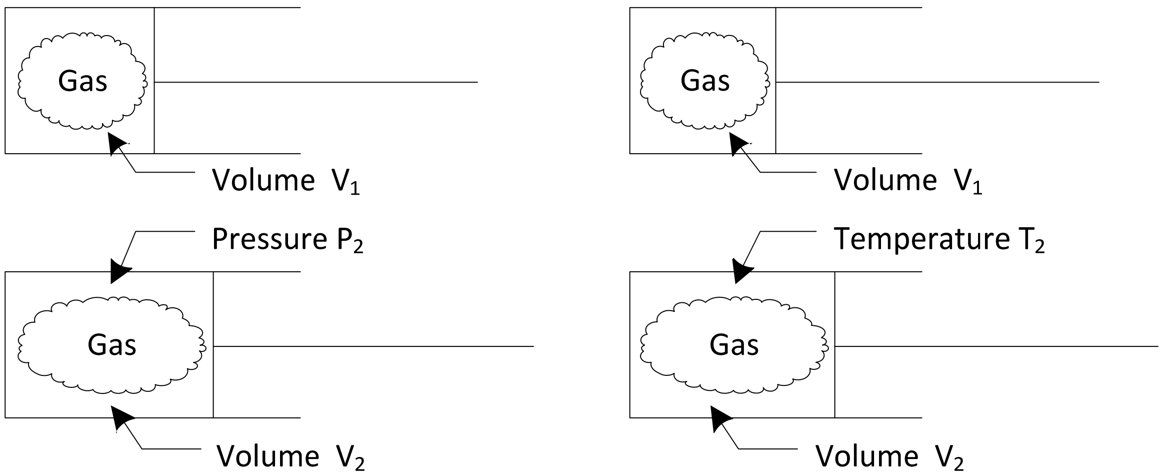
Ref. http://en.wikipedia.org/wiki/Charles's_law

Physics behind
– When Boyle meets Charles



Ref. http://en.wikipedia.org/wiki/Combined_gas_law

Physics behind
– When Boyle meets Charles



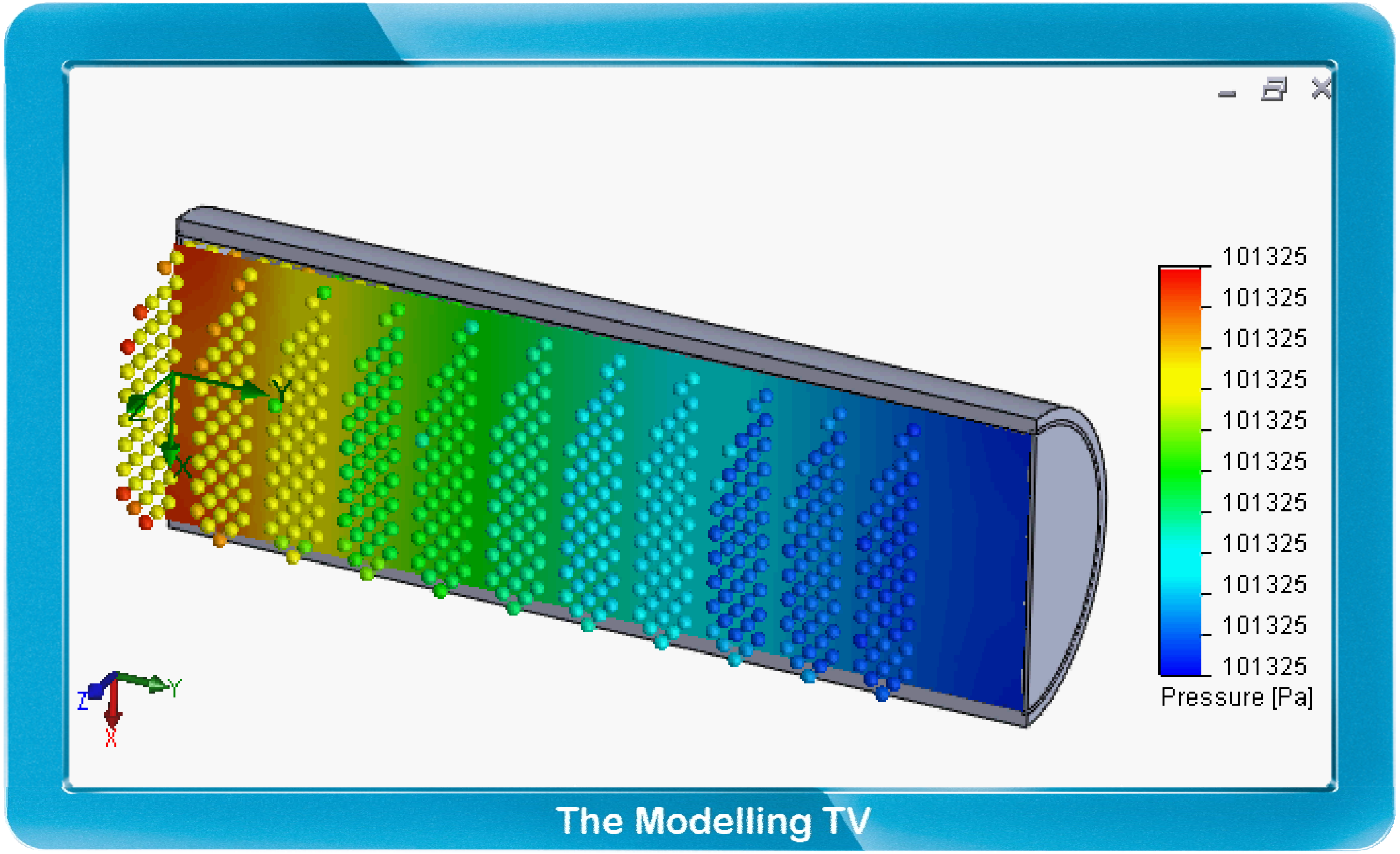
$P_1V_1 = P_2V_2$	Boyle's law	Charles' Law	$\frac{V_1}{T_1} = \frac{V_2}{T_2}$

Combined Gas law: $$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

Ref. http://library.thinkquest.org/12596/combined.html http://en.wikipedia.org/wiki/Combined_gas_law

Flow rate

Liquid moves due to pressure differences



Ref. http://www.freshgasflow.com/physics/flow/flow.html

Flow rate

Liquid Mass flow rate. Unit: Kg/s

$$\dot{m} = \frac{\text{Mass of liquid passing a given area}}{\text{Time}}$$

Liquid volumetric flow rate. Unit: m^3/s

$$q = \frac{\text{Volume of liquid passing a given area}}{\text{Time}}$$

Relations

$$\dot{m} = \rho q$$

Density. Unit: kg/m^3

$$\text{Mass flow rate} = \text{Density} \cdot \text{Volumetric flow rate}$$

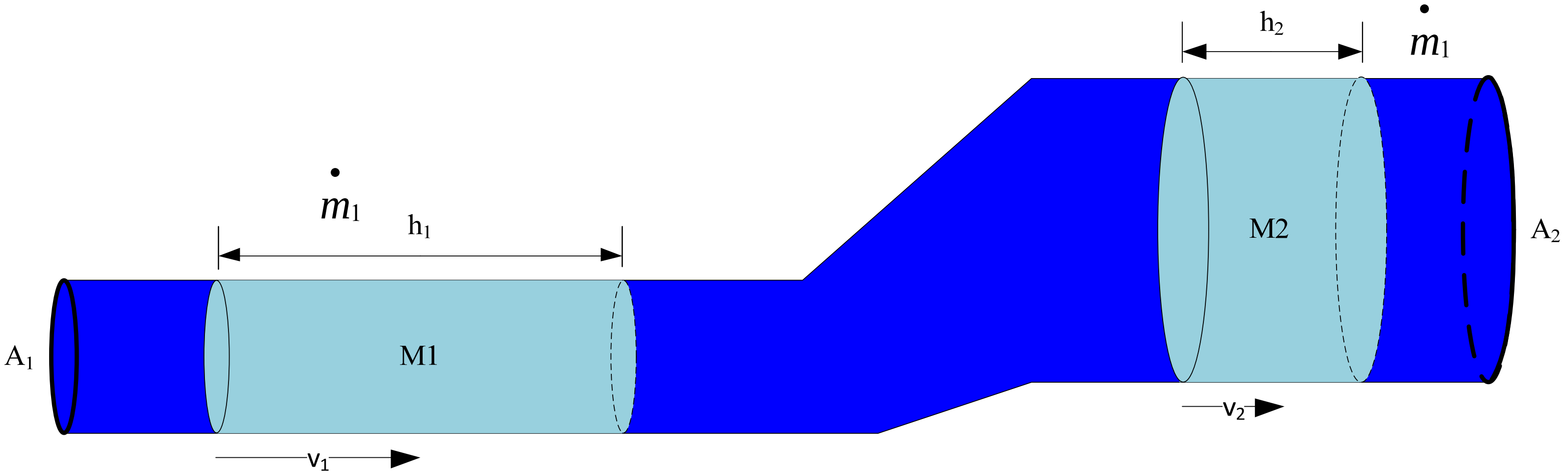
$$\frac{kg}{s} \qquad \frac{kg}{m^3} \cdot \frac{m^3}{s} = \frac{kg}{s}$$

Ref. http://www.freshgasflow.com/physics/flow/flow.html

Basic laws of fluid mechanics

Conservation of Mass

Conservation of mass	The mass of a closed system (in the sense of a completely isolated system) will remain constant over time



$$\dot{m}_1 - \dot{m}_2 = 0 \rightarrow \rho A_1 v_1 t - \rho A_2 v_2 t = 0 \rightarrow A_1 v_1 - A_2 v_2 = 0$$

Ref. http://www.grc.nasa.gov/WWW/K-12/airplane/mass.html

Case study: Garden spray

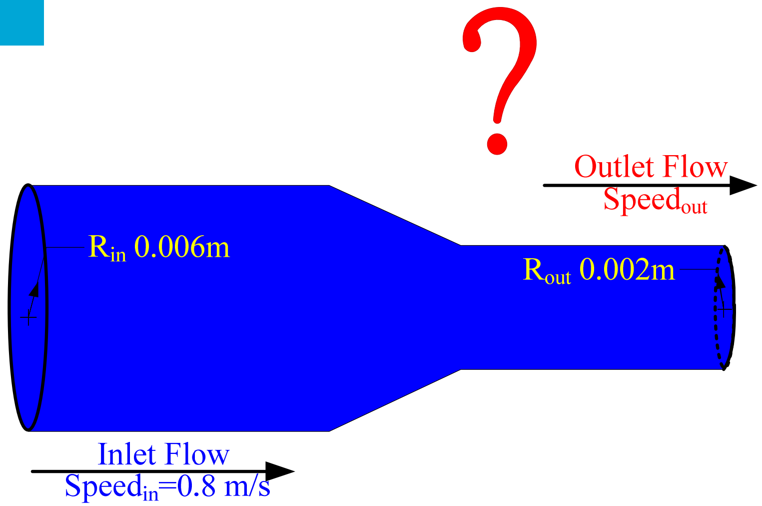
Inlet: Diameter 0.012m
Outlet: Diameter:0.004m

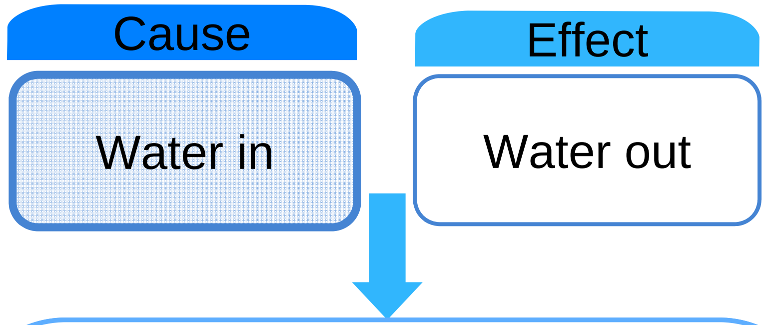
The inlet water speed is 0.8m/s
How about the outlet speed?

- Water
- Inlet
- Outlet

Courtesy of www.gardena.com

Case study: Garden spray





$$WaterOut - Waterin = 0$$

$$\dot{m}_{out} - \dot{m}_{in} = 0, \rho A_{out} v_{out} - \rho A_{in} v_{in} = 0$$

$$\rho \pi R_{out}^2 v_{out} - \rho \pi R_{in}^2 v_{in} = 0$$

$$v_{out} = v_{in} \frac{R_{in}^2}{R_{out}^2} = 7.2 m/s$$

Conservation of energy

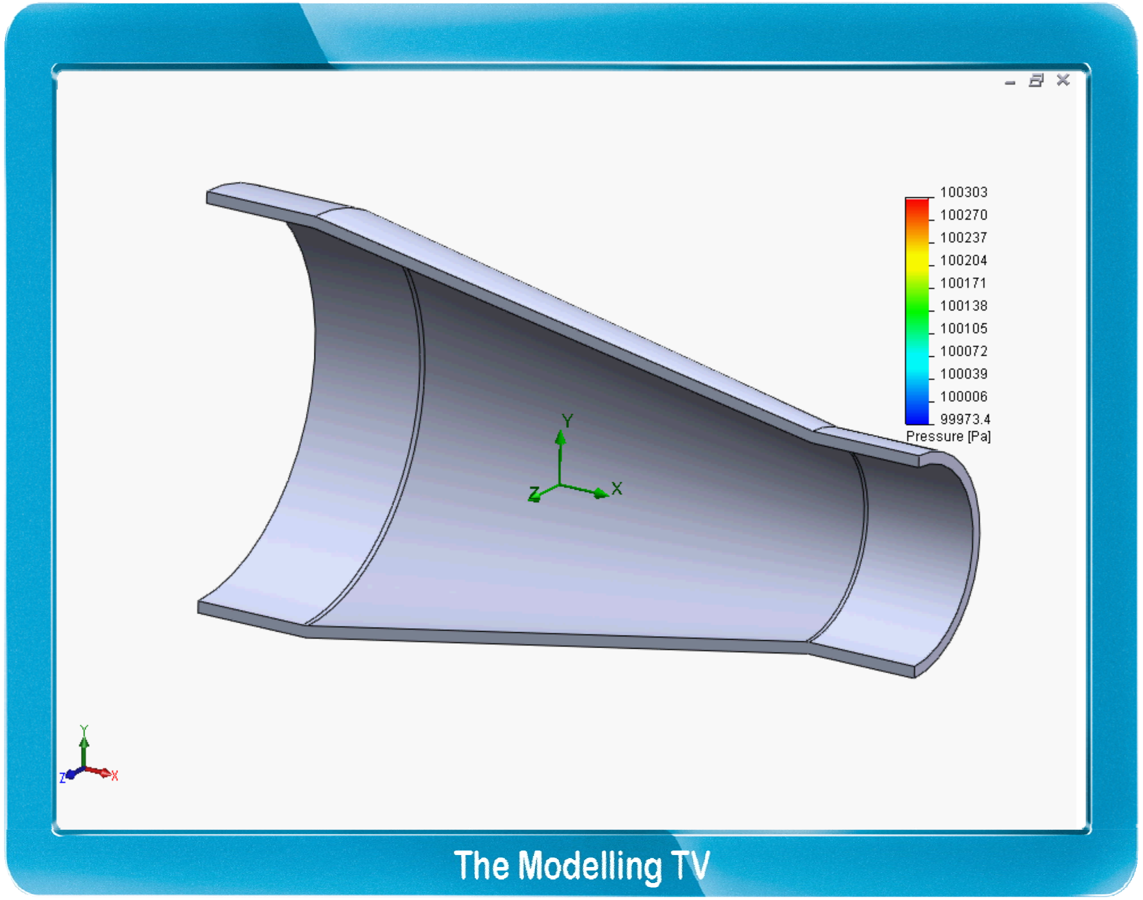
Bernoulli's Equation

$$\frac{p_a}{\rho} + gh_a + \frac{v_a^2}{2} = \frac{p_b}{\rho} + gh_b + \frac{v_b^2}{2} = \text{Constant}$$

For inviscid, adiabatic flow with no additional sources or sinks of energy,
the higher the speed, the lower the pressure

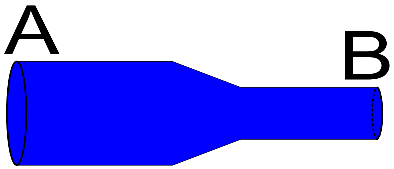
Ref. http://en.wikipedia.org/wiki/Bernoulli's_principle

Conservation of energy



Ref. http://www.freshgasflow.com/physics/flow/flow.html

Conservation of energy



Conservation of Energy	It states that the total amount of energy in an isolated system remains constant over time
Bernoulli's principle	For an inviscid flow, an increase in the speed of the fluid occurs simultaneously with a decrease in pressure or a decrease in the fluid's potential energy.

Flow Work

Kinetic Energy

Bernoulli Equations	$\frac{p_a}{\rho} + gh_a + \frac{v_a^2}{2} = \frac{p_b}{\rho} + gh_b + \frac{v_b^2}{2} = \text{Constant}$

Potential energy

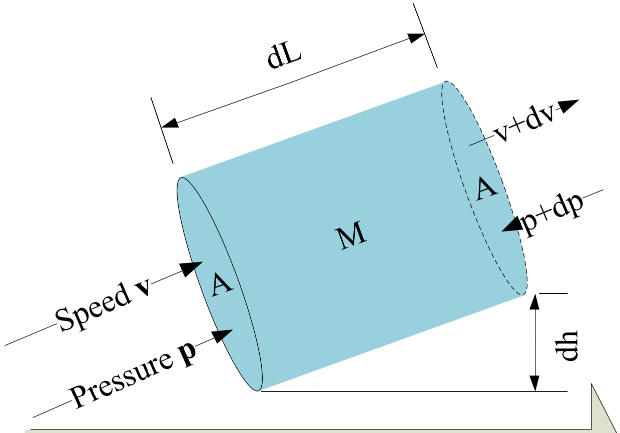
Ref. http://en.wikipedia.org/wiki/Bernoulli's_principle

Conservation of energy

$$\sum F = 0$$

$$F - m\frac{dv}{dt} = 0$$

$$p \cdot A - (p + dp) \cdot A - m \cdot g \cdot \frac{dh}{dL} - m \cdot \frac{dv}{dt} = 0$$



$$-A \cdot dp - \rho \cdot A \cdot dL \cdot g \cdot \frac{dh}{dL} - \rho \cdot A \cdot dL \cdot \frac{dv}{dt} = 0$$

$$-dp - \rho \cdot g \cdot dh - \rho \cdot dL \cdot \frac{dv}{dL} \cdot \frac{dL}{dt} = 0$$

$$\frac{dp}{\rho} + g \cdot dh + v \cdot dv = 0$$ Euler's Equation

Integrate Euler's equation along a streamline, between two points *a*, *b*

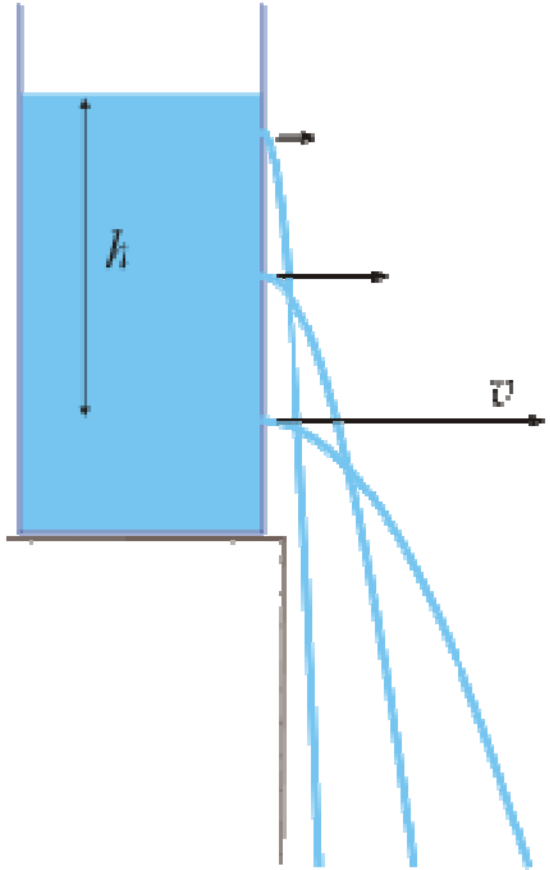
$$\int_a^b \frac{dp}{\rho} + \int_a^b g \cdot dh + \int_a^b v \cdot dv = 0$$

$$\frac{p_b - p_A}{\rho} + g \cdot (h_b - h_a) + \frac{v_b^2 - v_a^2}{2} = 0$$

Bernoulli's Equation

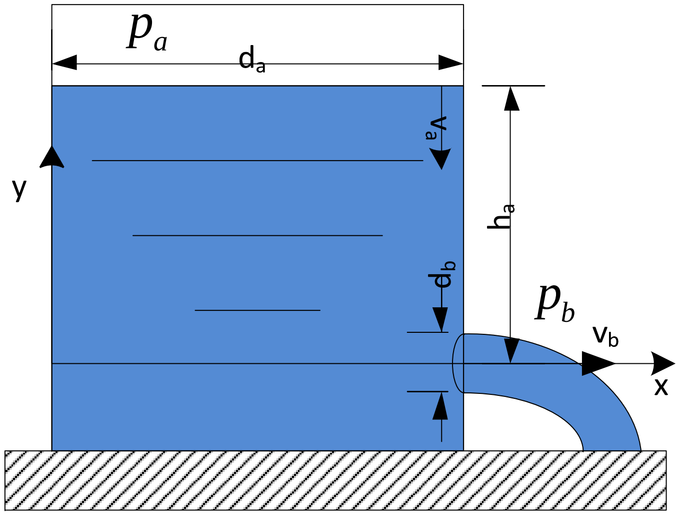
$$\frac{p_b}{\rho} + g \cdot h_b + \frac{v_b^2}{2} = \frac{p_b}{\rho} + g \cdot h_b + \frac{v_b^2}{2} = \text{Constant}$$

Case study: Torricelli's law



Ref. http://en.wikipedia.org/wiki/Torricelli's_law

Case study: Torricelli's law



$$\frac{p_a}{\rho} + gh_a + \frac{v_a^2}{2} = \frac{p_b}{\rho} + gh_b + \frac{v_b^2}{2}$$

$$p_a = p_b$$

$$h_b = 0$$

if $d_a >> d_b$,

v_a can be neglected

Torricelli's law:

$$v_b = \sqrt{2gh_a}$$

Ref. http://en.wikipedia.org/wiki/Torricelli's_law

Case study: Garden spray - pressure

Inlet: Diameter 0.012m
Outlet: Diameter:0.003m

The inlet water pressure is 1.3bar
How about the outlet speed?

- Water
- Inlet
- Outlet

Courtesy of www.gardena.com

Case study: Garden spray

Cause: Energy in

Effect: Energy out

Cause: Water in

Effect: Water out

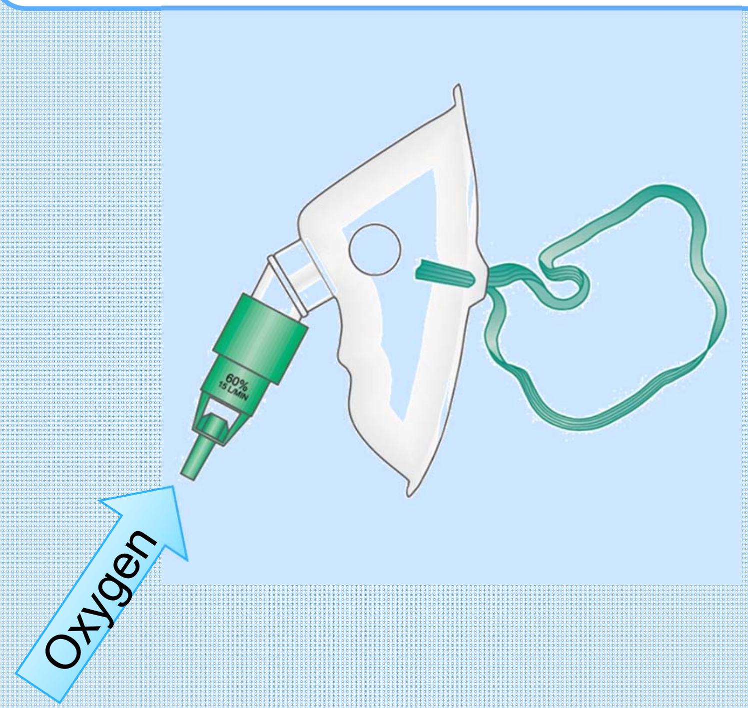
$$\frac{p_{in}}{\rho} + gh_{in} + \frac{v_{in}^2}{2} = \frac{p_{out}}{\rho} + gh_{out} + \frac{v_{out}^2}{2}$$

$$\dot{m}_{out} - \dot{m}_{in} = 0 \rightarrow v_{out} = v_{in}\frac{R_{in}^2}{R_{out}^2}$$

$$v_{in} = 0.87, v_{out} = 7.79$$

Product – Venturi Masks

The venturi mask is a medical device to deliver a known oxygen concentration to patients on controlled oxygen therapy.

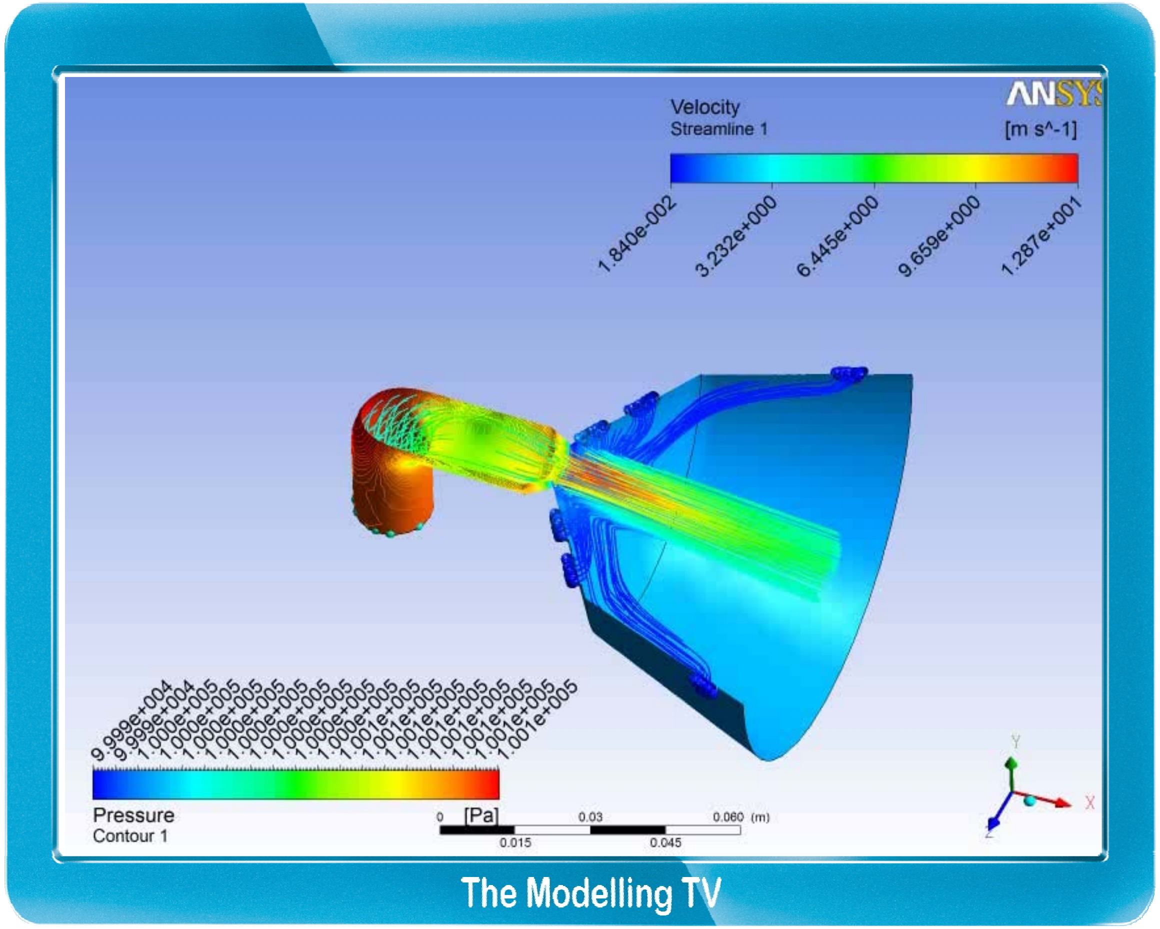


The color of the device reflects the delivered oxygen concentration, for example:

blue = 24%;
yellow = 28%;
white = 31%;
Green = 35%;
pink = 40%;
orange = 50%.
oxygen therapy.

Ref. http://en.wikipedia.org/wiki/Venturi_mask, http://www.flexicare.com/us/products/oxygen--aerosol-therapy/fixed-concentration/ventimask.aspx, http://boundtree.co.uk/index.php?route=product/product&product_id=1807

Product – Venturi Masks



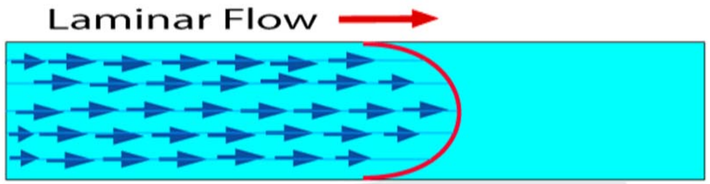
Ref. http://en.wikipedia.org/wiki/Venturi_mask, http://www.flexicare.com/us/products/oxygen--aerosol-therapy/fixed-concentration/ventimask.aspx,
http://boundtree.co.uk/index.php?route=product/product&product_id=1807

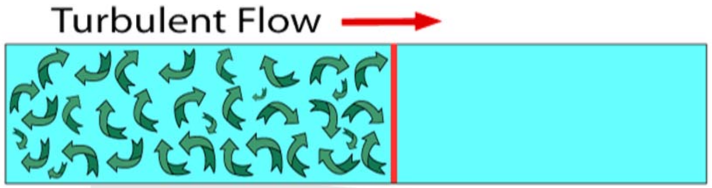
Resistances

Bullet train

Courtesy of http://bajansunonline.com/japans-new-bullet-train-debuts/

Laminar vs Turbulent flow





1. Laminar flow occurs in smooth tubes and at LOW flow rates.
2. Laminar is streamlined and there is no turbulence. The flow occurs in parallel layers, with minimal disruption between these layers.
3. Laminar is greatest at the centre and diminishes towards the periphery. This makes the laminar flow describe a bullet shaped "velocity profile".

1. Turbulent flow flows in rough tubes and at higher flow rates.
2. The flow is not streamlined. There is a lot of swirling (eddies) of the fluid.
3. The flow is not greatest at the centre. Thus, as shown in red, the "velocity profile" of turbulent flow is more flat than that caused by laminar flow.

Ref http://www.freshgasflow.com/physics/flow/flow.html

Reynolds number

In fluid mechanics, the **Reynolds number** (**Re**) is a dimensionless number that gives a measure of the ratio of inertial forces to viscous forces.

$$Re = \rho \frac{vL}{\mu}$$

- v: mean velocity of the object relative to the fluid
- L: characteristic linear dimension
- ρ: density of the fluid
- μ: dynamic viscosity of the fluid

Ref: http://en.wikipedia.org/wiki/Reynolds_number

The meaning of Reynolds number

$$Re = \rho\frac{vL}{\mu} = \rho\frac{vL}{\mu}\cdot\frac{vL}{vL} = \frac{\rho v^2 L^2}{\mu v L}$$

The inertia force

The viscous forces

Laminar

- when *Re* < *2300*

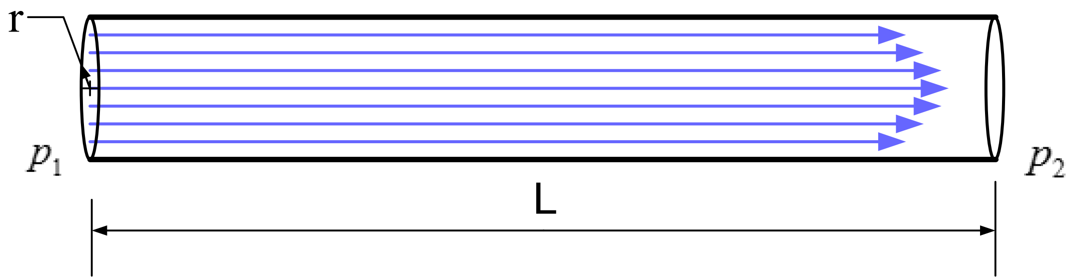
Transient

- when *2300* < *Re* < *4000*

Turbulent

- when *Re* > *4000*

Laminar flow resistances of a level tube - Hagen–Poiseuille equation

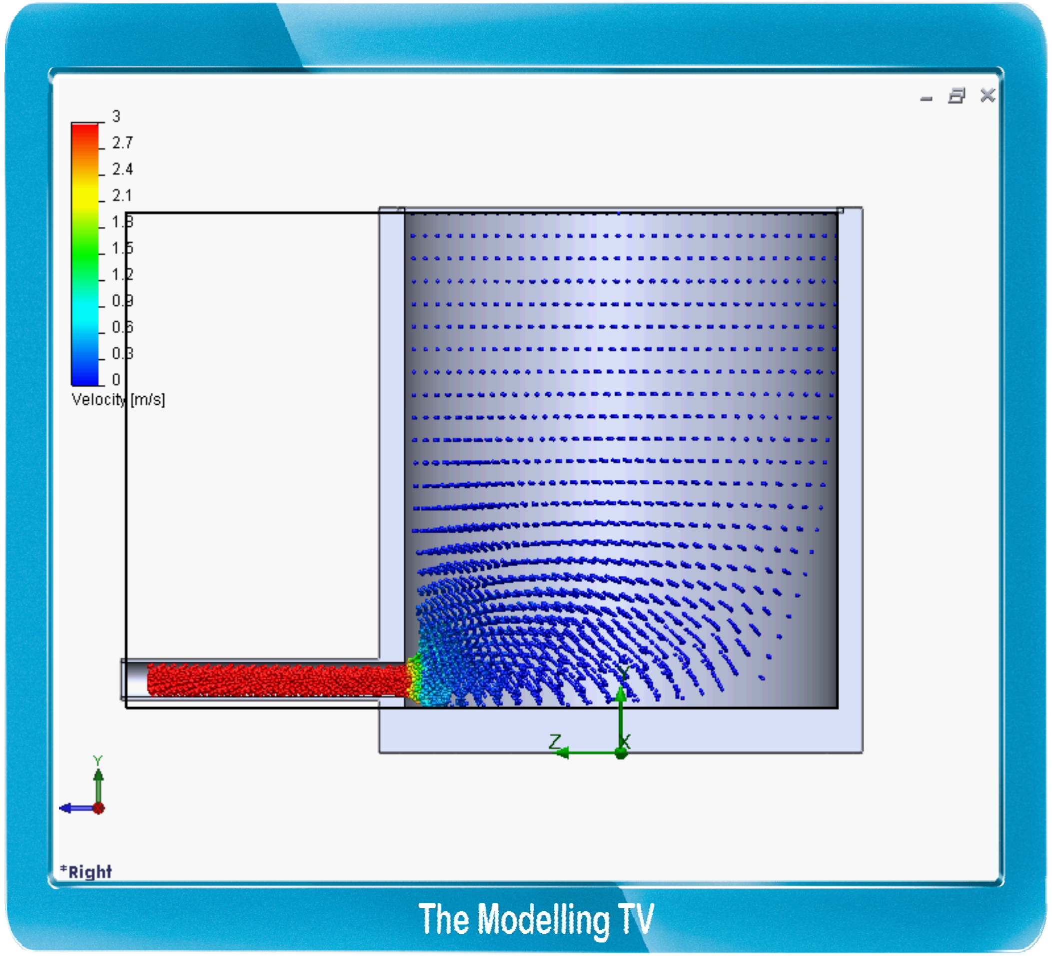


$$\text{Mass Flow Rate} = \frac{(p_1 - p_2)}{\text{Tube Laminar Resistances R}}$$

$$R = \frac{8 \cdot \text{Viscosity} \cdot \text{Tube Length}}{\pi \cdot \rho \cdot (\text{Tube Radius})^4}$$

Ref. http://en.wikipedia.org/wiki/Hagen–Poiseuille_equation

Orifice resistances

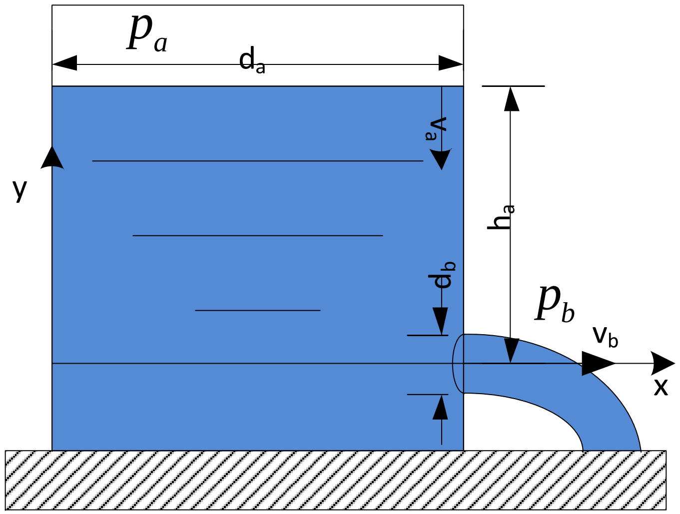


A discount from the ideal speed
(Torricelli's law)

Named:
Discharge coefficient

Ref. http://en.wikipedia.org/wiki/Torricelli's_law

Orifice resistances



Torricelli's law(http://en.wikipedia.org/wiki/Torricelli's_law)

$$v_b = \sqrt{2gh_a}$$

With discharge coefficient

$$v_b = c_d\sqrt{2gh_a}$$

Mass flow rate

$$\dot{m}_b = \rho A_b v_b = \rho A_b c_d \sqrt{2gh_a}$$

Generalize

$$\dot{m} = \sqrt{\frac{\rho \cdot g \cdot h}{\frac{1}{2 \cdot \rho \cdot C_d^{\,2} \cdot A^2}}} = \sqrt{\frac{p}{R}}$$

Ref. http://en.wikipedia.org/wiki/Torricelli's_law

Case study – Garden rain water barrels

Cylinder: Bottom area $0.2 m^2$
Water height: 0.5m
Orifice radius:0.02m
Discharge coefficient
of the orifice: 0.6

How long will the water stream last?

- Water in barrel
- Earth
- Orifice

Courtesy of http://mymontys.com/wordpress/?m=201008

Case study – Garden rain water barrels

	Cause	Effect
1	Gravity & Orifice	Water level drops in the barrel
2	Pressure	Water go out of orifice
3	Orifice	Slows the speed
4	Water lost in the barrel	Water out from the orifice

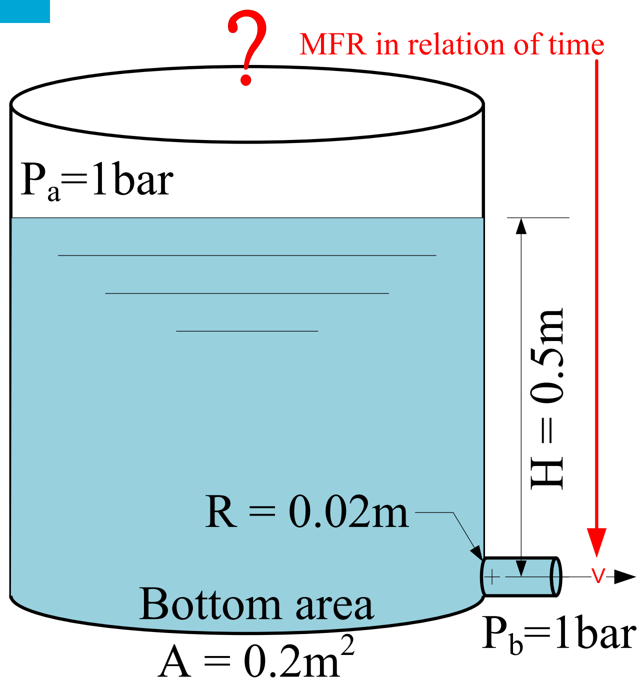
$$\dot{m}_1 = -\rho \frac{dh(t)}{dt} A_{barrel}$$

$$\dot{m}_2 = \rho C_d A_{orifice} \sqrt{2gh(t)}$$

$$\dot{m}_1 = \dot{m}_2$$

$$-\rho \frac{dh(t)}{dt} A_{barrel} = \rho C_d A_{orifice} \sqrt{2gh(t)}$$

Case study – Gar



Think about the question

Components in the system:

Water
Earth
Barrel
Orifice

Cause - effect

gravity-> pressure differences
pressure difference -> water moves
orifice-> resistances

Model the system

```
> restart :
> massflowrate := t→C_d·Ab·rho·sqrt(2·g·h(t));
```

$$massflowrate := t \rightarrow C_d\,Ab\,\rho\,\sqrt{2\,g\,h(t)} \qquad (2.1)$$

The consevation of mass

```
> ode := A·rho·diff(h(t), t) =-massflowrate(t)
```

$$ode := A\,\rho\left(\frac{d}{dt}\,h(t)\right) = -C_d\,Ab\,\rho\,\sqrt{2}\,\sqrt{g\,h(t)} \qquad (2.2)$$

Solve the problem

parameters

```
> g := 9.8 : C_d := 0.6 : rho := 1000 : Ab := 3.14·r^2; r := 0.02 : A := 0.2 :
```

$$Ab := 3.14\,r^2 \qquad (3.1)$$

Initial conditions, let's suppose h0 is the initial height since the drum is not always full

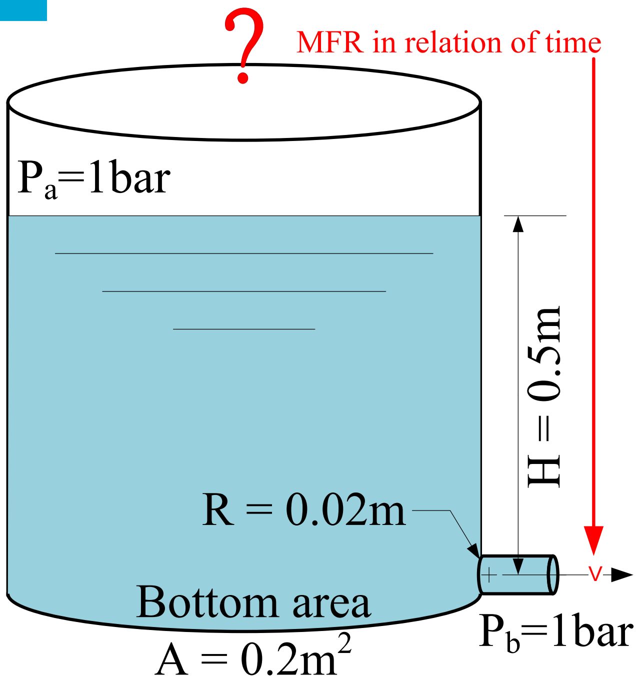
```
> ics := h(0) = 0.5 :
```

review the ode

```
> ode
```

(3.2)

Case study – Gar



$$200.0\left(\frac{d}{dt}h(t)\right) = -2.359141159\sqrt{2}\sqrt{h(t)} \tag{3.2}$$

solve

```
> dsol := dsolve({ode, ics}, h(t));
```

$$dsol := h(t) = \frac{5565547008087863281}{8000000000000000000000}t^2 - \frac{2359141159}{200000000000}t + \frac{1}{2} \tag{3.3}$$

change the solution to a function named h with parameter r and time t

```
> hh := rhs(dsol) : h := unapply(hh, t);
```

$$h := t \rightarrow \frac{5565547008087863281}{8000000000000000000000}t^2 - \frac{2359141159}{200000000000}t + \frac{1}{2} \tag{3.4}$$

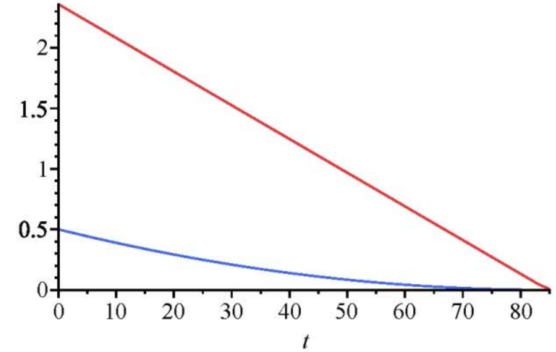
now the flow rate

```
> massflowrate(t)
```

$$0.7536000\sqrt{0.001363559017\,t^2 - 0.2311958336\,t + 9.800000000} \tag{3.5}$$

The flow rate

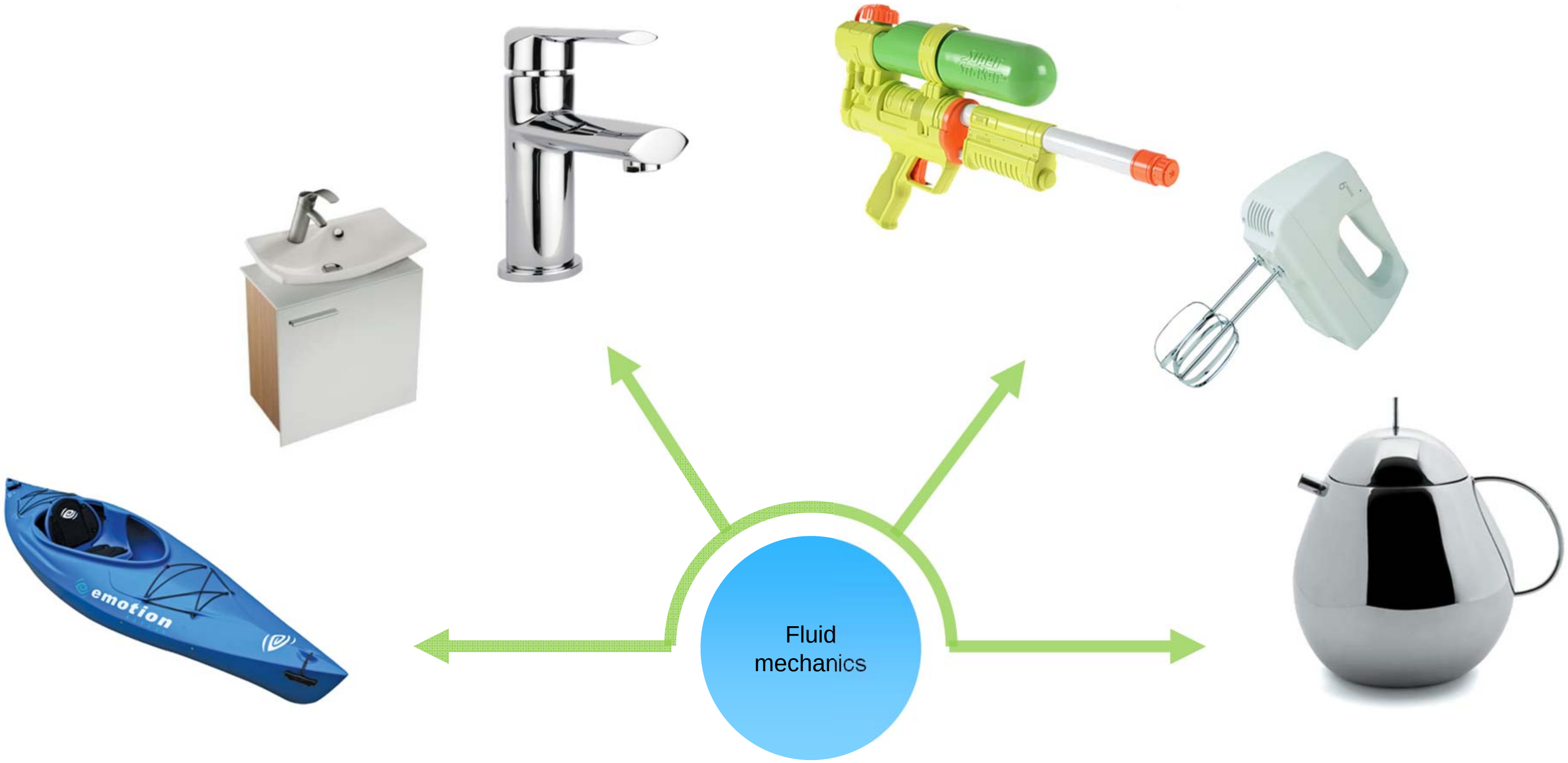
```
> plot([massflowrate(t), h(t)], t=0..85, color=[red, blue])
```




```
>
```

Industrial design applications

Applications



Courtesy of
http://www.ferrocompany.com/product-2884-BRT2.html
http://brokenjpg.net/?tag=water-guns
http://www.halebathrooms.co.uk/escale-wash-basin-cabinet-3220-p.asp
http://suriptotitl.wordpress.com/2012/06/02/mixer/
http://www.connox.com/categories/cooking/tea/fruit-basket-teapot.html
http://lakeannalife.com/outfitters.htm

Thank you

The Modelling Team
Department of Design Engineering
Faculty of Industrial Design Engineering
Delft University of Technology