

Pen and Paper Exercises - determinants

Theorem 1

- a) If A and B are $n \times n$ matrices, then $\det(AB) = (\det A)(\det B)$
- b) If A is an $n \times n$ matrix, then $\det A = \det A^T$
- c) An $n \times n$ matrix A is invertible if and only if $\det A \neq 0$

1. Use the relevant definitions and/or Theorem 1 to prove the following statements:

(a) If A is an invertible matrix, then $\det A^{-1} = \frac{1}{\det A}$

(b) If A and B are $n \times n$ matrices, then $\det(AB) = \det(BA)$

2. Use properties of determinants to evaluate the given determinant by inspection. Explain your reasoning.

(a) $\det \begin{bmatrix} 1 & 0 & 3 \\ 2 & 0 & 4 \\ 5 & 0 & 6 \end{bmatrix}$

(b) $\det \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 5 & 0 & 6 \end{bmatrix}$

(c) $\det \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 6 \end{bmatrix}$

(d) $\det \begin{bmatrix} 0 & 0 & 3 \\ 0 & 2 & 4 \\ 1 & -2 & 6 \end{bmatrix}$