## Pen and Paper Exercises - introduction to linear transformations

Theorem 1
A linear transformation $T: \mathbb{R}^{2} \rightarrow \mathbb{R}^{2}$ maps a straight line to a straight line or to a point.
Theorem 2
A linear transformation $T: \mathbb{R}^{2} \rightarrow \mathbb{R}^{2}$ maps parallel lines to parallel lines, a single line, a pair of points or a single point.

1. Prove Theorem 1.
2. Prove Theorem 2.
3. Let $A B C D$ we the square with vertices $(-1,-1),(-1,1),(1,1)$ and $(1,-1)$. Draw the image of $A B C D$ under the following transformations $T: \mathbb{R}^{2} \rightarrow \mathbb{R}^{2}, T \mathbf{x}=A \mathbf{x}$.
(a) $A=\left[\begin{array}{cc}-1 & 1 \\ 1 & 1\end{array}\right]$
(b) $A=\left[\begin{array}{cc}0.5 & 0 \\ 0 & 2\end{array}\right]$
(c) $A=\left[\begin{array}{cc}0.5 & -0.5 \\ -0.5 & 0.5\end{array}\right]$
(d) $A=\left[\begin{array}{cc}0 & 1 \\ -1 & 0\end{array}\right]$
