## Pen and Paper Exercises - introduction to linear transformations

Theorem 1

A linear transformation  $T: \mathbb{R}^2 \to \mathbb{R}^2$  maps a straight line to a straight line or to a point.

Theorem 2

A linear transformation  $T : \mathbb{R}^2 \to \mathbb{R}^2$  maps parallel lines to parallel lines, a single line, a pair of points or a single point.

- 1. Prove Theorem 1.
- 2. Prove Theorem 2.
- 3. Let ABCD we the square with vertices (-1, -1), (-1, 1), (1, 1) and (1, -1). Draw the image of ABCD under the following transformations  $T : \mathbb{R}^2 \to \mathbb{R}^2$ ,  $T\mathbf{x} = A\mathbf{x}$ .

(a) 
$$A = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}$$
  
(b)  $A = \begin{bmatrix} 0.5 & 0 \\ 0 & 2 \end{bmatrix}$   
(c)  $A = \begin{bmatrix} 0.5 & -0.5 \\ -0.5 & 0.5 \end{bmatrix}$   
(d)  $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$