Pen and Paper Exercises - systems of linear equations

1. Draw the graps corresponding to the equations of the linear system and use these graphs to determine the number of solutions of the linear system. Solve the linear system to confirm your answer.

   (a) $\begin{cases} x+y &= 1 \\ 2x+2y &= 1 \end{cases}$

   (b) $\begin{cases} x+y &= 1 \\ x+2y &= 1 \end{cases}$

   (c) $\begin{cases} x+y &= 1 \\ 2x+2y &= 2 \end{cases}$

2. Determine the augmented matrices corresponding to the linear systems of question 1.

3. Determine the linear systems corresponding to the following augmented matrices.

   (a) $\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{array}\right]$

   (b) $\left[\begin{array}{ccc|c} 1 & 0 & 1 & 3 \\ 0 & 0 & -1 & 2 \end{array}\right]$

4. Give all possible reduced row echelon forms of a 3×3 matrix.

5. The augmented matrix corresponding to a linear system is given. Row reduce the augmented matrix to echelon form for all values of h and determine the number of solutions. Motivate your answer.

   (a) $\left[\begin{array}{cc|c} 1 & h & 1 \\ h & 4 & 2 \end{array}\right]$

   (b) $\left[\begin{array}{ccc|c} h & 2 & -6 & 4 \\ 0 & h+1 & h-3 & 3 \\ 0 & 0 & h^2-4 & h-2 \end{array}\right]$

6. According to Anna a matrix with two rows is always row equivalent to a matrix with one zero row. She uses the following argument: add the first row of the matrix to the second row and and the second row of the matrix to the first row. The two rows of the matrix are now the same: subtract the first row from the second row to obtain a matrix with a zero row. Bart convinces that this is not correct; how does he convince her?

7. Give an example of three planes such that:

   (a) they have a common line of intersection (figure A);

(b) they intersect in pairs but have no common point of intersection (figure B);

(c) exactly two of them are parallel (figure C);

(d) they intersect in a single point (figure D).

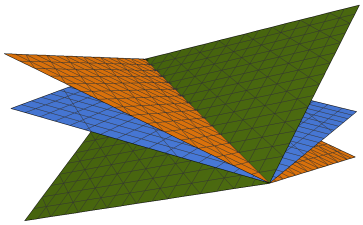

figure A

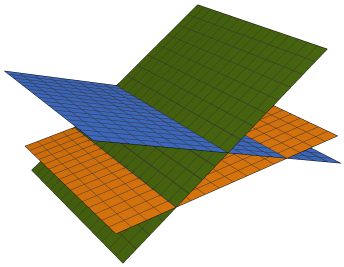

figure B

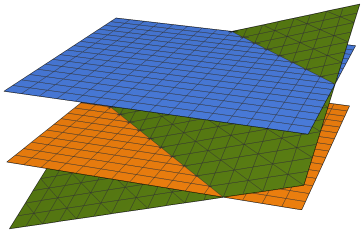

figure C

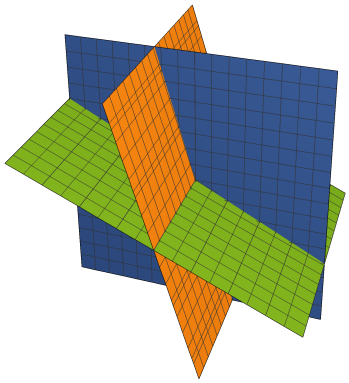

figure D