Post lecture questions Supply or Network Modelling 2017 plus answers

Uncongested networks

Consider the following statements concerning (shortest) paths on a network. Are they true or false?

- The number of links in a shortest path tree is always exactly N-1 (where N is the number of nodes)
- The path between any two nodes in a shortest path tree is again a shortest path
- For each OD Pair there is only a single shortest path

True, False, False

Consider the following statements concerning route choice. Are they true or false?

- Link flows and route travel times are outcomes of the traffic assignment problem.
- In the all-or-nothing assignment, all OD flows are assigned to the shortest route.
- A shortest-path all-or-nothing assignment can be computed by minimizing the total travel time subject to flow conservation and non-negativity constraints.
- Outcomes of a shortest-path all-or-nothing assignment are ‘stable’ outcomes; minor changes in the network structure only result in minor changes in the assignment outcomes.

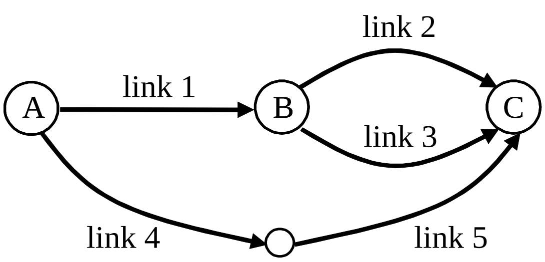
True, True and False, True, False

Consider the following statements concerning stochastic assignment. Are they true or false?

- In a shortest path AON assignment, the traveler chooses the objective shortest route, whereas in stochastic assignment the traveler chooses the subjective shortest route.
- The logit assignment is always based on route level, whereas the probit assignment is solved on the link level.
- The error term in stochastic assignment is the same for each route/link.
- Nested logit models and probit models are able to deal with overlapping route alternatives.

True, True, False, True and False

Consider the following transportation network with two origins (A and B) and two destinations (B and C).



Each link in the network is assumed identical and the travel time on each link a is given by t_a.

The travel demand trip matrix is given as follows:

from/to	**A**	**B**	**C**
A	-	500	2000
B	-	-	3500
C	-	-	-

A stochastic assignment can be determined using the logit model or the probit model.

(a) Determine the link flows in a stochastic assignment using the logit model.

For each OD-pair we can check the possible routes. This leads to the following route choice set:

OD-pair	route	links on route
AB	1	1
AC	1	1,2
	2	1,3
	3	4,5
BC	1	2
	2	3

For OD-pair AB only one route exists. Therefore all travellers will use this route. For OD-pair AC three routes exist, whereas all three routes are equal in length (and thus equal in travel time, since for the stochastic assignment the travel time is independent of the flow). Since all routes are equal, the travellers between origin A and destination C will be equally distributed over these three routes. Finally, for OD-pair BC two routes exist of equal length. Therefore, similarly, the travellers between origin B and destination C will be uniformly distributed over these two routes.

The link flows can then be computed from the route flows, by looking at the routes which use a certain link.

link 1: $500 + 1/3 * 2000 + 1/3 * 2000 = 1833.3$
link 2: $1/3 * 2000 + 1/2 * 3500 = 2416.7$
link 3: $1/3 * 2000 + 1/2 * 3500 = 2416.7$
link 4: $1/3 * 2000 = 666.7$
link 5: $1/3 * 2000 = 666.7$

(b) Will the flows on links 4 and 5 increase or decrease in case the probit model is used instead of the logit model? Explain your answer.

The logit model assumes independent alternatives. If more than two routes are available of which two are largely overlapping (in this case the routes using links 1-2 and links 1-3), all routes are still considered as independent alternatives. This results in an overestimation of traffic on the overlapping routes. The probit assignment can deal with overlapping route alternatives, since in the probit assignment the error terms are defined on link level (instead of route level). Hence, in case the probit model is used, traffic will shift from the (formerly overestimated) upper routes to the lower route consisting of links 4-5. The flows on these links will therefore increase.

(c) Use the following random numbers to apply the probit model to this network. Use 10 iterations.

	1	**2**	**3**	**4**	**5**
1	0.796	1.062	1.027	1.08	1.192
2	1.104	0.919	1.241	1.037	0.764
3	0.888	1.141	1.181	0.978	1.036
4	1.194	0.984	0.813	0.829	1.104
5	0.766	0.937	1.043	1.216	0.911
6	0.918	1.246	1.037	1.204	1.241
7	1.06	1.204	1.223	0.914	1.021
8	0.844	0.768	0.879	1.213	0.854
9	1.229	1.073	0.948	1.122	1.077
10	1.068	0.851	1.043	1.227	1.243

Determine for each 'network state' (i.e. iteration) the travel time for each route (link 1 + link 2, link 1 plus link 3, and link 4 plus link 5) and select the route having the shortest travel time. Count for each route how often is has the minimum travel time and divide this number by the total number of iterations. This yields 30%, 30%, and 40% (see table).

	1+2	**1+3**	**4+5**	**Min**
1	1.858	1.823	2.272	1.823
2	2.023	2.345	1.801	1.801
3	2.029	2.069	2.014	2.014
4	2.178	2.007	1.933	1.933
5	1.703	1.809	2.127	1.703
6	2.164	1.955	2.445	1.955
7	2.264	2.283	1.935	1.935
8	1.612	1.723	2.067	1.612
9	2.302	2.177	2.199	2.177
10	1.919	2.111	2.47	1.919
	30%	30%	40%	

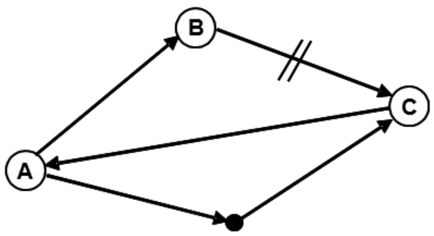
Congested networks

Consider the following statements concerning the DUE assignment. Are they true or false?

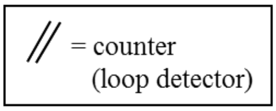
- In a deterministic user-equilibrium, all route travel times for each OD pair are equal.
- The solution to the DUE assignment problem is always unique.
- Similar to AON assignment, the DUE assignment aims at minimizing the total network travel time.
- In DUE assignment, most OD flow is assigned to the route with the lowest free-flow travel time.

False, False, False, False

Consider an island with three cities, A, B, and C. A road network between these cities exists, consisting of five uni-directional road segments. The travel demand is given by the OD trip matrix *T*. There is a counter in the network, counting the number of vehicles passing through the road segment from city B to city C.



$$T = \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{pmatrix} 200 & 50 & 1000 \\ 100 & 150 & 200 \\ 400 & 50 & 50 \end{pmatrix} \end{array}$$



Assume that all links are identical (that is, all links have the same length, maximum speed, capacity, etc.).

Suppose that each link *a* has a link travel time t_a depending on the link flow q_a given by the following function:

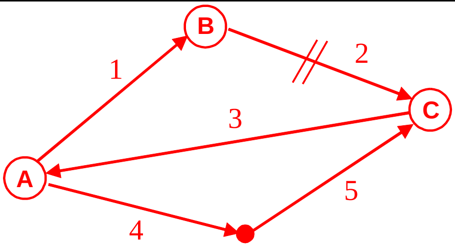
$$t_a(q_a) = 1 + \frac{q_a}{500}.$$

(a) Formulate Wardrop's first principle for a deterministic user equilibrium assignment.

Wardrop's first principle for a deterministic user-equilibrium assignment is that the travel time on any used route equals the travel time on any other used route between the same origin and destination and is no greater than the travel time on any unused route.

(b) Assuming that all travellers are assigned according to a deterministic user equilibrium assignment, how many travellers will be counted by the counter? Explain your answer.

First, number the links in the network, for example such as below.



Only travellers from A to C have a choice of route (route 1 consisting of links 1 and 2, and route 2 consisting of links 4 and 5), all other travellers only have a single route to choose from. Let AB be the number of travellers from A to B, CB from C to B, etc. Let AC1 be the number of travellers from A to C choosing route 1, and let AC2 be the number of travellers from A to C choosing route 2.

Number of travellers on link 1: q1 = AB + AC1 + CB = 50 + AC1 + 50

Number of travellers on link 2: q2 = BC + AC1 + BA = 200 + AC1 + 100

Number of travellers on link 4: q4 = AC2

Number of travellers on link 5: q5 = AC2

In a deterministic user equilibrium, routes 1 and 2 have equal travel time (assuming that both routes are used), such that:

Travel time on route 1

= (1+q1/500) + (1+q2/500)

= (1+(100+AC1)/500) + (1+(300+AC1)/500)

= 2.8 + AC1/250

Travel time on route 2

= (1+q4/500) + (1+q5/500)

= (1+(1000-AC1)/500) + (1+(1000-AC1)/500)

= 6 – AC1/250

If both travel times are equal:

2.8 + AC1/250 = 6 – AC1/250, which yields AC1 = 400.

Therefore, the total number of travellers counted on link 2 are 200 + 400 + 100 = **700 travellers**.

Now suppose that the road segment of the counter (link BC) is expanded with an extra lane, increasing the capacity, hence decreasing the congestion and travel time on this road segment.

(c) Again assuming that travellers choose their routes according to the deterministic user equilibrium principle, which OD-pairs will be better off, and which OD-pairs will be worse off? Explain your answer.

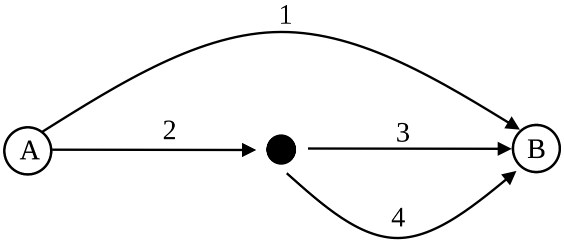
Link BC is being used by travellers:

- from B to C
- from A to C
- from B to A

By the increase in capacity, the travellers from B to C will be better off. Travellers from A to C will be **better off** as well, and so will the travellers from B to A.

The first route from A to C (using links 1 and 2) will become busier because it will attract more traffic now that the travel time has decreased. This means that also link 1 will become busier. Therefore, travellers from A to B will **be worse off**, as well as travellers from C to B.

Consider the following four-link transportation network, consisting of a single origin-destination pair (A,B), and three alternative routes from A to B. The link numbers a (=1,2,3,4) are indicated in the network. The travel demand from A to B is 1000 veh/h.



The link travel times (in minutes), denoted by t_a, depend on the car flows q_a (veh/h), and are given by the following functions:

$$t_1(q_1) = 2 + \frac{3q_1}{500}$$

$$t_2(q_2) = 1 + \frac{q_2}{1000}$$

$$t_3(q_3) = 1 + \frac{q_3}{500}$$

$$t_4(q_4) = 1 + \frac{q_4}{500}$$

(a) Determine analytically the link flows in a user equilibrium assignment using Wardrop's first principle.

Let the route flows for routes 1, 2, and 3 be denoted by f_1, f_2, and f_3.

The travel time on the first route is given by:

$$t_{route1} = 2 + \frac{3f_1}{500}$$

The travel time on the second route is given by:

$$t_{route2} = 1 + \frac{f_2 + f_3}{1000} + 1 + \frac{f_2}{500}$$

The travel time on the third route is given by:

$$t_{route3} = 1 + \frac{f_2 + f_3}{1000} + 1 + \frac{f_3}{500}$$

Furthermore, the conservation of flow equation is given by:

$$f_1 + f_2 + f_3 = 1000$$

According to Wardrop's first principle, assuming all routes will be used, the travel times should be equal. Hence, we have to solve the following system of equations:

$$\begin{cases} 2+\dfrac{3f_1}{500}=1+\dfrac{f_2+f_3}{1000}+1+\dfrac{f_2}{500} \\ 1+\dfrac{f_2+f_3}{1000}+1+\dfrac{f_2}{500}=1+\dfrac{f_2+f_3}{1000}+1+\dfrac{f_3}{500} \\ f_1+f_2+f_3=1000 \end{cases}$$

The second equation implies that $f_2 = f_3$, such that

$$\begin{cases} 2+\dfrac{3f_1}{500}=2+\dfrac{2f_2}{1000}+\dfrac{f_2}{500} \\ f_1+2f_2=1000 \end{cases}$$

which leads to

$$\begin{cases} 6f_1=4f_2 \\ f_1+2f_2=1000 \end{cases}$$

This results in the following route flows:

$f_1=250,\ f_2=375,\ f_3=375.$

Hence, the corresponding link flows are

$q_1=250, q_2=750, q_3=375, q_4=375.$

(b) Determine iteratively the route flows in a user equilibrium assignment using MSA. Perform 8 iterations.

Recall that we use AON in a DUE!

f_1	***f_2***	***f_3***	***t_1***	***t_2***	***t_3***	***w_1***	***w_2***	***w_3***	***alfa***
0	0	0	2	2	2	1000	0	0	1
1000	0	0	8	2	2	0	1000	0	2
500	500	0	5	3.5	2.5	0	0	1000	3
333.33	333.33	333.33	4	3.33	3.33	0	1000	0	4
250	500	250	3.5	3.75	3.25	0	0	1000	5
200	400	400	3.2	3.6	3.6	1000	0	0	6
333.33	333.33	333.33	4	3.33	3.33	0	1000	0	7
285.71	428.57	285.71	3.71	3.57	3.29	0	0	1000	8
250	375	375	3.50	3.50	3.50				

Note that you could also use the fact that f_2 equals f_3 and consider two routes only. In that case 4 iterations will be enough.

System optimal assignment

Consider the following statements concerning the System optimal assignment. Are they true or false?

- The only difference between the objective function of an AON assignment and a DSO assignment is, that in the latter case the link travel times depend on the link flows.
- In a DUE assignment, adding new links to a network can make the individual travel time higher.
- In a DSO assignment, adding new links to a network can make the total network travel time higher.
- In case all routes in a network have identical travel time functions and are non-overlapping, the DUE solution is equal to the DSO solution.

True, True, False, True

Consider a transportation network with a single origin-destination pair and two parallel routes, each consisting of a single link. The travel times of both routes are assumed to be determined by the BPR function,

$$t_a(q_a) = t_a^0 \left[1 + \alpha_a \left(\frac{q_a}{C_a} \right)^{\beta_a} \right], \quad a = 1, 2,$$

where t_a is the link (route) travel time of link (route) a, q_a is the link flow of link a, C_a is the capacity of link a, t_a^0 is the free-flow travel time of link a, and α_a and β_a are link-specific parameters of the function. In the following table the values for both links are given.

	link 1	link 2
t_a^0 [min.]	8	$7\frac{1}{5}$
C_a [veh/h]	600	600
α_a	$\frac{1}{6}$	$\frac{1}{27}$
β_a	2	2

The travel demand is 1200 vehicles per hour.

(a) Determine the system optimal flows numerically (iteratively). Perform a sufficient number of iterations such that convergence has been reached.

$$t_1(q_1) = 8\left[1 + \frac{1}{6}\left(\frac{q_1}{600}\right)^2\right], \text{ and } t_2(q_2) = 7\frac{1}{5}\left[1 + \frac{1}{27}\left(\frac{q_2}{600}\right)^2\right].$$

The marginal link cost functions are:

$$t_1^*(q_1) = t_1(q_1) + t_1'(q_1)q_1 = 8\left[1 + \frac{1}{2}\left(\frac{q_1}{600}\right)^2\right],$$

$$t_2^*(q_2) = t_2(q_2) + t_2'(q_2)q_2 = 7\frac{1}{5}\left[1 + \frac{1}{9}\left(\frac{q_1}{600}\right)^2\right].$$

The iterative process in which we aim to equate the marginal cost functions together:

q1	q2	t1	t2	w1	w2	α
0	0	8.0	7.2	0	1200	1
0	1200	8.0	10.4	1200	0	1/2
600	600	12.0	8.0	0	1200	1/3
400	800	9.8	8.6	0	1200	1/4
300	900	9.0	9.0			

After 5 iterations we have reached convergence. Route 1 should be used by 300 vehicles and route 2 by 900 vehicles in a system optimal assignment.

(b) What are the travel times experienced by travellers on both links (routes) in case of these system optimal flows?

t1(300) = 8.33
t2(900) = 7.8

(c) Why is such a system optimal state not a stable situation?

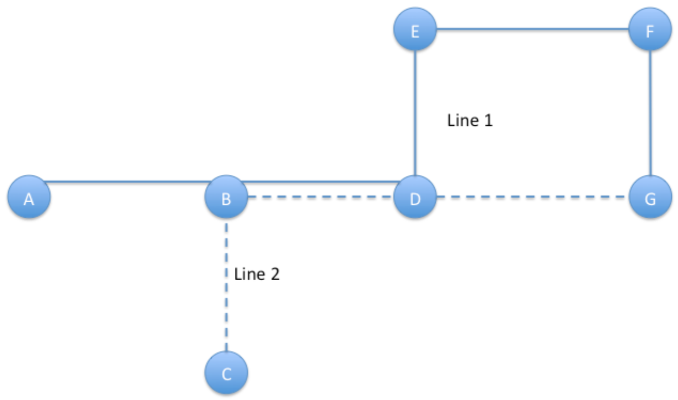
Travelers would like to derivate to the faster route, route 2.

(d) How can a road authority reach a stable system optimal state?

The road authority could introduce toll on route 2.

Public transport assignment

Given is the following public transport network:



When modelling a public transport network, three different network representations are possible.

(a) Give the main characteristics of these three network representations (or advantages and disadvantages) and illustrate the main principle for the given network. [3]

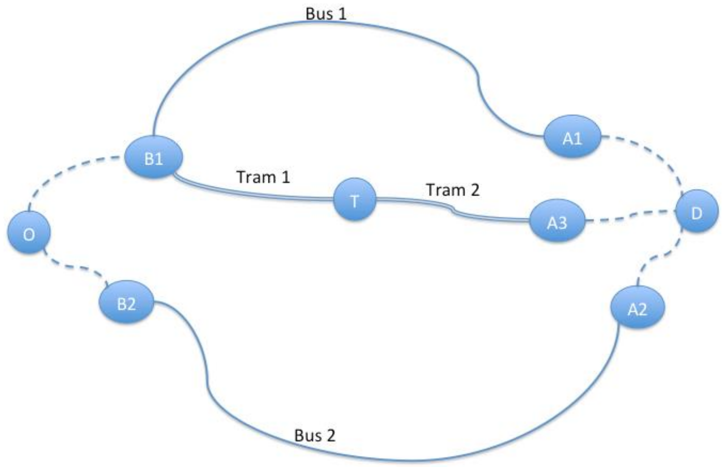
1. Trunk line: code per link which line use that link. Compact data storage, complex algorithm as it has to account for the transfer
2. Line specific: every link relates to a single line and transfers are explicitly coded. More links needed than for method 1. Problem with modelling PT quality for parallel lines.
3. Route section: links represent trips that can be made without a transfer. A sequence of *n* links in a route (or path) thus always implies *n-1* transfers. Parallel lines can be modelled. Consequence is substantially larger network.

For the public transport assignment different methods are possible. One of these assignment methods is based on the concept of strategies.

(b) What is meant with the concept of strategies? [2]

Definition O&W: a strategy is a set of rules that allows the traveller to reach his destination.
Alternative answer: A strategy is a set of routes from a starting stop for which it is assumed that travellers decide which line (and thus which route) they will use based on the actual conditions at the stop. The common, implicit, assumption is that they will board the first vehicle that arrives, so travellers are split over the lines (routes) according to their frequency.

A typical characteristic of public transport network assignment methods is that they use a backward search to determine paths or routes and assign the flow in a second step. For this assignment, again, different methods are possible. Consider the following situation:



For this OD-pair there are three routes having the following characteristics.

	Access time (min)	Frequency	In-vehicle time (min)	Transfer	In-vehicle time (min)	Egress time (min)	Fare (euro)
Route Bus 1	5	4	20			5	1
Route Bus 2	3	6	25			3	1,5
Route Trams 1 and 2	5	8	5	1	5	6	2

For the choice between these three routes a logit-model (MNL)can be used. Research showed the following utility function (generalised time expressed in minutes):

$$V_{OD} = -T_{access} - \frac{30}{Freq} - T_{in-veh} - Transfer \cdot P_{transfer} - T_{egress} - \frac{Fare}{VoT}$$

VOD = Utility for route between O and D
T_{access} = Access time
$Freq$ = Frequency
$T_{in\text{-}veh}$ = In vehicle time
$Transfer$ = Number of transfers
$P_{transfer}$ = Transfer penalty
T_{egress} = Egress time
$Fare$ = Fare
VoT = Value of time

Furthermore, it was found that the transfer penalty is 10 minutes, the Value of time is 10 euro/hour, and the scale parameter b is 0.3.

(c) What is the share for each route? [3]

	Utility	Exp(b* utility)	Percentage
Route Bus 1	-43,5	2,15009E-06	49,6%
Route Bus 2	-45	1,37096E-06	31,6%
Route Trams 1 and 2	-46,75	8,10998E-07	18,7%
		4,33205E-06	

Looking at the choice situation in more detail a different choice situation can be defined. Basically travellers have to opt to walk to stop B1 or to stop B2. From stop B1, they can choose to use Bus 1 or Tram 1. Therefore, there is overlap between route 1 and 3, which calls for the use of a nested logit-model. In this model the choice travellers make at stop B1 is modelled using the following utility function:

$$V_{B1D} = -\frac{30}{Freq} - T_{in-veh} - Transfer \cdot P_{transfer} - T_{egress} - \frac{Fare}{VoT}$$

The scale parameter l for this choice model was found to be 0.5.

As nothing changes for route 2, the utility function for the alternative via B2 is identical to that for question (c). The utility for travelling via B1 becomes:

$$V_{ODviaB1} = -T_{access} + \frac{1}{\lambda} \ln \sum_i \exp\left(\lambda \cdot V_{B1D,i}\right)$$

The scale parameter b for the choice between stop B1 or B2 is still 0.3 of the original MNL.

(d) Determine the proportion of travellers walking to stop B1. [4]

Choice model for choice at stop B1 yields:

	Utility	Exp(l* utility)	Percentage
Route Bus 1	-38,5	4.36346E-09	83.5%
Route Trams 1 and 2	-41,75	8.59217E-10	16.5%
		5.22268E-09	

The logsum thus becomes 1/ l *ln(5.22268E-09)=-38.1
The choice between stop B1 and B2 is modelled as follows:

	Utility	Exp(b* utility)	Percentage
Stop B1	-43,1	2.39492E-06	63.6%
Stop B2	-45	1,37096E-06	36.4%
		3.76588E-06	

(e) Determine the proportion of travellers using the tram alternative. [4]

This is the probability of stop B1 times the probability of route 1 given stop B1: 0.636*0.165=10.5%

(f) Explain why these results are different from those found in question (c), and what would happen if ζ equals 0.3? [4]

There's overlap between route 1 and route 3 which implies that using an MNL overestimates the attractiveness of both routes. The fact that there are two travel options available at stop B1 yields a limited benefit for stop B1 versus stop B2: the disutility decreases from -43.5 for route 1 and -46.75 for route 3 to -43.1. The value of ζ is quite important: the higher the values the smaller the impact. If ζ equals 0.3 the results of question (c) are obtained.