## Uncongested networks

Consider the following statements concerning (shortest) paths on a network. Are they true or false?

- The number of links in a shortest path tree is always exactly $\mathrm{N}-1$ (where N is the number of nodes)
- The path between any two nodes in a shortest path tree is again a shortest path
- For each OD Pair there is only a single shortest path

Consider the following statements concerning route choice. Are they true or false?

- Link flows and route travel times are outcomes of the traffic assignment problem.
- In the all-or-nothing assignment, all OD flows are assigned to the shortest route.
- A shortest-path all-or-nothing assignment can be computed by minimizing the total travel time subject to flow conservation and non-negativity constraints.
- Outcomes of a shortest-path all-or-nothing assignment are 'stable' outcomes; minor changes in the network structure only result in minor changes in the assignment outcomes.

Consider the following statements concerning stochastic assignment. Are they true or false?

- In a shortest path AON assignment, the traveler chooses the objective shortest route, whereas in stochastic assignment the traveler chooses the subjective shortest route.
- The logit assignment is always based on route level, whereas the probit assignment is solved on the link level.
- The error term in stochastic assignment is the same for each route/link.
- Nested logit models and probit models are able to deal with overlapping route alternatives.

Consider the following transportation network with two origins (A and B) and two destinations (B and C).


Each link in the network is assumed identical and the travel time on each link $a$ is given by $t_{a}$.

The travel demand trip matrix is given as follows:

| from/to | $\mathbf{A}$ | $\mathbf{B}$ | $\mathbf{C}$ |
| :---: | :---: | :---: | :---: |
| $\mathbf{A}$ | - | 500 | 2000 |
| $\mathbf{B}$ | - | - | 3500 |
| $\mathbf{C}$ | - | - | - |

A stochastic assignment can be determined using the logit model or the probit model.
(a) Determine the link flows in a stochastic assignment using the logit model.
(b) Will the flows on links 4 and 5 increase or decrease in case the probit model is used instead of the logit model? Explain your answer.
(c) Use the following random numbers to apply the probit model to this network. Use 10 iterations.

|  | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | 0.796 | 1.062 | 1.027 | 1.08 | 1.192 |
| $\mathbf{2}$ | 1.104 | 0.919 | 1.241 | 1.037 | 0.764 |
| $\mathbf{3}$ | 0.888 | 1.141 | 1.181 | 0.978 | 1.036 |
| $\mathbf{4}$ | 1.194 | 0.984 | 0.813 | 0.829 | 1.104 |
| $\mathbf{5}$ | 0.766 | 0.937 | 1.043 | 1.216 | 0.911 |
| $\mathbf{6}$ | 0.918 | 1.246 | 1.037 | 1.204 | 1.241 |
| $\mathbf{7}$ | 1.06 | 1.204 | 1.223 | 0.914 | 1.021 |
| $\mathbf{8}$ | 0.844 | 0.768 | 0.879 | 1.213 | 0.854 |
| $\mathbf{9}$ | 1.229 | 1.073 | 0.948 | 1.122 | 1.077 |
| $\mathbf{1 0}$ | 1.068 | 0.851 | 1.043 | 1.227 | 1.243 |

## Congested networks

Consider the following statements concerning the DUE assignment. Are they true or false?

- In a deterministic user-equilibrium, all route travel times for each OD pair are equal.
- The solution to the DUE assignment problem is always unique.
- Similar to AON assignment, the DUE assignment aims at minimizing the total network travel time.
- In DUE assignment, most OD flow is assigned to the route with the lowest free-flow travel time.

Consider an island with three cities, A, B, and C. A road network between these cities exists, consisting of five uni-directional road segments. The travel demand is given by the OD trip matrix $T$. There is a counter in the network, counting the number of vehicles passing through the road segment from city B to city C.


Assume that all links are identical (that is, all links have the same length, maximum speed, capacity, etc.).

Suppose that each link $a$ has a link travel time $t_{a}$ depending on the link flow $q_{a}$ given by the following function:
$t_{a}\left(q_{a}\right)=1+\frac{q_{a}}{500}$.
(a) Formulate Wardrop's first principle for a deterministic user equilibrium assignment.
(b) Assuming that all travellers are assigned according to a deterministic user equilibrium assignment, how many travellers will be counted by the counter? Explain your answer.

Now suppose that the road segment of the counter (link BC) is expanded with an extra lane, increasing the capacity, hence decreasing the congestion and travel time on this road segment.
(c) Again assuming that travellers choose their routes according to the deterministic user equilibrium principle, which OD-pairs will be better off, and which OD-pairs will be worse off? Explain your answer.

Consider the following four-link transportation network, consisting of a single origindestination pair ( $\mathrm{A}, \mathrm{B}$ ), and three alternative routes from A to B . The link numbers $a$ $(=1,2,3,4)$ are indicated in the network. The travel demand from A to B is $1000 \mathrm{veh} / \mathrm{h}$.


The link travel times (in minutes), denoted by $t_{a}$, depend on the car flows $q_{a}(\mathrm{veh} / \mathrm{h})$, and are given by the following functions:
$t_{1}\left(q_{1}\right)=2+\frac{3 q_{1}}{500}$
$t_{2}\left(q_{2}\right)=1+\frac{q_{2}}{1000}$
$t_{3}\left(q_{3}\right)=1+\frac{q_{3}}{500}$
$t_{4}\left(q_{4}\right)=1+\frac{q_{4}}{500}$
(a) Determine analytically the link flows in a user equilibrium assignment using Wardrop's first principle.
(b) Determine iteratively the route flows in a user equilibrium assignment using MSA. Perform 8 iterations.

## System optimal assignment

Consider the following statements concerning the System optimal assignment. Are they true or false?

- The only difference between the objective function of an AON assignment and a DSO assignment is, that in the latter case the link travel times depend on the link flows.
- In a DUE assignment, adding new links to a network can make the individual travel time higher.
- In a DSO assignment, adding new links to a network can make the total network travel time higher.
- In case all routes in a network have identical travel time functions and are nonoverlapping, the DUE solution is equal to the DSO solution.

Consider a transportation network with a single origin-destination pair and two parallel routes, each consisting of a single link. The travel times of both routes are assumed to be determined by the BPR function,
$t_{a}\left(q_{a}\right)=t_{a}^{0}\left[1+\alpha_{a}\left(\frac{q_{a}}{C_{a}}\right)^{\beta_{a}}\right], \quad a=1,2$,
where $t_{a}$ is the link (route) travel time of link (route) $a, q_{a}$ is the link flow of link $a, C_{a}$ is the capacity of link $a, t_{a}^{0}$ is the free-flow travel time of link $a$, and $\alpha_{a}$ and $\beta_{a}$ are linkspecific parameters of the function. In the following table the values for both links are given.

|  | link 1 | $\operatorname{link} 2$ |
| :--- | :---: | :---: |
| $t_{a}^{0}[\mathrm{~min}]$. | 8 | $7 \frac{1}{5}$ |
| $C_{a}[\mathrm{veh} / \mathrm{h}]$ | 600 | 600 |
| $\alpha_{a}$ | $\frac{1}{6}$ | $\frac{1}{27}$ |
| $\beta_{a}$ | 2 | 2 |

The travel demand is 1200 vehicles per hour.
(a) Determine the system optimal flows numerically (iteratively). Perform a sufficient number of iterations such that convergence has been reached.
(b) What are the travel times experienced by travellers on both links (routes) in case of these system optimal flows?
(c) Why is such a system optimal state not a stable situation?
(d) How can a road authority reach a stable system optimal state?

## Public transport assignment

Given is the following public transport network:


When modelling a public transport network, three different network representations are possible.
(a) Give the main characteristics of these three network representations (or advantages and disadvantages) and illustrate the main principle for the given network. [3]

For the public transport assignment different methods are possible. One of these assignment methods is based on the concept of strategies.
(b) What is meant with the concept of strategies? [2]

A typical characteristic of public transport network assignment methods is that they use a backward search to determine paths or routes and assign the flow in a second step. For this assignment, again, different methods are possible. Consider the following situation:


For this OD-pair there are three routes having the following characteristics.

|  |  |  |  |  |  |  | 运号 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Route Bus 1 | 5 | 4 | 20 |  |  | 5 | 1 |
| Route Bus 2 | 3 | 6 | 25 |  |  | 3 | 1,5 |
| Route Trams 1 and 2 | 5 | 8 | 5 | 1 | 5 | 6 | 2 |

For the choice between these three routes a logit-model (MNL)can be used. Research showed the following utility function (generalised time expressed in minutes):
$V_{O D}=-T_{\text {access }}-\frac{30}{\text { Freq }}-T_{\text {in-veh }}-$ Transfer $\cdot P_{\text {transer }}-T_{\text {egress }}-\frac{\text { Fare }}{\text { VoT }}$
$V O D=$ Utility for route between $O$ and $D$
$T_{\text {access }}=$ Access time
Freq = Frequency
$T_{i n-v e h}=$ In vehicle time
Transfer $=$ Number of transfers
$P_{\text {transfer }}=$ Transfer penalty
$T_{\text {egress }}=$ Egress time
Fare = Fare
VoT = Value of time
Furthermore, it was found that the transfer penalty is 10 minutes, the Value of time is 10 euro/hour, and the scale parameter is 0.3 .
(c) What is the share for each route? [3]

Looking at the choice situation in more detail a different choice situation can be defined. Basically travellers have to opt to walk to stop B1 or to stop B2. From stop B1, they can choose to use Bus 1 or Tram 1. Therefore, there is overlap between route 1 and 3, which calls for the use of a nested logit-model. In this model the choice travellers make at stop B1 is modelled using the following utility function:
$V_{B 1 D}=-\frac{30}{\text { Freq }}-T_{\text {in-veh }}-$ Transfer $\cdot P_{\text {transer }}-T_{\text {egress }}-\frac{\text { Fare }}{\text { VoT }}$
The scale parameter for this choice model was found to be 0.5 .
As nothing changes for route 2, the utility function for the alternative via B 2 is identical to that for question (c). The utility for travelling via B1 becomes:
$V_{\text {ODviaB1 }}=-T_{\text {access }}+\frac{1}{\lambda} \ln \sum_{i} \exp \left(\lambda \cdot V_{B 1 D, i}\right)$
The scale parameter for the choice between stop B1 or B2 is still 0.3 of the original MNL.
(d) Determine the proportion of travellers walking to stop B1. [4]
(e) Determine the proportion of travellers using the tram alternative. [4]
(f) Explain why these results are different from those found in question (c), and what would happen if equals 0.3? [4]

