Appendix: Solutions to Exercises

THIS BOOK includes 193 exercises, concentrating on a variety of problems relevant to the study of rock mechanics. Many of the exercises are subdivided into several sections, thus providing an indispensable resource for students and professionals who wish to deepen their understanding of the subject. Solutions to these exercises are given below. Some of the exercises require the construction of graphs, which are excluded here so that students can demonstrate independently how their solutions were obtained.

The vast majority of the exercises, which are compiled here, are original and published for the first time. However, a number of classical problems have been adopted from earlier work. These exercises have been modified to conform with the notation and approach followed in this book.

The sources from which exercises have been adopted are as follows: *Geodynamics* (1981, Wiley) by GERALD SCHUBERT and DONALD TURCOTTE (exercises 3-11, 3-12, 6-10, and 10-11 to 10-14); *Stress and Strain* (1976, Springer) by WINTHROP MEANS (exercises 5-14, 5-15, 10-3, 10-5, 10-7, 10-9, 12-9, and 15-5); *The Techniques of Modern Structural Geology. Volume 1: Strain Analysis* (1983, Academic Press) by JOHN RAMSAY and MARTIN HUBER (exercises 12-3, 12-13, 14-1, 14-2, 15-9, 15-10, 15-12, and 15-13); *Principles of Structural Geology* (1985, Prentice-Hall) by JOHN SUPPE (exercises 5-11, 6-6, and 6-9); *Structural Geology* (1973, 2nd ed., Wiley) by DONAL RAGAN (exercise 15-11); *Geological Structures and Moving Plates* (1988, Blackie) by GRAHAM PARK (exercise 8-13); *Stress and Deformation* (1996, Oxford University Press) by GERHARD OERTEL (exercise 10-16); *The Analysis of Strain in Folded Layers* (1971, Tectonophysics, vol. 11, pp. 329-375) by BRUCE HOBBS (exercise 12-12).

Chapter one

1-1: a) Obviously, knowledge of rock mechanics may be useful in engineering operations that commonly involve construction on, and excavation of, rock structures. It helps in assessing the stability of the host rock at past, present, and future construction sites.

b) A thorough understanding of rock behavior is essential for strategic planning in the petroleum and mining industry, in construction operations, and in locating subsurface repositories. The formation of geological structures or rock deformation patterns, including the folds, joints, and faults that are studied by structural geologists and tectonicians, is, also, governed by the principles of rock mechanics.

1-2: Knowledge of the physics of fluids (also termed *fluid mechanics* or *fluid dynamics*) can help in understanding and explaining the development of ductile deformation patterns in rocks, deformed by solid-state flow of their constituent mineral crystals.

1-3: The crystals of microcrystalline paraffin in candle wax and stearine lights can deform in a ductile fashion if their temperature is close to the melting point. The deformation occurs by solid-state flow of the crystals, through a process which is termed crystalline creep. Chapter 7 explains how the rearrangement of atoms in crystal lattices allows the creep of crystals in solid-state.

1-4: The rejection of the continental drift hypothesis at the 1928 Tulsa symposium is an interesting event in the history of science. The processes of crystalline flow had already been studied and described for a range of synthetic crystalline materials, such as metals and waxes. Also, it had been suggested to occur in natural minerals. However, this knowledge apparently went unacknowledged in geoscience, and the idea that huge continental plates could creep over a substratum was considered inconceivable. It, also, seems that Sir Harold Jeffreys statement that "*continents cannot move through solid rock like ships in the ocean*" made a serious impact through his established reputation in contemporary geophysics. This view was, therefore, apparently in accordance with what little was known about crystalline flow in the geoscientific community in 1928.

1-5: Rock salt can flow downhill in glacier-like fashion (e.g., see cover illustration of a namakier in the Zagros Mountains, Iran) by crystalline creep of the constituent salt crystals. Both halite and ice crystals are exceptional mineral compounds in that they can deform their crystal lattice under gravitational loading at the Earth's surface, largely without brittle failure. The ductile flow of glaciers may be accompanied by the formation of cracks and crevasses near the surface, where the physical conditions allow the occurrence of brittle failure together with ductile flow. In contrast, other principal rock-forming minerals can deform in a ductile fashion only when buried in the crust and mantle to depths where the elevated temperature activates atomic rearrangements of crystal lattices, if subjected to deformational stresses.

1-6: "*Panta rei*" or "*everything flows,*" according to Heraclitus (500 BC), only partly applies to solid rock. Undoubtedly, Heraclitus made his statement with little knowledge of rock deformation processes. More likely, this statement referred to the general motion that had been observed in the four basic elements considered by the Greek: air moves in the atmosphere, water flows in the oceans, fire leaks from volcanic eruptions, and soil washes away in water streams. Another classic statement is Deborah's observation reported in the Old Testament, "*The mountains flowed before the Lord... .*" Past prophets and historians most likely referred to volcanic eruptions when alluding to rock flow. Only in the past fifty years have we established a firm scientific basis for understanding the flow of solid rock under a variety of physical conditions.

1-7: The effective viscosity of water is 0.001 Pa s (read: Pascal second). For comparison, the mean viscosity inferred for the mantle is 10^{18} Pa s. Viscosity is, in fact, a measure of flow retardation (see, also, chapter 8). The viscosity values given here suggest that the Earth's mantle flows 10^{21} times slower than water if mantle rocks and water both were subjected to the same forces. But water commonly flows at maximum speeds at the order of m s^{-1} and mantle rocks at 10^{-15} m s^{-1}. Consequently, the forces on mantle rocks can be estimated to be about 10^6 times larger than the forces [measured in stress units of Pa (read: Pascal)] commonly occurring in flowing water. Assuming shear stresses in water are of the order of 1 Pa, shear stresses on creeping mantle rock are of the order of 1 MPa.

Chapter two

2-1: The systems of physical quantities *[MLT]*, *[FLT]*, *[PLT]*, and *[DLT]* all are physically equivalent. The unit of mass (kg) in the *[MLT]* system is, also, included in the units of force *[F]*, pressure *[P]*, and density *[D]*. Each of the above systems can be practical for measuring units of the physical quantities involved, and they are consistent with a series of measurement techniques.

2-2: The introduction of a new physical quantity for which no apparatus or technique exists to measure the units is highly speculative. If no apparatus can measure its effect or scale, then it is hard to prove that there is any new physical quantity involved at all.

2-3: Strain is an example of a quantity, measured in dimensionless units. Several measures of strain are used, according to a variety of definitions (for details, see section 11-1), but they commonly compare an initial distance between two material points before a deformation with the spatial separation of the same points after the deformation. All strain units are based on ratios of particular lengths, and, therefore, the unit of length vanishes to give way to a number, expressing the fractional change in length rather than an absolute measure.

2-4: The SI unit equivalent of one acrefoot is calculated as follows: 1 acre is equal to 4,046.9 m^2 and 1 foot corresponds to 0.3048 m. The volume of one acrefoot is equivalent to 1,233.5 m^3.

2-5: a) One mile is equivalent to 1,760 yards.

b) Non-metric units lack the convenience of use that metric units offer.

2-6: a) The tectonic strain-rate obtained by dividing plate velocity (ca. 10 cm y^{-1}) by the orogenic width (ca. 500 km) is 0.6 x 10^{-14} s^{-1}.

b) The amount of strain accumulated is roughly obtained by the product of strain-rate and the time available for the deformation. A strain-rate of 0.6×10^{-14} s^{-1} accumulates a maximum strain of 600 over 4 billion years. It must be noted, however, that an accurate estimate of the finite strain requires the integration of strain increments over time. For linear strains, the finite strain is computed by exp ($\dot{e}t$) [see eq. (12-6)], which gives a much larger finite strain than that obtained by the product of time and strain-rate.

2-7: The quantities listed in this exercise belong in the following categories: temperature, time, and length are scalar quantities; velocity, acceleration, and force are vector quantities; and stress, strain, strain-rate, and vorticity are tensor quantities.

2-8: Lateral changes in composition and texture of rocks complicate the continuum assumption as follows: The physical properties are likely to change across compositional boundaries. The continuum assumption must take into account the presence of these boundaries, which separate two continua. If properties change gradually, then the physical properties can be described by a spatial variable, accounting for the lateral changes. A continuum assumption is still possible, but the results of any physical modeling may differ from what transpires in the natural example if the continuum model is oversimplified. Most contemporary continuum models of geological and geophysical processes probably can be classified to include simplifying assumptions that neglect many features which affect deformations occurring in nature. In spite of their limitations, such model simulations still contribute to the improvement of our knowledge of the factors that play a role in the natural deformation of rock.

2-9: An unconsolidated, sedimentary rock, that deforms while compacting, undergoes volume change, loss of pore space, reduction of water content, increase of frictional surface between the grains, and, possibly, develops a mechanical anisotropy by settling of the mineral grains in a preferred orientation. These are time-dependent changes, which may affect the physical properties of the mechanical continuum. These changes can be accounted for, but the governing equations become time-dependent and require complex integration over time to describe the physical changes that occur during the deformation - a difficult task.

Chapter three

3-1: The surface gravities of the Moon (1.56 m s^{-2}), Mars (3.72 m s^{-2}), and Mercury (3.62 m s^{-2}) are, respectively, 0.16, 0.38, and 0.37 times that of Earth (9.78 m s^{-2}). Any construction works are subjected to body forces that are proportional to the gravitational force. However, the load-bearing capacity of such construction is determined by the strength of the materials of which they are made. The strength of these materials is a material property and is independent of the gravity field. But similar construction on the Moon, Mars, and Mercury can bear 6, 2.6, and 2.7 times the critical mass of a comparable Earth structure. Consequently, if the same mass is to be supported by similar construction on the different planetary bodies, then the construction thickness on the Moon, Mars, and Mercury can be 0.16, 0.38, and 0.37 times that of the reference construction on Earth.

3-2: The force at the 1 m^2 bottom of a 1,000-m-thick column of sandstone (ρ=2,500 kg m^{-3}) is $F=ma=V\rho g=24.5$ MN.

3-3: The solving of this exercise (on the gravity sliding forces of a rock block) requires about one hour for the average student. The time spent provides worthwhile deepening of understanding.

a) The block moves over a horizontal surface, provided that the horizontal force $F_S \geq \mu F_N$ (with $F_N = V\rho g$). Using the values provided in the exercise, i.e., $\mu=0.7$, V=4 m^3, ρ=2,700 kg m^{-3}, and g=9.8 m s^{-2}, a horizontal shear force, F_S, of 74,088 Newtons is the minimum required to move the block of rock over the horizontal surface.

b) If a single man of 100 kg mass can push his own weight of 980 Newtons, then at least 76 such men are needed to push this block forward. Incidentally, modern estimates of the number of men involved in the construction of the principal Egyptian pyramids amount to 10,000 men over a period of 25 years.

c) The two goniometric functions of equations (3-3a) and (3-3b) can be used to plot the variation of F_N and F_S. The vertical axis uses F_{net} as the unit, with $F_N=F_{net}$ for $\alpha=90°$. Other F_N values follow from the cosine function, which is zero for $\alpha=0°$. Conversely, $F_S=F_{net}$ for $\alpha=0°$, and other F_S values of the curve follow from the sine function, which is zero for α= 90°. The plot is symmetric about the vertical line through the α=45° mark on the horizontal scale.

d) The angle of internal friction, ϕ, is equal to $\tan^{-1}\mu$. For μ=0.7, the angle of internal friction is about 35°.

e) The acceleration, a_S, of the block in the direction of glide for subcritical angles, α, is: $a_S=(F_S-F_{S\,critical})/m$. Remember that $F_{S\,critical}=F_{net}\sin 35°=0.7F_{net}$ and $F_S=F_{net}\sin\alpha$. The acceleration can now be expressed as: $a_S=(F_{net}/m)(\sin\alpha-\sin 35°)=(\sin\alpha-\sin 35°)g$, for $\phi<\alpha\leq 90°$. The acceleration of the block is at maximum when it is nearly vertical. It is different from the acceleration in free fall, g, because it is still slowed down by the friction against the wall. In the case of α close to 90°, the acceleration of the block is close to 0.3g, but it will immediately become equal to g if the sliding contact with the wall is lost.

f) The additional push force, F_A, in the direction of glide for subcritical angles is as follows: The minimum force required for movement to occur is $(F_A+F_S)/F_N=0.7$. This condition is fulfilled if $F_A=0.7F_N-F_S=F_{net}(0.7\cos\alpha-\sin\alpha)$, for $0\leq\alpha\leq\phi$. It is seen that for $\alpha=35°$, $F_A=0$; that is, no additional force is required to cause motion. However, for $\alpha=0°$, the additional force required to move the block is $F_A=0.7F_{net}$, which is equal to 74,088 Newtons as calculated in exercise (3-3a).

3-4: The steep walls of the road cut of Figure 3-5 can be dangerous if joints in the walls provide potential slide surfaces.

a) If the friction coefficient, μ, is 0.5, then the angle of internal friction is 27°. If joints occur in the granitic wall of this hypothetical road cut at dips of 27° or steeper, then the walls are unstable and prone to sliding.

b) Pylons built on the shoulders of the road cut increase the load on the walls. Although this provides additional shear forces on any potential slide surface, the normal forces, also, increase. Consequently, the angle of internal friction will not be changed, and the stability is unaffected by the presence of the pylons. However, excessive loading of the shoulders of the road cut by adding pylons may incur new failure planes, and, thereby, it amplifies dangerous instabilities.

c) Pylons could better be placed on the floor of the road cut on either side of the road itself.

3-5: a) & b) Blocks of similar mass, but different shape, behave the same in gravity sliding, because the coefficient of friction, μ, remains the same, as well as do all the other force ratios required for the movement.

c) The critical slope for movement is equal to the angle of internal friction, that is, ϕ=35°.

d) The angle of internal friction, ϕ, is independent of both the volume and mass involved in the sliding. It is an intrinsic or internal material constant.

3-6: a) The roof of the tunnel in Figure 3-6 is unstable and caves in if the joints are continuous. The right wall of the tunnel is, also, unstable, because the joints dip at 45°, which is larger than the angle of internal friction of 27°.

b) An arched roof reduces the area of potential sliding but cannot stabilize this tunnel.

c) Tunnels through igneous rock are commonly more stable than in any other rock, because there are fewer discrete surfaces of separation. However, igneous rock can be expensive to cut through. For comparison, the Channel Tunnel, connecting Europe and the UK, could be cut in a relatively short time by following a soft chalk bed with low silica content, thus minimizing the wear on excavation tools.

3-7: The force at the 1 m^2 bottom of a 1000-m-thick column of sandstone was already calculated to be 24.5 MN (see exercise 3-2). The corresponding stress is 24.5 MPa. In fact, for estimates of the lithostatic pressure, it is convenient to assume a pressure gradient of about 25 MPa per km depth.

3-8: This is a controversial exercise that may stimulate a lively discussion on fluid pressure. The fluid pressure at the base of a block is:

a) For the block of Figure 3-2, the fluid pressure is 26.5 kPa.

b) For the tilted block of Figure 3-3, the fluid pressure would be raised by ($F_{net}/\cos\alpha$), due to the increase of the vertical thickness of the load upon tilting. The pressure is further enhanced by a decrease of the effective area on which the net force acts. Or is it?

3-9: The fluid pressures at locations A to D in the artesian well system of Figure 3-11 are: $P_A=0$, $P_B=2.5$ MPa, $P_C=2$ MPa, and $P_D=1$ MPa.

3-10: Bore mud exercise, referring to Figure 3-13:

a) At the time the drill hole, A, reaches a depth of 3 km, the fluid pressure of the bore mud should balance with the lithostatic load at that depth: 45 MPa. The mud density should be 1,500 kg m^{-3}. The bore hole must be cased in the upper 2 kilometers. This is because the fluid pressure would become too great for the drill hole section shallower than 2 km when the density of the bore mud is increased to 1,500 kg m^{-3} once the drill goes beyond 2 km depth.

b) The pressure of the mud column in the bore hole must balance the pressure of the gas reservoir (120 MPa) to prevent a blowout. The mud density would have to be: $\rho_{mud}=120$ MPa/gz=3,000 kg m^{-3}. Again, the upper section of the drill hole needs to be cased to prevent collapse of its walls at shallower depths.

3-11: The elevation of the continents above the ocean floor, in the isostasy model of Figure 3-14a, is equal to the length h-b. The pressure at continental depth, h, is $\rho_{cc}gh$. The pressure at oceanic depth, b, is $\rho_m gb$. If isostatic balance exists, then the two pressures are identical, and it follows that $h/b=\rho_m/\rho_{cc}$ or $b=(\rho_{cc}/\rho_m)h$. The elevation difference, h-b, can now be evaluated as follows: $h-b=h[1-(\rho_{cc}/\rho_m)]=6.7$ km.

3-12: a) The pressure condition at the base of the continent in the isostasy model of Figure 3-14b is as follows: $P_c=P_m+P_{oc}+P_w$ or $\rho_{cc}ghcc= \rho_m g(h_{cc}-h_{oc}-h_w)+\rho_{oc}gh_{oc}+\rho_w gh_w$. Evaluation for h_w yields: $h_w=[h_{cc}(\rho_m-\rho_{cc})-h_{oc}(\rho_m-\rho_{oc})]/(\rho_m-\rho_w)$.

b) The oceanic depth, h_w, is: 6.6 km.

c) The mean depth of the oceans would, after slow re-establishment of isostatic equilibrium in response to the rapid rise of global sea level, increase by 290 m.

Chapter four

4-1: The pressure at a point with major and minor principal stresses, $\sigma_1=-150$ MPa and $\sigma_3=-50$ MPa, is equal to the absolute mean stress: $P=|(1/2)(\sigma_1+\sigma_3)|=100$ MPa. The scaled sections show a pressure circle of unit radius superimposed on a stress ellipse with semi-axial lengths of 0.5 and 1.5, respectively.

4-2: a) The pressures at the various depths follow from the mean stress of the piezometer measurements. Pressures are: 0 at 0 km depth, 100 MPa at 4 km depth, 150 MPa at 6 km depth, and 175 MPa at 7 km depth. These pressures are similar to that following from the lithostatic load, assuming a pressure gradient of 25 MPa per km depth.

b) The deviatoric stresses are identical for all depths, as follows: τ_1=-50 MPa, τ_2=0, τ_3= +50 MPa.

c) The scaled ellipse sections of the principal planes of the deviatoric stress differ from those of the total stress by the pressure, if appropriately scaled.

d) The direction of σ_2 must be vertical, because σ_N is 0 at the surface. Consequently, the two other principal stresses are in the horizontal plane. The deviatoric stresses can be explained by a tectonic stress field in the horizontal plane.

4-3: This exercise addresses the basic difference between calculations with vector and tensor quantities. Because force is a vector quantity, the normal and shear forces, F_N and F_S, can be obtained from the two vector components of F_{net}. These are, according to equations (3-3a & b): $F_N = F_{net}\cos\alpha$ and $F_S = F_{net}\sin\alpha$. Unlike force, stress is a tensor quantity and not a vector quantity. The normal and the shear stress cannot be obtained as simple vector components of the principal stress. Assertions that $\sigma_N = \sigma_1 \cos \alpha$ and $\sigma_S = \sigma_1 \sin \alpha$, with α measured between σ_1 and the plane of action, as encountered in several established geology textbooks are simply wrong - be warned! The Mohr equations (4-5a & b) account for the tensor property of stress and relate the normal and shear stress to the major principal stress. For plane stress, the Mohr equations reduce to: $\sigma_N = \sigma_1 \cos 2\xi$ and $\sigma_S = \sigma_1 \sin 2\xi$ [eqs. (4-6a & b)], with ξ defined as in Figure 4-10b.

4-4: a) Gravity sliding of a sedimentary slope occurs when the inclination of the slope, α, is larger than the angle of internal friction, ϕ. This is the case when $F_S \geq \mu F_N$ or $\sigma_S \geq \mu\sigma_N$. However, if the effective normal stress is reduced by a pore pressure, P, then the gliding may occur even for subcritical angles, that is for slope angles, α, that are smaller than ϕ. This condition arises if $\sigma_S > (\sigma_N\text{-}P)\mu$. When the ground water table below an aquiclude (the mud cover) rises, then the pore pressure builds up inside the aquifer (the sand layer) as the water table continues to rise above the aquiclude. Sooner or later, sliding occurs for any slope, α, due to the reduction in the magnitude of the effective normal stress (σ_N-P), while the shear stress remains unaffected by the pore pressure.

b) Liquefaction occurs when $P > \sigma_N$, so that grains start to float past one another without any friction between them, except for some viscous retardation by the fluid suspension.

4-5: Hubbert & Rubey's (1959) classical theory of gravity sliding over a slope (Fig. 4-7) can be easily derived by yourself in this exercise.

a) A slab of rock with a coefficient of internal friction μ of 0.85 is stable for $\sigma_S < \mu\sigma_N$, is critical for $\sigma_S = \mu\sigma_N$, and is unstable for $\sigma_S > \mu\sigma_N$. The critical angle, $\alpha_c = \phi$ or $\alpha_c = \tan^{-1}\mu$, is equal to 40° for μ=0.85, provided the pore pressure is 0.

b) The pore pressure at the base of a layer contributes to destabilization as follows: Movement, in the presence of a pore pressure, occurs when: $\sigma_S \geq (\sigma_N\text{-}P)\mu$. The pore pressure (here normalized by the normal stress for convenience of calculation), P/σ_N, and the critical slope, α_c, are related as follows: $P/\sigma_N = 1\text{-}(\tan \alpha_c/\mu)$. This expression is valid only for the critical case of movement. It can be seen that, for $\alpha_c = \tan^{-1}\mu$, no extra pore pressure is required to initiate sliding. In other words, if P=0, then $\alpha_c = \phi$. However, $\alpha_c < \phi$ for P>0. When the pressure P increases, α_c decreases, with one extreme case occurring for $P = \sigma_N$ when α_c=0°.

The expression derived here can be easily graphed for pairs of values $[(P/\sigma_N), \alpha_c]$, with curves for a range of μ values. It is worth noting that, in many practical situations, $P/\sigma_N = \lambda$, that is, the coefficient of fluid pressure (cf. section 3-7). The above equation then can be rewritten as: $\mu = \tan \alpha_c/(1-\lambda)$, which is the condition under which sliding can occur.

c) To initiate gravity sliding on a slope of 2°, using μ of 0.85 as given here, the required normalized pressure is: $P/\sigma_N = 1-(\tan 2°/0.85) = 0.96$. In other words, to initiate gravity sliding on the slope of 2°, P must be equal to or larger than 0.96 times σ_N.

d) If the pore pressure remains zero, an additional tectonic shear stress, τ_S, can initiate gliding for subcritical angles when: $\tan \alpha_c + \tau_S/\sigma_N \geq \mu$. However, for supercritical angles, $\alpha_c \geq \phi$, sliding occurs even when $\tau_S = 0$.

e) The principal expressions derived in problems (b) and (d) can be combined into a single expression: $\mu \leq [\tan \alpha_c + (\tau_S/\sigma_N)]/[1-(P/\sigma_N)]$ or $\tau_S/\sigma_N \geq \mu[1-(P/\sigma_N)] - \tan \alpha_c$. This expression can easliy be graphed for values $[(\tau_S/\tau_N),(P/\sigma_N)]$, with curves for a range of critical angles, α_c. It is worth noting again that in many practical cases P/σ_N is equal to λ.

4-6: a) & b) The Mohr circle for calculating the stress on arbitrary planes through a point with principal stresses $\sigma_1 = 150$ MPa, $\sigma_2 = 100$ MPa, and $\sigma_3 = 50$ MPa can be constructed as follows: Use either equations (4-5a & b) for total stress or equations (4-6a & b) for deviatoric stress. The deviatoric stresses are $\tau_1 = 50$ MPa, $\tau_2 = 0$, and $\tau_3 = -50$ MPa. The pressure is 100 MPa, $\tau_N = 50\cos 2\xi$, and $\tau_S = 50\sin 2\xi$.

The Mohr circle for deviatoric stress is a circle about the origin with a radius of 50 MPa. The Mohr circle for total stress is drawn about point (100 MPa, 0) on the horizontal σ_N-axis as a circle of 50 MPa radius. For $2\xi = 90°$, $\sigma_N = 100$ MPa and $\sigma_S = 50$ MPa. For $2\xi = 60°$, $\sigma_N = 125$ MPa and $\sigma_S = 43.3$ MPa. The deviatoric stress components on planes at 30° and 45° to the τ_1-direction can be found by plotting the double angles, $2\xi = 60°$ and 90°, measured counterclockwise away from the horizontal axis. For $2\xi = 90°$, $\tau_N = 0$ and $\tau_S = 50$ MPa. For $2\xi = 60°$, $\tau_N = 25$ MPa and $\tau_S = 43.3$ MPa.

4-7: The completion of this exercise is easy if the principle of Mohr circles is understood. Two perpendicular planes through the same point have, at that point, respectively, stress states (σ_N, σ_S) as follows: (220,110) and (120, -110) in MPa. The two points can be plotted between the (σ_N, σ_S)-axes of a Mohr circle. The Mohr circle itself is not known at first. But, because the two points are for planes 90° apart, $2\xi = 180°$. Consequently, the center of the Mohr circle is found where the straight line (of $2\xi = 180°$), connecting the two data points, cuts the σ_N-axis in the (σ_N, σ_S)-plot. This straight line cuts the σ_N-axis at +170 MPa, which is equal to the pressure or mean stress: $(\sigma_1 + \sigma_3)/2 = 170$ MPa. The Mohr circle radius is equal to half the distance between the two points or 130 MPa, that is: $(\sigma_1 - \sigma_3)/2 = 130$ MPa. The principal stresses can now be evaluated as $\sigma_1 = 300$ MPa and $\sigma_3 = 40$ MPa.

4-8: a) The pair of normal and shear stresses in each of the two perpendicular planes plots as a point on the Mohr circle. Because the planes cross the same point, but are oriented 90° apart, they always lie in a straight line through the center of the Mohr circle (see, also, exercise 4-7). Consequently, the shear stresses on these so-called conjugate planes are always equal in magnitude but of opposite sign.

b) The sum of the two normal stresses is, physically, equal to twice the pressure at the point where the two planes cross. If the line connecting the pairs of (σ_N, σ_S) for the two planes is vertical, the sum of the normal stress will be 2P (see Mohr circle, Fig. 4-8b). If this synthetic line is horizontal, the pairs of (σ_N,σ_S) on the σ_N-axis, also, add up to give a sum for normal stress equal to 2P. All mutually perpendicular planes of other orientations to σ_1, have, also, normal stresses, the sum of which is consistently equal to 2P.

4-9: After comparison of graphs of F_N and F_S against α (see exercise 3-3c) and graphs of τ_N and τ_S against $\alpha=2\xi$ (Fig. 4-9b), the questions posed in this exercise can be answered as follows:

a) The shear stress at a plane always is at maximum for $\alpha=90°$, which means that the plane is at 45° to the major principal stress axis, as follows from $\alpha=2\xi$ (cf. Fig. 4-11). The shear force on a slope of $\alpha=0°$ is equal to the weight of the slide mass, i.e., $F_S=F_{net}$ (cf. Fig. 3-3).

b) The normal stress at a plane is at maximum for $\alpha=0°$. The normal force is, also, at maximum for $\alpha=0°$, that is, $F_N=F_{net}$ (cf. Fig. 3-2).

c) F_{net} and σ_1 are parallel only for $\alpha=0°$. F_{net} and σ_1 are 45° apart for $\alpha=90°$. The full relationship is fixed by equation (4-8), using the angles, α and ξ, as defined in Figure 4-11.

4-10: The stress trajectories in the Shiprock area of Figure 4-14 were, at the time of intrusion, similar to those given in Figure 4-12b. The dikes are intruded along σ_1 trajectories, which delineate vertical surfaces normal to σ_3. The fact that the radial dikes of Shiprock are connected at the surface to the central stock implies that a few kilometers of overburden has been removed by erosion since the injection of the dikes. If the exposed dikes had been intruded at surface conditions, then the stress field of Figure 4-12c would apply, rather than that of Figure 4-12b. However, the alternative stress field of Figure 4-12c precludes a connection of the central stock and the radial dikes in any plan view at shallow depths.

4-11: a) The Richat dome (Fig. 4-15), if overlying a buoyant granitic pluton, must have been stressed approximately as outlined by the stress trajectories of Figures 4-12a to c.

b) Pegmatite dikes, originating from a granitic infrastructure, would have intruded the suprastructure layers of the dome. The fractures would be controlled by the stress patterns, illustrated in Figures 4-12a to c. The stress trajectories that would apply to the Richat dome at shallow erosion depths are given in Figure 4-12c. Pegmatites would inject as cone sheets within the central region but be intruded as radial dikes outside the circular section of neutral stress. The sketch of Figure 4-13 may help to visualize the anticipated pegmatite orientations.

4-12: a) The stress trajectories suggested by the fracture pattern over the Clay Creek Dome, Texas (Fig. 4-16a), are more or less radially symmetric, similar to those of Figure 4-12c.

b) The stress trajectories suggested by the fracture pattern of the Bell Isle Dome, Louisiana (Fig. 4-16b), are not strictly radially symmetric, but they are similar to those of Figure 4-17. (Figure 4-17 illustrates horizontal stresses around a salt dome, subjected to a regional stress field that is superimposed on the stresses generated by the pressure of the salt dome intrusion itself.)

c) The overburden of the Bell Isle Dome probably became fractured as it rose during a regional compression with a NE-SW trend. However, the Clay Creek Dome fractured its overburden in the absence of any regional tectonic background stress.

4-13: The stress field around the salt dome of Figure 4-17 is completely dominated by regional tectonic stresses, which are here much larger than the stresses induced by the buoyant salt dome itself. The salt plug represents a mechanical anomaly, which deflects the regional stress trajectories, as indicated by the in-situ stress measurements.

Chapter five

5-1: The qualitative graph of force against time for the elasto-plastic unit with peak strength (Fig. 5-1d) has the following characteristics: The force increases linearly when the spring is pulled toward the left, but it decreases abruptly when the plug pops out. The friction of the plastic unit may result in another linear increase of the force until the plastic unit starts to slip over the surface. The force then remains at a constant value equal to the frictional resistance of the frictional plastic unit. Two force maxima occur in the plot, equivalent to the resistances of the plug unit and the frictional plastic unit, respectively. The relative magnitude of the two maxima varies, commensurate with the relative strengths of both mechanical units.

5-2: a) In Figure 5-5a, the strain remains stationary at a plateau value during the loading, but it recovers completely after removal of the load (at time t_x).

b) In Figure 5-5b, the stress will remain constant if the strain-rate ceases after time $t=t_x$, and the strain remains fixed at its final value.

c) The stress-time graph for a stress-relaxation test in an ideal elastic material shows instantaneous establishment of a particular stress upon the emplacement of the load. Subsequently, the stress remains at a constant value, indicated by a horizontal line [as in Figure 8-5, top right diagram (p. 129)].

5-3: a) In a creep or constant load test, the stress level is kept constant at all times. In fact, this may require different load forces, unlike what is suggested by the term "constant load" test. A creep test on the mechanical unit of Figure 5-6, applying a stress that exceeds the plug strength, is of short duration, because the plug will pop out instantaneously. The strain-time graph shows instantaneous increase of elastic strain at time zero (Fig. 5-5a), which immediately drops back to strain-rate zero when the plug pops out.

b) A constant strain-rate test on a Hookean elastic requires controlled increase of the stress in order to keep the strain-rate constant. If applied to the plug-elastic unit of Figure 5-6, the stress-time graph looks as shown in Figure 5-5b, but the stress drops instantaneously to zero once the stress reaches the peak strength of the plug unit.

5-4: The stress-time graph of a plastic unit in a constant load test shows a stress constant over time and is equal to the frictional resistance of the plastic unit.

5-5: The stress-time plot for a constant strain-rate test on the elasto-plastic unit with plug peak strength, as illustrated in Figure 5-1d, shows linear build-up of elastic stress in the spring at the onset of the test. The stress drops abruptly when the plug's peak strength is reached but immediately puts the frictional unit into motion, because its frictional resistance stress is only half of that required to pull out the plug. Consequently, after the initial instantaneous drop in stress, the stress remains constant at a level required to move the plastic unit at a constant strain-rate.

5-6: The Poisson ratio is defined as $\nu=-e_3/e_1$. Rock volume remains constant when $e_1+e_2+e_3=0$. The free lateral boundaries in the Poisson test on a homogeneously straining, elastic rock imply that the shortening, e_1, must be compensated for by proportional extensions in the plane normal to e_1. The condition of no volume change is, therefore, fullfilled if $e_2=e_3=-0.5e_1$. Using the values of this constant volume case in the definition of the Poisson ratio gives: $\nu=0.5$.

5-7: The Poisson ratio is defined only for small or infinitesimal strains, where linear elasticity is valid. The initial principal strain values of the constant strain-rate test on the Berea sandstone sample are given in Table 5-2 and plotted in the graph of Figure 5-8. The values of the Poisson ratio during the initial and subsequent load increments are: $\nu_1=0.87$, $\nu_2=0.17$, $\nu_3=0.20$, $\nu_4=0.24$, $\nu_5=0.28$, $\nu_6=0.33$, $\nu_7=0.37$, $\nu_8=0.41$, $\nu_9=0.48$, and $\nu_{10}=0.54$. The initial and final values are larger than 0.5 and may be due, initially, to the settling of the sample and, finally, to the onset of microcracking. Initial average values of ν are between 0.17 and 0.28, which is within the range quoted for sandstone in Table 5-1.

5-8: This is a tricky question. The bulk modulus, κ, is given for air at atmospheric pressure. However, ideal gases obey the thermodynamical equation of state: PV=nRT (P-pressure, V-volume, n-moles, R-gas constant, T-absolute temperature). Thus P=nRT/V and $(\delta P/\delta V)_T=nRT/V^2$ or $\beta_T=V/(nRT)=1/P$. Consequently, β is inversely proportional to P. The conclusion is that the bulk modulus for air is proportional to P. The pressure rises when the air pump is used, but so does the bulk modulus. The volume of the air in the pump decreases indeed when the pressure increases, but it does so according to P=nRT/V. Consequently, the bulk modulus is of little use to monitor the volume change of air in the pump.

5-9: a) The elastic elongation of foam rubber with E=100 Pa, subjected to a tensional stress of 1 kPa, is: $e_1=\tau_1/E=1kPa/100Pa=10$.

b) The principal stress in a foam rubber which is extended 100%, that is, $e_1=1$, is: $\tau_1=Ee_1=100$ Pa x 1=100 Pa.

5-10: Estimates of the Young modulus for Berea sandstone, using $E=\tau_1/e_1$ with the experimental data of Figure 5-8 and Table 5-2, vary between 14 and 18 GPa. This is within the range quoted for sandstone in Table 5-1.

5-11: A cylinder with Young modulus E=100 GPa, that is uniaxially compressed by -10 MPa, will have a principal elongation of: $e_1=\tau_1/E=-10$ MPa/100 GPa=-10^{-4}. If the cylinder was initially 25 cm long (L_0), it will shorten so that the new axial length, L_1, is: $L_1=L_0+e_1L_0$ [acc. to eq. (5-1)]. Substituting the values provided, $L_1=(25-10^{-4}x25)$ cm= 24.9975 cm. The transverse expansions, e_2 and e_3, follow from the Poisson ratio: $e_3=-\nu e_1=0.25x10^{-4}$. The new diameter is: $L_3=L_0+e_3L_0=[10+(0.25x10^{-4}\ x10)]$ cm=10.000025 cm.

5-12: The removal of an elastic angular shear strain of $\gamma=2.5x10^{-15}$ from the walls of the San Andreas fault system, which has a shear modulus, G, of 30 GPa, reduces the stress by: $\tau_S=30$ [GPa]x$2.5x10^{-15}$=7.5 MPa.

5-13: According to equations (5-10) and (5-11), the ratio of G/κ is equal to: $(3-6\nu)/(2+2\nu)$. Substituting $\nu=0.5$ gives: $G/\kappa=0$. Physically, this means that materials of Poisson ratio 0.5 are much easier to shear than to compress, because $\kappa>>G$.

5-14: The vertical lithostatic stress associated with elastic deformation of buried rock is: $\tau_1=-[(2-4\nu)/(3-3\nu)]\rho gz$. The stress at 10 km depth, using the values provided, is $\tau_1=-118$ MPa. Relaxation of such elastic stresses upon rapid exhumation may be large enough to cause joint development in rocks.

5-15: a) The equations for the three principal deviatoric stresses are, using equations (5-16a & b): $\tau_1=-11.76$ x z (in km) [MPa] and $\tau_{2,3}=-5.88$ x z (in km) [MPa]. Both these expressions can be plotted as linear functions of depth.

b) If a tectonic compression of -100 MPa is superimposed on the τ_2-direction, as illustrated in Figure 5-15, then the total deviatoric stress in the former τ_2-direction becomes larger than the vertical stress for rocks shallower than 5 km deep. The level of isotropic points occurs where $\tau_1=\tau_2=\tau_3=0$.

5-16: The principal stresses on the buried surface can be computed using equations (5-16a & b): τ_1=-54.5 MPa and $\tau_{2,3}$=27.2 MPa.

5-17: The strain-time graph for a stress-relaxation test on a Kelvin-Voigt unit is similar to that shown in Figure 5-16a, except for the initial elastic strain, which cannot occur in the Kelvin-Voigt unit. The correct graph shows the building up of initial anelastic strain followed by complete anelastic recovery of this anelastic strain after some time upon the removal of stress.

Chapter six

6-1: The contours in Figure 6-2b are curves of constant maximum shear stress magnitude. The maximum shear stress *at each point* in the material is given by: $\tau_{Smax}=(\tau_1-\tau_3)/2$. The stress contour values vary between 0 and 2 in Figure 6-2b (with contour interval 0.2), and the stresses are normalized by the vertical load stress. The maximum shear stress occurs at the crack tip, where the normalized shear stress peaks to twice the magnitude of the principal stress that is applied to the bulk of the material.

6-2: The single Hookean spring element of the mechanical analog, portrayed in Figure 6-7, represents the elastic mode of distortion (stage B in Figure 6-6). The Kelvin-Voigt unit represents the anelastic behavior when the microcracking occurs (stages C & D in Figure 6-6). The plug unit represents the peak strength and subsequent drop of stress with anelastic recovery (stages E & F in Figure 6-6). The residual strength is represented by the frictional plastic unit.

6-3: Rock tests in triaxial deformation apparatus can increase the confining pressure, unlike that in uniaxial deformation rigs, where the pressure remains atmospheric (at 1 atmosphere or 0.1 MPa). At higher pressures, the strain softening, that causes a secondary drop in the differential stress in tests with low confining pressures, does not occur at pressures above 100 MPa (in the example of Figure 6-9). This means that the plug in the arrangement of Figure 6-7 can be removed in mechanical analogs for the stress-strain behavior of rocks in cold press tests at higher confining pressures (cf. Fig. 6-9).

6-4: a) A depth scale can be plotted along the horizontal pressure scale of Figure 6-10, dividing pressure values by the mean pressure gradient of 25 MPa/km for crustal depths.

b) According to the generalized experimental data of Figure 6-10, at 150 MPa pressure, or about 6 km depth and beyond, anhydrite will break at strains lower than those required to break sandstone. This behavior is reversed at shallower depths, where sandstone breaks at strains lower than those required to break anhydrite.

c) During progressive strain, according to the generalized experimental data of Figure 6-10, sandstone breaks before limestone between confining pressures of 65 and 170 MPa (2.5 to 7 km deep). But limestone breaks before sandstone at depths that are shallower than 2.5 km and deeper than 7 km. However, it must be noted that ductile creep is likely to occur beyond 7 km depth, precluding any brittle faulting, because the elevated temperatures activate crystalline creep processes (see chapter 7).

6-5: The intermediate and minor principal stresses, σ_2 and σ_3, are equal in uniaxial deformation tests. This generally, also, applies to triaxial tests, where σ_2 and σ_3 are still equal and utilized only to impose a confining pressure larger than the ambient atmospheric pressure that occurs in uniaxial deformation tests. The critical surface of circular shear failure of Figure 6-12a can occur when the rock specimen is perfectly homogeneous and when σ_2 is equal to σ_3 throughout the sample. However, in practice, minor perturbations of these conditions lead to specimen failure with conjugate shear surfaces that are planar, as illustrated in Figures 6-11b and 6-12b (rather than circular as in Fig. 6-12a).

6-6: a) The coefficient of thermal expansion for basaltic lava is: $\alpha=\delta V/(V\delta T)=2.5\text{x}10^{-6}$ °C^{-1}. Lowering of the temperature from 1000° C to 25° C will cause a shortening of: $\delta V/V=\alpha\delta T=-2.5\text{x}10^{-6}\text{x}975=-2.44\text{x}10^{-3}$ or -0.25%. The three principal lengths will all shrink by 0.08 %. Over a length of 100 meters, this means a shortening of 8 cm has to be accommodated. Traction on the floor of the flow prevents such shortening from occurring through homogeneous horizontal slip, and that is why discrete columnar joints develop.

b) The jointing is principally due to the thermal stresses, which are classified as body forces, but which would not occur without the traction at the base of the lava sheet. Those traction stresses are classified as surface forces.

6-7: The location of tension and shear joints can be found by using the model of Figure 6-12b to trace the joints at key positions in Figure 6-17b, away from the neutral surfaces.

6-8: The peak strength for pyroxene, inferred from Figure 6-9, is 220 MPa (or σ_1-σ_3) for a confining pressure, P, of 20 MPa. The radii of each of the Mohr circles are: 280 MPa (for P=40 MPa), 315 MPa (for P=60 MPa), 340 MPa (for P=80 MPa), 340 MPa (for P=80 MPa), 380 MPa (for P=140 MPa), and 410 MPa (for P=180 MPa). These values give a well-defined Mohr-Coulomb plot. The Mohr-Coulomb envelope is approximately linear.

6-9: a) The equation for the Mohr-Coulomb envelope is: $\sigma_S=229$ [MPa]$+1.4\sigma_N$[MPa].

b) The effective pressure for Westerly granite at 5 km depth is given by: $P_{eff}=\lambda\rho gz$, with λ denoting the coefficient of fluid pressure: (1) for hydrostatic state, $\lambda=0.42$, and $P_{eff}=52$ MPa; (2) for $\lambda=0.8$, $P_{eff}=100$ MPa; (3) for lithostatic pressure, $\lambda=1$, and $P_{eff}=125$ MPa.

c) The pressure is plotted along the horizontal σ_N-axis of the Mohr diagram. For each pressure, only one circle that is exactly tangential to the Mohr envelope can be found. The single intersection point of the Mohr envelope and the circle is connected by a straight line (for 2ξ) to the center of the circle. The angle 2ξ can now be measured. The critical horizontal stress, σ_1, can be read where the Mohr circle intersects the σ_N axis.

d) The angles, ξ, between σ_1 and the normal to the fault surfaces become smaller and smaller for larger effective pressure (cf. Fig. 6-20). Consequently, the dips of the faults, given by (90°-ξ), increase with increasing pressure. The failure planes show reverse fault movement, but they may be termed thrusts if (90°-ξ) becomes sufficiently small.

6-10: Amonton's law is as follows: $\sigma_S=\mu\sigma_N$. The rock mass of Figure 6-23 slides when $\sigma_S\geq\mu\sigma_N$. It can be demonstrated that $(\sigma_S/\sigma_N)=\tan\alpha$, with α as the slope of the slide mass. Consequently, the mass is stable when $\tan\alpha<\mu$, and the critical angle of $\alpha=\phi$ occurs when α is equal to the angle of internal friction, $\tan^{-1}\mu$ (cf. section 3-3).

6-11: a) The requested Byerlee failure envelope is already drawn in Figure 6-21b.

b) The center of your Mohr circles must lie along the horizontal σ_N-axis. Different centers correspond to different magnitudes of the confining pressure.

c) At higher confining pressures, the angle between the failure planes and σ_1 increases, as illustrated separately in Figure 6-20.

d) The deviatoric and total, normal stresses differ by the pressure, P. The increase of the deviatoric stress with pressure suggests that energy, required to move the fault plane at higher pressure, is higher. However, this relationship is not so simple, as becomes clearer when expressing the relationship between pressure and σ_S and σ_N.

6-12: The plot of Figure 6-25 scales the differential stress required to initiate movement on reverse, normal, and strike-slip faults. If reverse faulting occurs (under compression), then the stress difference cannot exceed the values outlined by line a. If normal faulting occurs (under tension), then the stress difference cannot exceed the values outlined by line b. If strike-slip faulting occurs, then the stress difference cannot exceed the values outlined by line c. An increased pore pressure may activate fault movement at much lower stresses than required in dry rocks, as follows by comparing the differential stress scales for $\lambda=0$ and $\lambda=0.9$, indicated along the horizontal strength scale of Figure 6-25. (For more pressure effects, see, also, Figs. 6-26a to c and 6-27a & b.)

6-13: The strength profile for the brittle section of the crust, with variations in fluid pressure, as specified in Figure 6-27b, can be quickly drawn onto the graph of Figure 6-27a. Each time the coefficient of fluid pressure changes, the actual strength for that particular depth is indicated by the strength envelope for the corresponding fluid pressure. This means instantaneous drops in the crustal strength occur when the fluid pressure increases. Conversely, the crustal strength increases when the fluid pressure decreases.

Chapter seven

7-1: Some videotapes, suitable for studying crystalline creep, are listed in the references on p. 123 to 124. They can be viewed, described, and discussed, with or without supervision of your course instructor.

7-2: a) "Strain memory" is a term referring to the ability of strain markers, such as crystal grains, to pick up the finite strain of a deformation. The strain memory of truly passive markers is perfect, because such markers record and preserve all the strain that has occurred. However, other strain markers, such as crystal grains, are not passively recording the strain; they are active strain markers. They do not record the bulk strain of the total rock volume, because they interact with neighboring grains. Figures 7-3a to c effectively illustrate that the strain memory of mineral grains in a deformed rock or tectonite varies and generally is less than perfect.

b) Static recrystallization can remove any preferred orientation of a crystal fabric (e.g., quartz). After static recrystallization, mineral crystals no longer preserve any sign of deformation, but the outside boundaries of the rock volume generally retain the deformation patterns attained during dynamic recrystallization. Static recrystallization does not remove the strain of the bulk rock; that is, the deformation attained by dynamic recrystallization is definitely inelastic.

7-3: The rate of grain-shape fabric formation during dynamic crystallization is counteracted by a rate of static recrystallization. The latter rate is likely to be independent of the former, because static recrystallization occurs even when the rate of dynamic recrystallization vanishes. At lower rates of dynamic recrystallization, the intensity of the developing grain-shape fabric is likely to be reduced by the effect of static recrystallization. The process of static recrystallization is least effective at higher rates of dynamic recrystallization, and it is most effective when the dynamic deformation ceases.

7-4: Pressure solution seams commonly are oriented normal to the direction of lithostatic loading (cf. Figs. 7-7a & b). This can be understood better by studying the deviatoric stress patterns in granular aggregates, acting as rock analogs (cf. Fig. 4-3). The stress at the grain contacts, where the overlying grains impinge, is large. Obviously, the grain contacts are locations where pressure solution is most effective.

7-5: The effective viscosity of minerals in Coble creep is proportional to the third power of the grain size. In Nabarro-Herring creep, the viscosity is proportional to the second power of the grain size. Grain size reduction during dynamic recrystallization, therefore, reduces the effective viscosity of the rock and may lead to concentration of the flow in regions with lowered effective viscosity.

7-6: When the principal compressive stress is parallel to the glide plane of micas, plastic bending is achieved by the movement of edge dislocations along the glide planes (cf. Fig. 7-16a to c). The plastic bending results in a gradual distortion of the crystal lattice, which is responsible for the undulous extinction, observed along the arc of the folded minerals (cf. Figs. 7-17 and 7-18), as viewed with an optical polarizing light microscope.

7-7: a) This process is illustrated in Figures 7-16a to c.

b) Subgrains form at locations where the crystal lattices locally mismatch, due to the piling up of dislocations and impurities. If the mismatch in the crystallographic orientations becomes larger than about 7°, the subgrain boundary will have sufficiently mismatched as to be regarded as a new crystal boundary (Fig. 7-16d). In this fashion, initial grain boundaries may become lobate and may bifurcate and migrate to form new subgrains, some of which develop into new individual grains with distinct grain boundaries.

7-8: The log stress/log strain-rate graphs for Yule marble, using the steady-state flow data of Figures 7-21a to c, plot with slopes less than unity (that is, $n>1$, according to $n=\cot\alpha$, as defined in Fig. 7-23). This indicates that dislocation creep is the dominant mechanism of deformation at the strain-rates used in hot press tests.

7-9: a) According to the data of Figure 7-25, if the grain size of quartz reduces from 1 mm to 0.01 mm at a constant strain-rate of 10^{-14} s^{-1}, then the dominant deformation mechanism changes from dislocation creep to Coble creep. In Coble creep, the viscosity is proportional to the third power of the grain size. Within the regime of Coble creep, the grain size dropped from approximately 0.1 mm to 0.01 mm (using Fig. 7-25). If the grain size reduces to 0.1 times the initial value, then the stress in those grains (in Coble creep) drops three orders of magnitude. According to Figure 7-25, the stress was still about 1 MPa when the grain size reduced from 1 mm to 0.1 mm in the dislocation creep regime. However, the stress must drop to 0.001 MPa when the grain size reduces one order of magnitude further, to 0.01 mm, in the Coble creep regime.

b) According to Figure 7-25, if the stress is maintained at 1 MPa, the grain-size reduction, described in exercise 7-9a, will result in a strain-rate softening, because the strain-rate increases two orders of magnitude from 10^{-14} to 10^{-12} s^{-1}.

Chapter eight

8-1: a) The units of the dynamic shear viscosity, η, follow from the definition: $\eta = \tau(u/d)$ [Pa m $s^{-1}m^{-1}$], and they are equivalent to Pa s [Pascal second].

b) The effective viscosity is a scalar quantity, provided it is isotropic in the material point(s) concerned.

8-2: a) The units of the kinematic viscosity, ν, follow from its definition as: $\nu=\eta/\rho$ [Pa s kg^{-1} m^3]. Using Table 2-1, section B: 1 Pa s kg^{-1} m^3=1 kg m^{-1} s^{-2} kg^{-1} m^3=1 m^2 s^{-1}.

b) The kinematic viscosity is a scalar quantity, provided it is isotropic in the material point(s) concerned.

8-3: The rheology of some of the materials listed in Table 8-1 can be tested qualitatively by a stirring movement in air, water, olive oil, honey, and silicone gum. For qualitative comparison, the viscosity of chewing gum commonly ranges between 10^3 and 10^4 Pa s.

8-4: Comparison of the three graphs in Figure 8-5 for each mechanical analog reveals that creep tests provide time/strain-rate graphs that are distinctive for each type of mechanical behavior. In contrast, constant strain-rate and stress-relaxation tests provide graphs that are not necessarily unique for a particular type of mechanical behavior.

8-5: In order to determine the n-value of the power-law body in Figure 8-8, pairs of viscosity readings and strain-rate need to be plotted in a log-stress/log strain-rate diagram. Obtain a flow curve, such as illustrated in Figure 8-7. The points plotted can be matched to segments of straight lines. The cotangent of the slope of these line segments is equal to their n-value. The data of Figure 8-8, when transferred to Figure 8-7, yield an n of 10.

8-6: The general trend of the effective viscosity graph for granite must be similar to that of other rocks, as illustrated in Figure 8-11.

8-7: a) All materials that have n-values between unity and infinity are strain-rate softening. A theoretical strain-rate hardening material would have n-values smaller than unity. However, the author is unaware of the existence of any such material in the real world. [see, also, the concluding remark under (b).]

b) Strain-rate hardening materials deform faster when lower deviatoric stresses are applied, and their strain-rates are slower when higher deviatoric stresses are applied. This means such materials, if existing, would be the ultimate solution to the quest to find sources of unlimited energy.

8-8: a) The oblique lines in the plot of Figure 8-10 are isoviscosity contours. The effective viscosity is defined as $\eta_{eff}=\tau_c/\gamma_c$ [cf. eq. (8-7)].

b) The effective viscosity of 243 K ice, flowing at 10^{-11} s^{-1}, is (10^4 Pa/10^{-11} s^{-1}) or 10^{15} Pa s, using values from Figure 8-10.

c) The effective viscosity of 243 K ice, flowing at 10^{-7} s^{-1}, is (10^6 Pa/10^{-7} s^{-1}) or 10^{13} Pa s, using values of Figure 8-10. The lowering of the viscosity at increased strain-rates [compare (b) and (c)] is termed a strain-rate softening behavior.

8-9: The flow curves for rocks listed in Table 8-2 are similar in appearance to those of Figure 8-11. They can be calculated using equation (8-8b), substituting the parameter values given in Table 8-2. Remember that the deviatoric stress, τ_1, is equal to: $\tau_1=(\sigma_1-\sigma_3)/2$.

8-10: It is well-known that more competent or "stiffer" layers in a multilayer sequence will buckle upon layer-parallel shortening. At 200° centigrade, Carrara marble has a much lower strength (or lower viscosity) than quartzite, according to Figure 8-12. In conclusion, the quartzite layers are likely to fold and they largely control the rate of the buckling process.

8-11: a) Crustal strength in quartzitic crust with a hydrostatic pore pressure is graphed in Figure 8-13b for two possible geothermal gradients (30 K/km and 15 K/km). According to the map of Figure 8-14, location A has a geothermal gradient of 15 K/km. The brittle-ductile transition in an ultra-deep borehole at A would not occur until a depth of 15 km has been reached. For location B, where the heat flow is larger (30K/km), the brittle-ductile transition occurs at about 8.5 km depth, according to the curves of Figure 8-13b.

b) If the rocks are dry, rather than wet, then the brittle-ductile transitions for holes A and B occur at depths of 13 and 7 km, respectively, according to the graph of Figure 8-13a.

8-12: The lithospheric strength profiles for a Precambrian shield area with a 40 km deep MOHO and that of the adjacent platform with a 30 km deep MOHO are similar to that shown in Figure 8-15. The detailed values for the strengths of the crustal section of assumed quartzitic composition follow from the plots of Figures 8-13a to c. The crustal part of the continental lithosphere below the colder shield area is, obviously, much stiffer than that below the hotter platform. The strength of the mantle parts of the continental lithosphere are similar for both areas. The platform margin deforms much faster than the Precambrian interior, simply because the total resistance of the platform sequence to any internal deformation is much lower than that of the Precambrian interior, as follows by comparing their respective lithospheric strength profiles.

8-13: The Flannan Fault is likely to be mostly a ductile shear zone, rather than a brittle fault. This is so because the synthethic strength profile for that crustal section suggests that the crust is mostly ductile. The Flannan Fault extends without soling out, because there is no obvious zone of weakness to sole into. The Outer Isles Fault is, also, likely to be a ductile shear zone at depth. It soles into the base of layer 1, because a zone of low viscosity occurs there. The other normal faults are soling out at the base of layer 2, because another zone of relatively low crustal viscosity occurs there, according to the suggested strength profiles. Although idealized strength profiles are highly conjectural, they provide a possible explanation for the crustal structure observed. However, one important concern is that the age of the faults is likely to be Cretaceous, and the modern heat flow data used in the synthetic strength profiles may not be valid for the remote past, when the faults were formed.

Chapter nine

9-1: a) MDCCLXI times CCCIV is equivalent to 1761 times 304.

b) MDCIV minus LIX is equivalent to 1604 minus 59. The Roman notation and the Arabic notation used here are both constituted by metric numbers. However, most of us would prefer the Arabic number notation, because we are more familiar with it and it is more compact. Nonetheless, the Roman notation utilizes only seven different symbols (i.e., M, D, C, L, X, V, and I), whereas the Arabic notation uses ten different symbols (i.e., 1, 2, 3, 4, 5, 6, 7, 8 , 9, and 0). For comparison, binary numbers are comprised of only two symbols (i.e., 0 and 1). Binary numbers are most practical for use in computers, but they are, apparently, less suitable for the perception of quantities by the human brain.

9-2: The velocity field for the simple shear deformation of Figure 9-4 is: $v_x=\partial\psi/\partial z=\partial[(\gamma/2)z^2]/\partial z=\dot{\gamma}z$. The velocity gradient is: $\partial^2\psi/\partial z^2=\partial(\gamma z)/\partial z=\dot{\gamma}$.

9-3: a) & b) The pure shear deformation of Figure 9-5 has a velocity vector, **v**, of $(\dot{e}_1x, 0, \dot{e}_3z)$.

The *velocity-gradient tensor*, $\nabla\mathbf{v}$, according to equation (9-7) is:

$$\nabla v = \begin{bmatrix} \dot{e}_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \dot{e}_3 \end{bmatrix}$$

The normal elements of the velocity gradient tensor are equal to the three principal strain-rates: $\dot{e}_1$, 0, and $\dot{e}_3$.

The *divergence of the velocity*, $\nabla.\mathbf{v}$, according to equation (9-8), is: $[\partial(\dot{e}_1x)/\partial x]+[\partial(\dot{e}_3z)/\partial z]=\dot{e}_1+\dot{e}_3$. It is obvious that for pure shear deformations $\dot{e}_1=-\dot{e}_3$, so that $\nabla.\mathbf{v}=0$. This means that the deformation involves *no volume change*.

The *curl vector of the velocity*, $\nabla x\mathbf{v}$, according to equation (9-9), is: (0,0,0). The elements of the vorticity vector vanish, which indicates that the deformation is *irrotational.* The velocity gradient or deformation rate, $\nabla\mathbf{v}$, for irrotational deformations with $\nabla x\mathbf{v}=(0,0,0)$ is equal to the strain-rate tensor (see, also, exercise 9-7c).

9-4: a) The deviatoric stress tensor for the simple shear of exercise (9-2), using the differential equation (9-11), is: $\tau_{ij}=\eta(1/2)\,[(\partial v_i/\partial x_j)+(\partial v_j/\partial x_i)]=\eta/2$ times the following tensor:

$$\begin{matrix} 2\partial v_1/\partial x_1 & (\partial v_1/\partial x_2)+(\partial v_2/\partial x_1) & (\partial v_1/\partial x_3)+(\partial v_3/\partial x_1) \\ (\partial v_2/\partial x_1)+(\partial v_1/\partial x_2) & 2\partial v_2/\partial x_2 & (\partial v_2/\partial x_3)+(\partial v_3/\partial x_2) \\ (\partial v_3/\partial x_1)+(\partial v_1/\partial x_3) & (\partial v_3/\partial x_2)+(\partial v_2/\partial x_3) & 2\partial v_3/\partial x_3 \end{matrix}$$

Substitution of $v_x=\dot{\gamma}z$, $v_y=0$, and $v_z=0$ yields:

$$\tau_{ij} = \begin{bmatrix} 0 & 0 & \eta\dot{\gamma}/2 \\ 0 & 0 & 0 \\ \eta\dot{\gamma}/2 & 0 & 0 \end{bmatrix}$$

b) The magnitude of the stress tensor for a thrust movement with angular shear strain-rate of $\dot{\gamma}=10^{-14}$ s^{-1} and dynamic shear viscosity of $\eta=10^{22}$ Pa s is:

$$\tau_{ij} = \begin{bmatrix} 0 & 0 & 50 \\ 0 & 0 & 0 \\ 50 & 0 & 0 \end{bmatrix} \quad [\text{MPa}]$$

9-5: The stream function of a pure shear flow with a velocity field, given by $v_x=\dot{e}_1x$, $v_y=0$, $v_z=\dot{e}_3z$, can be determined by integration, according to expression (9-12):

$$\psi=\int_z v_x dz+\int_x v_z dx+c=\dot{e}_1xz+c$$

The boundary condition is that $\psi=0$ for $(x,z)=(0,0)$, so that $c=0$. The stream function for pure shear flow, $\psi=\dot{e}_1xz$, is discussed further in chapter 13 (cf., exercise 13-2).

9-6: a) Deformation tensor (a,b,c,d) = (1,0,1,1) represents a unit simple shear deformation ($\gamma=1$) with shear along the Y-axis.

b) Deformation tensor (a,b,c,d)=(1,0,0,1) is a self-copy tensor, which effectuates neither deformation nor translation.

c) Deformation tensor (a,b,c,d)=(1,1,0,1) is a unit simple shear ($\gamma=1$) with shear along the X-axis.

9-7: a) The strain-rate tensor for the pure shear deformation of exercise 9-3 follows from differentiating the velocity components, $v_x=\dot{e}_1x$, $v_y=0$, and $v_z=\dot{e}_3z$, according to expression (9-16b):

$$D_{ij} = \begin{bmatrix} \dot{e}_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \dot{e}_3 \end{bmatrix}$$

It is worth noting that the tensor elements, $\dot{e}_{11}$ and $\dot{e}_{33}$, are expressed in terms of the principal strains, $\dot{e}_1$ and $\dot{e}_3$.

b) The expression for the vorticity tensor [cf. eq. (9-16a)] is:

$$\begin{matrix} 0 & (1/2)[(\partial v_1/\partial x_2)-(\partial v_2/\partial x_1)] & (1/2)[(\partial v_1/\partial x_3)-(\partial v_3/\partial x_1)] \\ (1/2)[(\partial v_2/\partial x_1)-(\partial v_1/\partial x_2)] & 0 & (1/2)[(\partial v_2/\partial x_3)-(\partial v_3/\partial x_2)] \\ (1/2)[(\partial v_3/\partial x_1)-(\partial v_1/\partial x_3)] & (1/2)[(\partial v_3/\partial x_2)-(\partial v_2/\partial x_3)] & 0 \end{matrix}$$

For pure shear, $v_1=\dot{e}_1x$ and $v_3=\dot{e}_3z$, so that all elements in the vorticity tensor vanish.

c) The sum of the strain-rate and vorticity tensors is equal to the velocity gradient tensor. For the special case of pure shear deformation considered here, all elements of the vorticity tensor are zero [according to equation (9-16a)]. Consequently, the strain-rate tensor is equal to the velocity gradient tensor. This relationship is valid for all irrotational or coaxial deformations in 3D, including pure shear deformations, which are restricted to 2D (see, for example, Fig. 12-1a).

9-8: Given the tensor (in units of MPa):

$$\begin{bmatrix} 0 & 0 & 100 \\ 0 & 0 & 0 \\ -100 & 0 & 0 \end{bmatrix}$$

a) The determinant of this tensor, following the definition of equation (9-18b), is: $I_3=0$.

b) All tensors with vanishing third invariants represent a state of plane stress. For the particular tensor studied here, the first invariant, I_1, is, also, 0. This means that the pressure is 0 and the tensor represents a deviatoric stress rather than a total stress. More specifically, this is a state of simple shear stress. But, if the coordinate axes are rotated over 90° about the Y-axis, then the tensor becomes equal to that given in equation (9-20), which is a pure shear stress. This subject is further elaborated in exercises 10-8 and 10-10.

9-9: The real and imaginary parts of the complex functions given are as follows:

a) $W=f(z)=z=x+iy$. The real part is x; the imaginary part is y.

b) $W=f(z)=-z=-(x+iy)$. The real part is -x; the imaginary part is y.

c) $W=f(z)=z^2$. The answer is given by equations (9-21b & c).

Chapter ten

10-1: a) The stress trajectories for a uniformly oriented stress field consist of a set of straight lines.

b) A heterogeneous stress field has curved stress trajectories.

c) The stress trajectories are everywhere parallel to the principal stresses. A uniform stress field of straight trajectories requires the stress orientation to be constant, but gradients in the magnitude of the stress may occur.

10-2: $\mathbf{v}_i=n_i\mathbf{v}$ is the index notation for the velocity vector: $(v_1,v_2,v_3)=(n_1\mathbf{v},n_2\mathbf{v},n_3\mathbf{v})=(\mathbf{v}\cos\alpha, \mathbf{v}\cos\beta, \mathbf{v}\cos\gamma)$.

10-3: The angles of the direction vector on the plane considered, to be used for the direction cosines, are $\alpha=60°$, $\beta=45°$, and $\gamma=60°$. The magnitudes of the principal stresses are: $\sigma_1=-600$, $\sigma_2=-400$, and $\sigma_3=-200$ MPa.

a) The magnitude of the effective stress on the plane follows from equation (10-6a): $\sigma_{eff}=(\sigma_1^2\cos^2 60°+\sigma_2^2\cos^2 45°+\sigma_3^2\cos^2 60°)^{1/2}=(-)424$ MPa. Compressive stress, which has a negative sign, would give a positive σ_{eff}, according to equation (10-6a). The sign of σ_{eff} must be adjusted if a sign convention is used that attribute a negative sign to compresive stresses.

b) The magnitude of the normal stress on the plane considered follows from equation (10-6b): $\sigma_N=\sigma_1\cos^2 60°+\sigma_2\cos^2 45°+\sigma_3\cos^2 60°=$ -400 MPa.

c) The magnitude of the shear stress on the plane considered follows from equation (10-6c): $\sigma_S=(\sigma_{eff}^2-\sigma_N^2)^{1/2}=(424^2-400^2)^{1/2}=141$ MPa.

10-4: a) $\sigma_{12}=\sigma_{21}$, $\sigma_{13}=\sigma_{31}$, and $\sigma_{23}=\sigma_{32}$.

b) The explanation is as follows: For a continuous body to be in equilibrium, the resultant forces and the resultant moment about any axis must vanish. The cubic volumes of Figures 10-7 a & b are infinitesimally small. The magnitude of the normal stresses on opposing surfaces of the cube must be identical for a continuous body to be in equilibrium. The force moment, resulting from the shearing stresses, must, also, vanish, according to the equilibrium of moments. The shear stresses act at the midpoint of each face of the infinitesimal cubes in Figures 10-7a & b. The sum of the moments about an axis parallel to the X-axis must be: $\sigma_{yz}-\sigma_{zy}=0$, so that $\sigma_{yz}=\sigma_{zy}$. Similarly, the vanishing moments about the Y-axis and the Z-axis require that $\sigma_{xz}=\sigma_{zx}$ and $\sigma_{xy}=\sigma_{yx}$.

10-5: The stress tensor elements for a situation with σ_2 parallel to the Y-axis and σ_1 at 45° to the X-axis can be determined as follows: Because σ_2 is normal to the XZ-plane, $\sigma_{yy}=\sigma_2$, and the shear stresses vanish so that $\sigma_{yx}=\sigma_{yz}=0$. Because σ_2 and σ_3 both are at 45° to the XY and YZ planes, the normal stress vanishes; σ_{xx} and σ_{zz} are both 0. The remaining shear stresses are at maximum: $\sigma_{zx}=\sigma_{xz}=(\sigma_1-\sigma_3)/2$. The stress tensor, expressed in terms of principal stresses, reads:

$$\sigma_{ij}=\begin{bmatrix} 0 & 0 & (\sigma_1-\sigma_3)/2 \\ 0 & 0 & 0 \\ (\sigma_1-\sigma_3)/2 & 0 & 0 \end{bmatrix}$$

10-6: a) The Kronecker delta matrix reads:

$$\delta_{ij}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

b) The tensor elements of P_{ij} are:

$$P\delta_{ij}=\begin{bmatrix} P & 0 & 0 \\ 0 & P & 0 \\ 0 & 0 & P \end{bmatrix}$$

c) The tensor elements of $\sigma_{ij}=\tau_{ij}+P_{ij}$ are:

$$\sigma_{ij}=\begin{bmatrix} P+\tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & P+\tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & P+\tau_{zz} \end{bmatrix}$$

10-7: The stress tensor is given. The direction cosines of plane ABC, also, are given: (0.7, 0.5, 0.5).

a) The pressure is $P=|\sigma_{mean}|=|-15/3|=5$ MPa. The deviatoric stress tensor [e.g., see solution to exercise (10-6c)] is $\sigma_{ij}-P_{ij}$ or:

$$\tau_{ij}=\begin{bmatrix} -45 & -40 & -35 \\ -40 & 40 & -50 \\ -35 & -50 & -25 \end{bmatrix}$$

b) The components of the effective stress on plane ABC can be calculated, according to the Cauchy formula of equation (10-11a to c): $\sigma_x=-66$ MPa, $\sigma_y=-31$ MPa, and $\sigma_z=-60$ MPa.

c) The magnitude of the effective stress on the plane follows from equation (10-6a): $\sigma_{eff}=(\sigma_x^2+\sigma_y^2+\sigma_z^2)^{1/2}=(-)94$ MPa. In order to enable the calculation of the normal stress, using equation (10-6b), the principal stresses are needed first. According to equations (10-5a to c), these are:

$$\sigma_1=\sigma_x/l=-66/0.7=-94.3 \text{ MPa}$$
$$\sigma_2=\sigma_y/m=-31/0.5=-62 \text{ MPa}$$
$$\sigma_3=\sigma_z/n=-60/0.5=-120 \text{ MPa}$$

The normal stress follows from equation (10-6b): $\sigma_N=\sigma_1 l^2+\sigma_2 m^2+\sigma_3 n^2=-92$ MPa. This result demonstrates that the plane ABC of this study is nearly normal to σ_1.

The shear stress follows from equation (10-6c): $\sigma_S=(|\sigma_{eff}^2-\sigma_N^2|)^{1/2}=(|94^2-92^2|)^{1/2}=19.3$MPa.

10-8: The tensor of exercise 9-8 is:

$$\begin{bmatrix} 0 & 0 & 100 \\ 0 & 0 & 0 \\ -100 & 0 & 0 \end{bmatrix}$$

a) The cubic equation (10-14) for this tensor simplifies to: $\sigma_i^3+100^2\sigma_i=0$. The solutions for the principal stresses, σ_i (for i=1, 2, or 3), are: $\sigma_1=100$ MPa, $\sigma_2=0$ MPa, and $\sigma_3=-100$ MPa. Obviously, the above stress tensor, after alignment of the coordinate axes with the principal axes of stress, can be rewritten within the new coordinate axes, as:

$$\sigma_{ij}=\begin{bmatrix} -100 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -100 \end{bmatrix}$$

b) The tensor of exercise 9-8 is referred to axes that are oriented at 45° to the principal stresses.

10-9: The change of coordinate axes is governed by the tensor transformation formula [eq. (10-16)]. The direction cosines are:

	X_1	X_2	X_3
X_1^*	cos 90°	cos 0°	cos 0°
X_2^*	cos 0°	cos 90°	cos 0°
X_3^*	cos 0°	cos 0°	cos 90°

Remember that cos 0°=1 and cos 90°=0. The tensor transformation formula gives:

$$\sigma_{11}^*=\sigma_{22}+\sigma_{23}+\sigma_{32}+\sigma_{33}$$
$$\sigma_{22}^*=\sigma_{11}+\sigma_{13}+\sigma_{31}+\sigma_{33}$$
$$\sigma_{33}^*=\sigma_{11}+\sigma_{12}+\sigma_{21}+\sigma_{22}$$

$$\sigma_{12}^*=\sigma_{21}+\sigma_{23}+\sigma_{31}+\sigma_{33}$$
$$\sigma_{13}^*=\sigma_{21}+\sigma_{22}+\sigma_{31}+\sigma_{32}$$
$$\sigma_{23}^*=\sigma_{11}+\sigma_{12}+\sigma_{31}+\sigma_{32}$$

$\sigma_{21}^*=\sigma_{12}^*$, $\sigma_{31}^*=\sigma_{13}^*$, and $\sigma_{32}^*=\sigma_{23}^*$.

10-10: Given is the following deviatoric stress tensor:

$$\tau_{ij}=\begin{bmatrix} -100 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -100 \end{bmatrix}$$

a) Counter-clockwise rotation of the coordinate axes of 45° about the old X_2-axis gives the direction cosines, as follows:

	X_1	X_2	X_3
X_1^*	cos 45°	cos 90°	cos 135°
X_2^*	cos 90°	cos 0°	cos 90°
X_3^*	cos 45°	cos 90°	cos 45°

The transformation matrix is:

$$\begin{bmatrix} 0.7 & 0 & -0.7 \\ 0 & 1 & 0 \\ 0.7 & 0 & 0.7 \end{bmatrix}$$

Using these direction cosines in the tensor transformation formula (10-16) yields: $\tau_{11}^*=0.5\tau_{11}-0.5\tau_{13}+0+\tau_{22}+0+0.5\tau_{31}+0.5\tau_{33}=0$, $\tau_{33}^*=0$, $\tau_{12}^*=0$, $\tau_{13}^*=+100$ MPa, and $\tau_{31}^*=-100$ MPa. In conclusion, the new stress tensor is:

$$\tau_{ij}^*=\begin{bmatrix} 0 & 0 & 100 \\ 0 & 0 & 0 \\ -100 & 0 & 0 \end{bmatrix}$$

b) The normal stresses have vanished from the new stress tensor, due to the rotation of the coordinate axes to align with the directions of maximum shear stress.

c) The maximum shear stress is 100 MPa.

10-11: The stresses at a point on a fault plane are $\sigma_{zz}=-150$ MPa, $\sigma_{xx}=-200$ MPa, and $\sigma_{xz}=0$. The coordinate axes are oriented as follows: Z is vertical, X points westward, and, consequently, Y points northward.

a) The normal stress on a fault plane of $\theta=35°$ is given by equation (10-19a): $\sigma_{xx}^*=-183.7$ MPa.

b) The shear stress is given by equation (10-19b): $\sigma_{xz}^*=23.5$ MPa.

10-12: If σ_1 (=-80 MPa) is horizontal and σ_3 (=-45 MPa) is vertical, than a practical choice of coordinate axes renders $\sigma_{xx}=\sigma_1$, $\sigma_{zz}=\sigma_3$, and all the shear stresses vanish from the stress tensor. The normal and shear stresses on a bedding surface inclined at $\theta=30°$ follow from applying equations (10-19a & b): $\sigma_N=-55.75$ MPa and $\sigma_S=-15.16$ MPa.

10-13: The overcoring stress test in a mine at a depth of 1.5 km gives normal stresses of -62 MPa in the N-S direction, -48 MPa in the E-W direction, and -51 MPa in the NE-SW direction. One is free to choose coordinate axes. If the X-axis is fixed at E-W and the Z-axis at N-S, then $\sigma_{xx}=-48$ MPa, $\sigma_{zz}=-62$ MPa, and $\sigma_{xz}=-51$ MPa. The principal stresses follow from equations (10-21a & b). The orientation of the principal stresses follows from angle θ in equation (10-20).

10-14: The maximum shear stress follows from applying equation (10-24) to the stresses given. They are 0.5, 3, 5, and 5.5 MPa at distances of 2, 4, 22, and 34 km to the San Andreas Fault. Obviously, at the time these stress measurements were taken, movement of the San Andreas Fault had relieved much of the elastic shear stress accumulated in its vicinity, due to the relative plate movement in opposite directions at either side of the fault trace.

10-15: The coefficients of the cubic equation (10-26) are given as follows: $I_1=4$ MPa, $I_2=-11$ MPa, and $I_3=-30$ MPa. Substitution of these invariants in equation (10-26) yields:

$$\sigma_i^3-4\sigma_i^2-11\sigma_i+30=0$$

Cubic equations always have three possible solutions, each solution corresponding to one principal stress. The principal stresses, σ_i (for i=1, 2, or 3), are here: 50 (σ_1), 20 (σ_2), and -30 (σ_3) MPa.

10-16: a) The cubic equation (10-26), for plane deviatoric stress simplifies to:

$$\tau_i^3+\tau_i I_2=0 \quad \text{or} \quad \tau_i(\tau_i^2+I_2)=0$$

This is so because the first and third invariants vanish for tensors of plane deviatoric stress.

b) It is now obvious that solutions for the principal stresses, σ_i (for i=1, 2, or 3), are: $\tau_1=(-I_2)^{1/2}$, $\tau_2=0$, and $\tau_3=-(-I_2)^{1/2}$.

Chapter eleven

11-1: The ellipsoid of Figure 11-1 has the following characteristics:

Direction	S_1	S_2	S_3
Length, L	2 cm	1.2 cm	0.6 cm
Stretch, S	1.66	1	0.5
Elongation, e	0.66	0	-0.5
Quadratic elongation, λ_e $(=S^2)$	2.75	1	0.25

(There is volume loss; no volume loss occurs here only if S_1 is 2.4 cm rather than 2 cm.)

11-2: A line with a stretch, S, of 0.7 has an elongation, e, of -0.3, and a quadratic elongation, λ_e, of 0.49.

11-3: Remember that the stretch $S=1+e=L/L_0=R/R_0$. The normalized change in volume is: $\delta V=(V_1-V_0)/V_0=S^3-1$. The normalized change in surface area is: $\delta A=(A_1-A_0)/A_0=S^2-1$.

a) Consider a shrunken sphere with a radial stretch of 0.5. The fractional change in volume is -0.875, which is equivalent to 87.5% shrinkage. The associated fractional change in surface area is -0.75, which is equivalent to 75% shrinkage.

b) Consider an expanded sphere with a radial stretch of 2. The fractional increase in volume is $\delta V=2^3-1=7$, or 700%. The corresponding increase in surface area, δA, is 3 or 300%.

11-4: The derivation showing that the maximum shear strain is $(e_1-e_3)/2$ is analogous to that derived for the maximum shear stress in section 10-10. Another important conclusion is that, for plane strain, the maximum shear strain magnitude is equal to e_1 [because, for plane strains, $e_3=-e_1$, so that $(e_1-e_3)/2=e_1$]. If the maximum shear strain is expressed in terms of the engineering shear strain, γ, then $\gamma=2e_1$.

11-5: Parallel lines with different stretches that are initially crossed by perpendicular marker lines will bend those marker lines. Consequently, the deformation is heterogeneous in all such cases.

11-6: This drawing procedure can be easily completed on Figures 11-10b & c. The deformation of Figure 11-10b is homogeneous, but that of Figure 11-10c is heterogeneous.

11-7: The relationship between rock fabric or foliation and finite strain is considered a complex issue. Mineral grains tend to align during deformation to form a foliation, defined by a grain shape fabric of uniformly oriented and elongated mineral grains. The long axis of the mineral grains will tend to align with the direction of maximum stretch. However, mineral grains are not passive strain markers (refer to section 7-2), and the finite strain recorded by deformed mineral grains may differ from the bulk strain experienced by the rock volume as a whole (cf. Figs. 7-3a to c). The foliation in Figure 11-13 can be interpreted to trace the S_1-trajectories. The spacing of the foliation tightens in the central zone of shear, which indicates that the strain magnitude is larger in the central zone. The shear zone is clearly recording a component of differential simple shear (cf. Fig. 11-11a). The steep gradient of the shear strain can be explained simply by the occurrence of strain softening in the central shear zone, due to grain-size reduction and shear heating during progressive shear (see, also, exercises 7-5 and 7-8).

11-8: Porous rock that closes pore space upon deformation undergoes a volume change that is equal to the former porosity. The first invariants of the strain tensor would be $J_1=-0.32$ and $J_1=-0.17$ for the compactional deformations of the rocks in Figures 11-14a & b.

11-9: a) The hypothetical normal stress, τ_1, of the stress tensor for the case given is: $\tau_{11}=2Ge_{11}=2 \times 10$ GPa $\times 0.5=10$ GPa.

b) Firstly, elastic strains with elongations of 0.5 do not occur in rocks, because the deviatoric stresses are commonly ranging between 10 and 100 MPa. The corresponding elastic elongations are two to three orders of magnitude smaller than 0.5.

c) Secondly, elastic deformation with elongations of 0.5 or a stretch of 1.5 do not occur in rocks, because the brittle failure strength is two to three orders of magnitude smaller than 10 GPa (see Table 6-1, p. 82).

11-10: The tensor elements of Lamé's expression (11-16b) are:

$$T_{ij}=\begin{bmatrix} 2Ge_{11}+\lambda e_{11} & 2Ge_{12} & 2Ge_{13} \\ 2Ge_{21} & 2Ge_{22}+\lambda e_{22} & 2Ge_{23} \\ 2Ge_{31} & 2Ge_{32} & 2Ge_{33}+\lambda e_{33} \end{bmatrix}$$

11-11: a) The maximum shear stress in a rock, subjected to a stress field of 100 MPa, is: 100 MPa (cf. exercise 10-10). The associated maximum shear strain, for a modulus of G=10 GPa is: $e=\tau/2G=0.005$.

b) The angular shear strain, caused by the elastic shears, is less than half a degree.

c) Large deformations such as folding occur principally by ductile flow under stresses that are subcritical to that required for causing brittle failure. (Brittle failure stresses are given in Table 6-1.)

11-12: A characteristic geological strain-rate follows from $\dot{e}_{ij}=\tau_{ij}/2\eta$. Characteristic tectonic stress is on the order of 100 MPa, and crustal viscosities range from 10^{20} to 10^{24} Pa s. The corresponding strain-rates range between 5 x 10^{-13} and $5x10^{-17}$ s^{-1}. The most commonly adopted geological strain-rate of 10^{-14} s^{-1} occurs, for example, when (τ, η) is (10 MPa, $5x10^{20}$ Pa s) or (100 MPa, $5x10^{21}$ Pa s) or (200 MPa, 10^{22} Pa s).

Chapter twelve

12-1: a) A deformation with principal stretches $S_1=2$, $S_2=1$, and $S_3=0.5$ is a perfect plane strain.

b) A deformation with principal stretches $S_1=4$, $S_2=0.5$, and $S_3=0.5$ is a perfect prolate strain.

c) A deformation with principal stretches $S_1=2$, $S_2=2$, and $S_3=0.25$ is a perfect oblate strain.

d) No, volume change was not involved in any of the above deformations, because the product of the principal stretches, $S_1.S_2.S_3$, remains unity for all cases.

12-2: Draw these sections to scale, and you will understand more of strain ellipses.

12-3: a) *The deformation sequence of Figure 12-5a [Series (a) in Table 21-1]* is a progressive pure shear deformation, because it is an irrotational deformation, involving perfectly plane strains only. No volume change is involved, because the product of the principal stretches remains unity for all stages of the illustrated deformation. *The deformation sequence of Figure 12-5b [Series (b) in Table 21-1]* is a progressive, general oblate strain with less than one percent volume change. *The deformation sequence of Figure 12-5c [Series (c) in Table 21-1]* is a progressive deformation with ten percent volume change in each of the stages 2, 3, and 4. The final deformation involves 30 percent reduction in volume. Although the intermediate stretch remains at unit length throughout the deformation, this is neither a pure shear deformation nor a progressive plane strain distortion. The shape parameters indicate a general oblate strain. *The deformation sequence of Figure 12-5d [Series (d) in Table 21-1]* is a progressive deformation, showing 50 percent compactional volume change in the first increment of deformation. The first increment is a perfect oblate strain. A shortening, subsequently superimposed in the future S_3-direction, maintains a general oblate strain regime, but without any further volume change, as $S_1.S_2.S_3$ remains consistently at 0.5.

b) Compose the deformation tracks in a Flinn-Ramsay plot for each of the progressive deformation series (a) to (d).

c) Also, compose the deformation tracks in a Hsu plot for each of the progressive deformation series (a) to (d).

12-4: a) The three major stretches in Figures 12-6c to e are: 1.33, 1.43, and 1.67.

b) The three minor stretches in Figures 12-6c to e are: 0.75, 0.7, and 0.6.

The stretch values obtained in (a) and (b) can be plotted in the graph of Figure 12-7. It follows that, for pure shear deformation without volume change, $S_1=1/S_3$ and S_2 remains unit length throughout the deformation. For pure shear deformation, combinations of S_1 and

S_3, other than those graphed in Figure 12-7, cannot occur.

12-5: a) The ultimate orientation of the major stretching axis, after an infinitely large simple shear, is parallel to the direction of maximum shear stress, which is parallel to the reference plane in Figure 12-8a.

b) The height of the deformed unit cube remains unchanged during simple shear deformation.

c) The material lines, that retain unit stretch throughout the deformation, are horizontal in Figure 12-8a, and are parallel to the Y-axis or parallel to the X-axis, or they may have any other orientation within XY-planes.

12-6: a) An angular shear strain of unity occurs for $\psi=45°$. The corresponding value of θ is 31.72°, which is the angle between the principal stretching axis and the direction of shear movement. The principal stretches for a unit simple shear are as follows: $S_1=1.63$, $S_2=1$, and $S_3=0.61$.

b) The S_1 and S_3 values of (a) can be plotted in Figure 12-9a. The θ values can be plotted in Figure 12-9b.

c) Figure 12-8e is indeed consistent with a unit simple shear deformation, according to the data in the plots of Figure 12-9a & b.

12-7: a) The values of (ψ,θ) are (0°, 45°), (22°, 39°), (31°, 36°), and (45°, 32°). These values can be plotted in (ψ,θ)-space and fitted to a curve.

b) The correct mathematical realtionship between θ and ψ is: $\theta=[\tan^{-1}(2/\tan\psi)]/2$.

c) The formula $\theta=(90°-\psi)/2$ [unlike the solution given in (b)] would imply that the major stretching axis remains parallel to the diagonal material line, that connects the corners of the deforming square in all subsequent parallelograms. This is not the case, although the mistake is often made by inexperienced geologists. The major stretching axis rotates more slowly than the material line that is the diagonal of the deforming cube. However, these two lines concerned coincide only at the onset of deformation.

12-8: In the absence of the fault reference surface, the original orientation of the deformed object would remain unknown. In that case, the finite deformation pattern may have been produced by either of the two shears, simple or pure, as outlined in Figures 12-13a & b.

12-9: The deformation tensor for the pure shear deformation of Figure 12-17a is:

$$F_{ij}=\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}$$

The deformation tensor for the compactional deformation of Figure 12-17b is:

$$F_{ij}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}$$

12-10: The finite elongations, outlined by the strain ellipses in Figures 12-18a & b, are identical. However, the orientation of the finite strain ellipses in Figures 12-18a & b is different, because rigid body rotation occurred after the elongation in Figure 12-18b. If the coordinate axes are allowed to rotate with the rigid body rotation after pure elongation, then the finite deformations of Figures 12-18a & b become indistinguishable.

12-11: The stretches for the deformations of Figures 12-17a & b can be calculated by substituting the tensor elements, as determined in exercise 12-9, in equations (12-12a & b): $S_1=2$ and $S_3=0.5$ in Figure 12-17a, and $S_1=1$ and $S_3=0.5$ in Figure 12-17b. $S_2=1$ for both cases. In conclusion, the deformation of Figure 12-17a is a perfect plane strain; Figure 12-17b illustrates a perfect oblate strain.

12-12: The major principal stretch, S_1, of the infinitesimal strain ellipse, for each location in the fold, can be calculated, substituting the deformation tensor elements (F_{11},F_{13},F_{31},F_{33}) in equation 12-12a. The corresponding tensor elements are: (0.5,0,0,2) for $x_0=0$, (0.5,0,2,2) for $x_0=2$, (0.5,0,4,2) for $x_0=4$, (0.5,0,8,2) for $x_0=8$, and (0.5,0,10,2) for $x_0=10$. The orientations of the same strain ellipses can be calculated from equation (12-13).

12-13: All the values of Table 12-2 are consistent with their mutual interrelationships, according to equations (12-12a & b) and (12-13).

Chapter thirteen

13-1: a) Comparison of Figure 13-3a & b indicates that the streamlines are everywhere parallel to the velocity vectors of the flow. The length of the velocity vectors scales the magnitude of the local flow velocity. Streamlines outline streamtubes, and the flux of fluid flow across any section of a particular streamtube remains constant. If flowlines diverge, the flux remains the same, but the flow rate slows down. If flowlines converge, the flux, also, remains constant, but the flow rate speeds up.

b) The flow tube can be chosen as wide or as narrow as required. Ten percent of the total fluid flux occurs in the central streamtube between the streamlines for $\psi=0$ and $\psi=0.1$. Obviously, the streamlines are, here, normalized by the total fluid flux through the top of the flow box in Figure 13-3b. The wider streamtube between streamlines for $\psi=0$ and $\psi=0.5$, for example, encloses 50% of the total fluid flux in the right half of the section. The total fluid flux through the horizontal top of the outlined area is 2x100%, because the central line of flow symmetry has been taken as a reference surface (normal to the plane of drawing) with $\psi=0$.

c) The particles of an initial strain marker move along the streamlines outlined in Figure 13-3b. Consider a small cube with initial coordinates (x_0, z_0) for its four corners as follows: (0, 0), (0, 0.5), (0.5, 0.5), and (0.5, 0). The (0, 0) corner remains stationary during the flow. The (0, 0.5) corner moves vertically upward along $\psi=0$. The (0.5, 0.5) corner moves along $\psi=0.25$ and maintains the same Z-coordinate as the (0, 0.5) corner, now outlining a rectangle, rather than a square. The (0, 0.5) corner slides along the horizontal base of the box, which, also, is a streamline for $\psi=0$, and this corner will reach the origin at time infinity.

d) Equipotential lines for the pure shear flow of Figure 13-3b are illustrated in Figure 13-5.

13-2: a) The velocity components for a pure shear flow, with a coordinate system chosen such that $\psi=\dot{e}_1xz$ [m^2 s^{-1}], are [differentiating according to eqs. (13-1a & b)]: $v_x=\partial\psi/\partial z=\dot{e}_1x$ and $v_z=-\partial\psi/\partial x=-\dot{e}_1z$.

b) The streamlines for $\psi=0$ coincide with the two coordinate axes, X and Z.

c) All of the shear strain-rates vanish from the strain-rate tensor, using the coordinate axes implied in $\psi=\dot{e}_1xz$ [m^2 s^{-1}]. (However, refer to exercise 10-10, and realize that the normal strain-rates would vanish and shear strain-rates would become equal to the principal strain-rates if the coordinate system is rotated 45° about the Y-axis.)

d) The numbers along the box outlined in Figure 13-3b, are the non-dimensional lengths of the flow box. The fractional numbers along the streamlines are flux rates of fluid flow across any observation line between a point on the normalized streamline for $\psi=0$ and another point on any one of the adjacent streamline(s).

13-3: The properties of flownets are summarized in the first paragraph of page 217. Equipotential lines are surfaces, normal to the plane of section, across which matter moves. Equipotential lines are, everywhere, perpendicular to streamlines. A streamline or flowline represents surfaces, also normal to the plane of section, across which no matter can move, as explained in the first paragraph of page 216. Because the streamlines in Figure 13-5 are symmetric about the X and Z axes, the axes themselves represent streamlines, as well. No material moves across them, and, because the streamlines cross at the origin, the X and Z-axes must have $\psi=0$.

13-4: The stream function for the pure shear flow of Figure 13-5 is: $\psi=\dot{e}_1xz$ [$m^2\ s^{-1}$]. The equipotential function, Φ, is: $\Phi=(\dot{e}_1/2)(x^2-z^2)$ [$m^2\ s^{-1}$] [cf. eq. (13-9b)]. Solutions for $\Phi=0$ occur at points that satisfy $x^2=z^2$, which is for $x=y$ and $x=-y$. This pair of orthogonal, conjugate lines is dashed (stippled) in Figure 13-5.

13-5: a) The normal and shear strain-rates can be written as functions of the flow asymptote angle, α, as follows: $\dot{e}_{xx}=\dot{e}_1\cos(90°-\alpha)$ and $\dot{e}_{xz}=\dot{e}_1\sin(90°-\alpha)$.

b) The ratio $\dot{e}_{xz}/\dot{e}_{xx}=\tan(90°-\alpha)$. The angles α, indicated in Figure 13-8, effectively fix the rate of the shear and normal strain-rates as follows:

$\alpha=0°$ has $\dot{e}_{xz}/\dot{e}_{xx}=\infty$
$\alpha=30°$ has $\dot{e}_{xz}/\dot{e}_{xx}=1.73$
$\alpha=60°$ has $\dot{e}_{xz}/\dot{e}_{xx}=0.58$
$\alpha=90°$ has $\dot{e}_{xz}/\dot{e}_{xx}=0$
$\alpha=-30°$ has $\dot{e}_{xz}/\dot{e}_{xx}=-1.73$
$\alpha=-60°$ has $\dot{e}_{xz}/\dot{e}_{xx}=-0.58$

In conclusion, the strain-rate tensors, normalized by $\dot{e}_{xx}$, for the flows of Figure 13-8 (and Figs. 13-9 & 13-11) can be written as:

$$\begin{bmatrix} 1 & 0 & \infty \\ 0 & 0 & 0 \\ \infty & 0 & 1 \end{bmatrix} \quad \text{for } \alpha=0°$$

$$\begin{bmatrix} 1 & 0 & 1.73 \\ 0 & 0 & 0 \\ 1.73 & 0 & 1 \end{bmatrix} \quad \text{for } \alpha=30°$$

$$\begin{bmatrix} 1 & 0 & 0.58 \\ 0 & 0 & 0 \\ 0.58 & 0 & 1 \end{bmatrix} \quad \text{for } \alpha=60°$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{for } \alpha=90°$$

$$\begin{bmatrix} 1 & 0 & -1.73 \\ 0 & 0 & 0 \\ -1.73 & 0 & 1 \end{bmatrix} \quad \text{for } \alpha=-30°$$

$$\begin{bmatrix} 1 & 0 & -0.58 \\ 0 & 0 & 0 \\ -0.58 & 0 & 1 \end{bmatrix} \quad \text{for } \alpha=-60°$$

13-6: a) The velocity-gradient tensor, L_{ij}, for a pure shear flow of $\psi=\dot{e}_1xz$, can be calculated, using equation (13-17), and is as follows:

$$L_{ij}=\begin{bmatrix} \dot{e}_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\dot{e}_1 \end{bmatrix}$$

b) The deformation tensor, F_{ij}, can be determined, using equations (13-20a to d) and (13-21a & b):

$$F_{ij}=\begin{bmatrix} \exp(\dot{e}_1t) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \exp(-\dot{e}_1t) \end{bmatrix}$$

c) The principal stretches can be calculated, if the deformation tensor elements are known, according to equations (12-12a & b) [see, also, eq. (12-16)]: $S_1=\exp(\dot{e}_1 t)$ and $S_3=\exp(-\dot{e}_1 t)$. If the major principal strain-rate, $\dot{e}_1$, is 0.315 Ma^{-1} or 10^{-14} s^{-1}, then $S_1=2$ after $t=(\ln 2/0.315)$ Ma=2.2 Ma.

d) & e) The requested graphs are similar to that shown in Figure 12-7, because $t=\ln S_1/\dot{e}_1$.

13-7: The velocity gradient tensor, L_{ij}, for a simple shear flow of $\psi=\dot{e}_{xz}z^2$, can be calculated, using euation (13-17), and is as follows:

$$L_{ij}=\begin{bmatrix} 0 & 0 & 2\dot{e}_{xz} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

13-8: The vorticity tensor, W_{ij}, in terms of angular velocity is:

$$W_{ij}=\begin{bmatrix} 0 & \Omega_z & -\Omega_y \\ -\Omega_z & 0 & \Omega_x \\ \Omega_y & -\Omega_x & 0 \end{bmatrix}$$

The vorticity and angular velocity or rotation rate are sometimes confused in the geoscience literature.

13-9: The strain-rate tensor, D_{ij}, for a plane deformation with coordinate axes, as illustrated in Figure 13-10a, is as follows:

$$D_{ij}=\begin{bmatrix} \dot{e}_{xx} & 0 & \dot{e}_{xz} \\ 0 & 0 & 0 \\ \dot{e}_{xz} & 0 & -\dot{e}_{xx} \end{bmatrix}$$

The vorticity tensor, W_{ij}, is as follows:

$$W_{ij}=\begin{bmatrix} 0 & 0 & -\dot{\omega}_y/2 \\ 0 & 0 & 0 \\ \dot{\omega}_y/2 & 0 & 0 \end{bmatrix}$$

The velocity-gradient tensor, L_{ij}, is equal to the sum of the strain-rate and the vorticity tensors, that is, $L_{ij}=D_{ij}+W_{ij}$:

$$L_{ij}=\begin{bmatrix} \dot{e}_{xx} & 0 & 2\dot{e}_{xz} \\ 0 & 0 & 0 \\ 0 & 0 & -\dot{e}_{xx} \end{bmatrix}$$

This implies that the magnitude of $\dot{\omega}_y/2=-\dot{e}_{xz}$ (or $\dot{\omega}_y/2=\dot{e}_{xz}$, depending on the sign convention followed for the vorticity).

Chapter fourteen

14-1: a) The calc-silicate of Figure 14-5 has developed boudins, and, therefore, the calc-silicate, at the time these boudins were formed, was more competent than the adjacent marble.

b) Generally, the folding is due to initial layer-parallel shortening. The tight fold, shown here, has a component of bulk shortening normal to the axial surface. Extension occurred parallel to the axial surface and normal to the fold hinge.

c) Megascopic and parasitic folds occur in the calc-silicate layers, where the enveloping surface is parallel to the direction of bulk shortening. Obviously, boudins must have formed after the limbs of the folds rotated into parallelism with the direction of principal extension. The calc-silicate resists extension more than the marble, and the calc-silicate layers on the fold limbs have separated into individual boudins.

14-2: The ptygmatic folds in the competent pegmatite veins of Figure 14-8a are due to bulk shortening normal to the axial surface of the folds. The competent vein can adjust to ductile compressive deformation in the host rock only through folding in order to accommodate the component of layer-parallel shortening.

The folded boudins in the amphibolite gneiss of Figure 14-8b require an explanation different from that given to explain the folds and boudins of Figure 14-5. It is obvious that, here, the boudins formed first. Subsequently, the separate boudin sections and the host rock were, together, folded into harmonic, similar folds. The boudin-segments became imbricated during the folding. The dark amphibolite layers are more competent than the gneiss.

The initial extension and subsequent shortening of material lines and layers cannot occur in progressive simple shear or pure shear deformation. The opposite mechanism of deformation, where material lines rotate from an initial shortening direction into an extension direction, is more common and is discussed in detail in sections 14-4 and 14-6. The initial shortening and subsequent extension are graphed in Figure 14-13d for a range of initial orientations. However, the structure of Figure 14-8b can be explained best by a change in the regional deformation regime, rather than by a single episode of progressive deformation with constant shear directions. A change in the regional orientation of the shortening and extension directions could account for the final structures of Figure 14-8b.

14-3: a) The two imaginary lines, A, in the incremental strain ellipse for pure shear deformation (Fig. 14-9) are consistently 90° apart and at 45° to the direction of principal shortening.

b) Any material lines, that become oriented at 45° to the shortening direction, will rotate further away from the shortening direction during the next increment of deformation and, consequently, sweep into the field of extension. However, the imaginary lines, A, separating the fields of shortening and extension of incremental strain, always remain at 45° to the shortening direction throughout the deformation.

14-4: a) All the material lines in the cubic rock volume of Figure 14-10a will have stretched and rotated into the direction of extension after very large finite strain, except for those material lines that are exactly parallel to either the Z-axis or the Y-axis (normal to the plane of section) or any other lines oriented parallel to the YZ-plane.

b) In the progressive pure shear deformation of Figures 14-10a & b, any material lines that are perfectly parallel to the X-, Y-, or Z-axis do not rotate at all.

c) The fastest rotation of material lines occurs when they are in the XZ-plane (Fig. 14-10a) close to the Z-axis, but with α_0 slightly less than 90°. In contrast, for lines with $\alpha_0=90°$, no rotation occurs at all.

14-5: Although the rate of stretching is not included in Figure 14-10d, the lines which stretch fastest are parallel to the X-axis in Figure 14-10a & b, with $\alpha=0°$ at all times. Such lines plot along the S_1-axis in the graph of Figure 14-10d.

14-6: a) The stretching history of material lines with $\alpha_0=70°$, 80°, and 89° can be plotted in Figure 14-10d, using equation (14-3b). It will follow that these lines first shorten ($S_\alpha<1$) before reversal of the stretching direction occurs ($S_\alpha>1$) for larger bulk stretches of S_1.

b) The requested explanation is as follows: Material lines that have $\alpha_0>45°$ first shorten and then extend when they rotate into the field of incremental extension (cf. Fig. 14-9). Even the material lines with $\alpha_0=60°$ will initially shorten for a brief moment, but the effect is negligible, as illustrated in Figure 14-10d, because these lines soon sweep into the extension field.

14-7: a) The two imaginary lines, A, in the incremental strain ellipse for simple shear deformation (Fig. 14-12) are consistently 90° apart. One of the lines is parallel to the direction of shear movement; the other line is normal to the direction of shear.

b) One of the material lines, B^*, that will become a direction of no-finite-strain, B, after some finite strain (Figs. 14-12 & 14-19), coincides with the line, A, that is parallel to the direction of shear motion. This is so because material lines that are parallel to the direction of shear movement cannot rotate at all and remain in that orientation with constant length throughout the simple shear deformation.

14-8: The rotation history of material lines with $\alpha_0=0°$ plots along the γ-axis in Figure 14-13c. Such lines maintain $\alpha=0°$ throughout the simple shear deformation.

14-9: The finite strain ellipse of Figure 14-15 illustrates a major principal stretch of $S_1=1.5$. The material lines, A^*, were initially at 45° to the shortening direction in the incremental strain ellipse at the first instant of pure shear deformation (Fig. 14-9). Subsequently, these material lines, A^*, have rotated into the field of extension, and, for $S_1=1.5$, their orientation is $\alpha_{A*}=30°$, according to the plot of Figure 14-11. Angles α_{A*} are measured between line A^* and the X-direction of principal extension.

14-10: a) The plot of $(\alpha_{B*}-\alpha_B)$ against S_1 can be easily constructed, using equations (14-7) and (14-8) or applying the solutions given in Figure 14-16.

b) The requested plot shows that the difference between the lines B and B^* is zero at the onset of deformation. But for S_1 equal to 6, the difference has grown to 80°. This means that the volume of rock, containing material lines that are still shortening at that moment in the deformation, has become very small, and, thus, most of the material lines in the rock are extending. This, for some geologists, may be counter to intuition of what is expected to happen to material lines in pure shear deformation. Extension of material surfaces inside rocks is much more common than their shortening.

14-11: The major principal stretch, S_1, for the finite simple shear deformation of Figure 14-17 is 1.5. It follows from Figure 12-9a that the angular shear strain is slightly less than unity, that is, $\gamma=0.84$, according to equation (12-7a). The material lines, A^*, were initially parallel and normal to the direction of shear movement in the incremental strain ellipse at the first instant of simple shear deformation (Fig. 14-12). The material line, A^*, that was parallel to the direction of shear, remains fixed in that orientation throughout the deformation. However, the material line, A^*, that was originally normal to the direction of shear movement, progressively rotates toward the direction of shear movement. For $S_1=1.5$ and $\gamma=0.84$, the orientation of $\alpha_{A*}=50°$, according to the plot of Figure 14-14 and equations (14-6).

14-12: The major principal stretching axis, S_1, starts at 45° away from the direction of shear movement and progressively rotates towards the shear direction as S_1 increases. The minor stretch axis initially is at 135° or -45° to the direction of shear movement, but it progressively rotates toward 90° or -90°, that is, normal to the direction of shear movement.

14-13: a) The plot of $(\alpha_{B*}-\alpha_B)$ against S_1 can be best produced by first plotting $(\alpha_{B*}-\alpha_B)$ against γ and then adding an alternative scaling of the strain axis with S_1, according to equation (12-7a).

b) The answer here is entirely similar to that given in exercise 14-10b for pure shear deformation.

14-14: Pure shear deformation: a) to c) The 40% flattening in the strain ellipse of Figure 14-22 implies $S_3=0.6$ and $S_1=1/S_3=1.67$. The dimensional length of the short semi-axis is 2.1 cm. By definition $S_3=L_3/L_0$, and this implies that $L_0=L_3/S_3=2.1/0.6=3.5$ cm. Thus, the radius of the undeformed unit circle is 3.5 cm. The intersection of the unit circle and the finite strain ellipse determines the orientation of lines, B, of no-finite-strain (cf. Fig. 14-15). They occur at 30° to the extension axis and are drawn as two conjugate lines, that are 120° apart. The angle α_B of 30°, thus obtained graphically for $S_1=1.67$, is compatible with the data plotted in Figure 14-16. The lines, A, of no infinitesimal strain are 90° apart and, also, symmetric about the shortening direction. Initial material lines, A^*, of no infinitesimal strain are, for $S_1=1.67$, oriented at 20° to the extension direction, according to equation (14-4) and Figure 14-11. The initial material lines, B^*, of no finite strain are, for $S_1=1.67$, oriented at 78° to the direction of extension, according to equation (14-8) and Figure 14-16.

d) The various deformation sectors of the finite strain ellipse for the pure shear deformation of Figure 14-22, when completed, are similar to those indicated in Figure 14-19a. The deformation structures, that are likely to occur in each sector, are systematically discussed in the text of page 248.

14-15: Simple shear deformation: a) to c) For the simple shear deformation of Figure 14-23, the dimensional length of the short semi-axis is 1.5 cm. The radius of the undistorted strain circle is $L_0=L_3/S_3=1.5/0.6=2.5$ cm. The intersection of the undistorted unit circle and the finite strain ellipse determines the orientation of lines, B, of no finite strain. One of the lines, B, is horizontal; the other is at 65° (counter-clockwise) to the direction of shear movement. The angle of $\alpha_B=65°$, for $\gamma=1$, is compatible with the data plotted in Figure 14-18. It, also, follows from equation (14-10). The two lines-of-no-infinitesimal-strain, A, are oriented as follows: One of the lines remains parallel to the horizontal direction of shear movement at all times. The complementary line is consistently normal to the direction of shear movement. One of the initial material lines-of-no-infinitesimal-strain, A^*, is, for $\gamma=1$, oriented at 45° to the direction of shear movement, according to equation (14-6) and Figure 14-14. The complementary line, A^*, is parallel to the direction of shear at all times (cf. Fig.14-19b). One of the initial material lines-of-no-finite-strain, B^*, is, also, parallel to the direction of shear motion at all times. The complementary line, B^*, is 116° away from the direction of shear movement, according to equation (14-10) and the plot of Figure 14-18.

d) The various deformation sectors of the finite strain ellipse for the simple shear deformation of Figure 14-23, when completed, are similar to those indicated in Figure 14-19b. The deformation structures, that are likely to occur in each sector, are systematically discussed in the text of page 248.

14-16: Incompetent single layers, hosted in more competent matrix rock, may develop mullions and inverse folds. Mullions form in incompetent layers that are parallel to the shortening direction. Inverse folds are thought to look similar to pinch-and-swell structures and form when extended normal to the shortening direction (cf. Fig. 14-20).

14-17: a) For a pure shear deformation without volume change, the following conditions apply: $S_2=1$ and $S_1.S_2.S_3=1$ and, also, $S_1=1/S_3$. Equations (14-12a to c) then simplify to:

$$(\tan\theta_1/\tan\theta_{1*})=S_1^{-2}$$
$$(\tan\theta_2/\tan\theta_{2*})=S_1^{-1}$$
$$(\tan\theta_3/\tan\theta_{3*})=S_1^{-1}$$

b) Consider a plane of final orientation $(\theta_1, \theta_2, \theta_3)=$ (10°,30°,10°) after a pure shear strain with $S_1=2$. The initial orientation $(\theta_{1*},\theta_{2*},\theta_{3*})=$ (35°,49°,19°) has been solved, using expressions obtained in exercise 14-17a.

Alternatively, the nomogram of Figure 14-26c can, also, be used to solve this exercise. To find θ_1, use S_3/S_1 (or S_1^{-2}) of 0.25. The vertical line above $S_3/S_1=$ 0.25 crosses a curve for

$\theta_1 = 10°$. That curve can be followed further back to the vertical axis, where it crosses at $\theta_{1*} = 35°$ (for $S_3/S_1 = 1$). Likewise, the angles θ_{2*} and θ_{3*} can be recovered from the nomogram of Figure 14-26c.

c) The deformed and undeformed orientations of the plane can be illustrated in a fashion similar to that of Figures 14-26a & b.

14-18: Using equation (14-13), the stretches of the lines, for pure shear in the XZ plane, are: a) $S_{(90°,90°,0°)} = S_3$, b) $S_{(90°,0°,90°)} = 1$, c) $S_{(0°,90°,90°)} = S_1$, and d) $S_{(90°,30°,60°)} = 0.75 + 0.25S_3^2$.

14-19: a) Truly *unidirectional boudins* form only under *perfect plane strain* in competent layers that coincide with the S_1-S_2 surface (Fig. 14-24b). The separation line between the individual boudins coincides with lines of no finite strain (Fig. 14-28b). All other material lines suffer extension, causing them to boudinage. Truly *single folds* form only under *perfect plane strain* in competent layers that coincide with the S_2-S_3 surface (Fig. 14-24b). The fold axis coincides with lines of no finite strain (Fig. 14-28b). All other material lines suffer shortening and may buckle. *Chocolate-tablet boudinage*, with equally strongly developed boudins in all directions, develops only in *perfect oblate deformation* in competent layers that coincide with the S_1-S_2 plane, as illustrated in Figure 14-24d. There are no material lines of no finite strain (Fig. 14-28d). All material lines will suffer extension. *Dome-and-basin folds*, with equal amplitude wavelengths in the different lateral directions, develop only under *perfect prolate deformation* in competent layers that coincide with the S_2-S_3 plane, as illustrated in Figure 14-24c. There are no material lines of no finite strain (Fig. 14-28c). All material lines will suffer shortening.

b) The directions of the separation "channels" of chocolate-tablet boudins in perfect oblate strain are impossible to predict. This is so because all material lines in the S_1-S_2 plane of the boudinaging surface extend equally in theory (Fig. 14-28d). However, in nature, physical imperfections or perturbations determine which directions develop in the separation "channels." The problem is rather similar to that of predicting the orientation of mud cracks in drying clay or columnar joints in cooling basalt. It is to be expected, therefore, that boudins, formed under perfectly oblate strain in the S_1-S_2 plane, display a hexagonal pattern, rather than the chocolate-tablet pattern, a term coined by J.G. Ramsay.

c) The direction of the egg-carton grooves of dome-and-basin folds in perfect prolate strain is impossible to predict. This is so because all material lines in the S_2-S_3 plane of the folding surface shorten equally in theory (Fig. 14-28c). However, again, in nature, imperfections or perturbations determine which directions develop into the egg-carton grooves of dome-and-basin folds. It is worth mentioning that many dome-and-basin type folds in nature have not formed during one episode of deformation, but rather by two distinct superimposed deformations occurring at separate times.

14-20: a) Despite the difference between pure and simple shear in 2D deformation analysis, it appears that their respective ellipses of superimposed strain histories contain sectors with strain evolutions that are rather similar (cf. Fig. 14-19a & b). The pure shear deformation possesses a symmetry that is absent in simple shear deformations, which possess only point symmetry. The surface-of-no-finite-elongation for rotational or coaxial 3D deformations will be different from that illustrated for the irrotational 3D deformations in Figure 14-28a to d. However, the similarities, that occur in the strain histories of rotational and irrotational 2D deformations (that is, pure and simple shear) are, also, expected to exist for rotational and irrotational 3D deformations.

b) Understanding non-coaxial 3D deformations is more complex than coaxial 3D deformations, because the planar symmetry, existing in Figures 14-28a to d, will vanish. However, surfaces of no-finite-strain will possess a point symmetry similar to that observed for strain histories of simple shear deformation (cf. Fig. 14-19b).

Chapter fifteen

15-1: a) & b) See answers to exercise 7-2a.

c) The measurement or gauging of strain is possible when strain markers are available. Such markers can be mechanically active or passive. Active strain markers have a viscosity different from that of the host rock. Passive strain markers have the same viscosity as the host rock. The material outline of passive strain markers deforms together with the host rock in a passive fashion. The total strain is duly recorded by passive strain markers, which have a perfect strain memory. However, most strain markers are active and may record more (if incompetent) or less (if competent) strain than the host rock. Examples of strain markers are cited in the text on page 267. It still remains largely unclear how the strain memory of the rock inclusions, commonly used for practical strain analysis, compares to that of the bulk deformation, experienced by their host rock.

15-2: a) A general prolate deformation is likely to generate an L-tectonite with a strong stretching lineation in the direction of S_1 (cf. Fig. 15-6b). It, also, develops a progressively better defined foliation fabric in the S_1-S_2 plane if the shape factor, K, is close to unity. If K=1, the rock becomes an L-S tectonite.

b) A general oblate deformation is likely to generate an S-tectonite with a well-defined foliation fabric in the S_2-S_3 plane (cf. Fig. 15-6a). It is very unlikely that any stretching lineation develops, unless the shape factor nears unity, and a weak lineation develops (cf. Fig. 15-5c). In the latter case, it becomes an L-S tectonite.

15-3: A shear zone, if caused by simple shear, is still an example of perfect plane strain deformation. The resulting fabric is likely to be an L-S tectonite, with the stretching lineation aligned with the S_1 direction and a foliation fabric in the S_1-S_3 plane of finite strain.

15-4: Figure 15-8a illustrates an L-tectonite. Such a fabric can be formed in supracrustal sink flows between buoyant plutons or in the tail of a rising pluton (Fig. 15-7b). L-tectonites are, also, generated in the tectonic scenario of Figure 15-6b. Figure 15-8b illustrates an L-S tectonite (weak lineation, L, strong foliation, S). Such L-S fabrics indicate that the strain shape factor K is slightly larger than, or close to, unity. Such fabrics are generated by progressive pure shear deformation (cf. Fig. 15-5a to c), but, also, by progressive simple shear deformation that is commonly dominating the deformation regime in shear zones below thrust nappes. This sample was taken from a Caledonian nappe complex in Scandinavia.

15-5: Refer to Figure 15-10:

a) The dimensional lengths of the semi-axes of the strain ellipse, outlined by the deformed ooid are: L_1 =1.2 mm and L_3=0.7 mm. The assumption of plane strain implies: $S_1=1/S_3$ or $L_1/L_0=L_0/L_3$ or $L_1.L_3=L_0^2$. It follows that L_0=0.92 mm. The assumption of plane strain implies that S_2 must be unity.

b) $S_1=1.2/0.92=1.3$ and $S_3=0.7/0.92=0.76$.

c) The ellipticity, according to equation (15-4), is: $R_S=S_1/S_3=S_1^2=1.69$.

d) The absolute area of the strain ellpse is: $2\pi L_1L_3=5.28$ mm^2. The normalized area, according to equation (15-5), is: $A=S_1.S_3=1.28$.

15-6: Refer to Figure 15-10, and see, also, exercise 15-5:

a) If the actual measurement of L_2 on the sample of Figure 15-10 reveals that $S_2=2$, then the assumption of plane strain, on which the calculations of exercise 15-5 were based, is wrong. The true length of the intermediate axis can be computed, using equation (15-7b): $L_2=[(L_1L_3)^{1/3}/S_2]^{3/2}=1.2$ mm. The normalized area of the strain ellipse's S_1-S_3 section now becomes: $A=(L_1L_3)^{1/3}/L_2^{2/3}=0.84$. This area is much smaller than the 1.28 that earlier followed from the plane strain assumption (exercise 15-5d).

b) The three principal stretches are, after recalculation, dropping the assumption of plane strain, due to the new evidence against this assumption: Firstly, $L_0=1$ mm (from $S_2=L_2/L_0$). Secondly, $S_1=1.2$, $S_2=1.2$, and $S_3=0.7$. The ooid is perfectly oblate.

15-7: Equation (15-6) states: $L_0=(L_1L_2L_3)^{1/3}$. The condition of no volume change is: $S_1S_2S_3=1$. The definitions of the stretches are: $S_1=L_1/L_0$, $S_2=L_2/L_0$, and $S_3=L_3/L_0$. Substitution of the stretches in dimensional lengths into the condition of no volume change gives equation (15-6).

15-8: Exercise 12-1 gives three principal stretches, (S_1, S_2, S_3), as follows: (a) (1, 1, 0.5), (b) (4, 0.5, 0.5), and (c) (2, 2, 0.25). It was established in exercise 12-1 that these stretch values represent: (a) perfect plane strain, (b) perfect prolate strain, and (c)perfect oblate strain. Equation (15-9a), valid only for plane strain, states $S_1=1/S_3$. Yes, this is compatible with case (a); just substitute (1, 1, 0.5). Equation (15-9b), valid only for prolate strain, states $S_1=1/S_3^2$. Yes, this is compatible with case (b); just substitute (4, 0.5, 0.5). Equation (15-9c), valid only for oblate strain, states that $S_1=1/S_3^{1/2}$. Yes, this is compatible with case (c); just substitute (2, 2, 0.25).

15-9: Applying the (R_f,θ)-method of strain analysis on elliptical markers:

a) The (R_f,θ)-plot for the strain ellipses in Figure 15-16a is bell-shaped, because $R_S<R_i$; that is, $R_S=1.34$ and $R_i=2.7$. The average orientation of S_1 is at 20°, measured in a counterclockwise direction away from a vertical reference line.

b) The (R_f,θ)-plot for the strain ellipses in Figure 15-16b can be approximated by a teardrop-shape, because $R_S>R_i$; that is, $R_S=2.3$ and $R_i=1.65$. The average orientation of S_1 is at 32°, measured counterclockwise from a vertical reference line.

c) The (R_f,θ)-plot for the deformed ooids in Figure 15-16c can, also, be approximated by a teardrop shape, because $R_S>R_i$; that is, $R_S=1.97$ and $R_i=1.52$.

15-10: Applying the stretched line method to the two stretched belemnites of Figure 15-7: The respective stretches and their angles, α and β, with the S_1 direction, indicated by the stretching lineation, can be measured on Figure 15-7. The longer belemnite has $\alpha=30°$ and $S_\alpha=1.3$. The shorter belemnite has $\beta=60°$ and $S_\beta=1.2$. These values can now be substituted in equations (15-12a & b) to evaluate for S_1 and S_3. It follows that $S_1=1.35$ and $S_3=1.1$. Assuming no volume change has occurred, S_3 should be smaller than unity, so the conclusion is that S_3 should be renamed to S_2, as follows: $S_1=1.35$ and $S_2=1.1$. The condition of no volume change, that is, $S_1S_2S_3=1$, yields $S_3=0.67$. The conclusion is that the deformation is not by plane strain, because $K>1$. It is general oblate strain, but it is still close to plane strain and that is why the rock of Figure 15-17 is still an L-S tectonite. The ellipticity of the finite strain ellipse, R_S, for the S_1-S_3 plane, is $R_S=S_1/S_3=1.35/0.67=2$.

15-11: Applying the Wellman method on the outcrops of Figures 15-21a & b yields the following ellipticities: (a) $R_S=S_1/S_3=1.5$ and (b) $R_S=L_1/L_3=1.25$.

15-12: Applying the Breddin method to the trilobite specimens of Figures 15-23a & b yields the following ellipticities for the finite strain ellipses: (a) $R_S=2.08$ and (b) $R_S=2.15$.

15-13: Strain analysis on the oolitic limestone of Figure 15-27: (a) Fry method: $R_S=1.7$; and (b) tieline method: $R_S=1.79$.

(c) The ooids that have accommodated a bulk deformation of the rock volume by pressure solution along the grain boundaries. Some ooids have suffered more pressure solution than others. The finite strain, inferred by the two methods applied, may well be representative for the bulk strain. As a population, the ooids may have a good strain memory. However, individual ooids that did not suffer from pressure solution, have no strain memory at all.

15-14: The true or maximum ellipticity of an L-tectonite, if assumed due to perfect prolate strain, that gave an $R_S=2$ in a section at 45° to L_1, can be calculated, using equation (15-15): $R_S=2.65$.

15-15: The maximum ellipticity of a section through the oblate ellipsoid is: $R_S=L_1/L_3$. An oblique section has: $R_{S\ REDUCED}=L_1/L_{3\ INCREASED}$.

Combining the two equations yields:

$$R_S=R_{S\ REDUCED}L_{3\ INCREASED}/L_3.$$

This expression can be simplified by the substitution of expression (15-16) for $L_{3\ INCREASED}$:

$$R_S=R_{S\ REDUCED}(L_1^2\cos^2\phi+L_3^2\sin^2\phi)^{1/2}/L_3= R_{S\ REDUCED}(R_S^2\cos^2\phi+\sin^2\phi)^{1/2}.$$

Further evaluation gives:

$$R_S=[(R^2_{S\ REDUCED}\sin^2\phi)/(1-\cos^{2\phi})]^{1/2}.$$

It can be seen that the maximum ellipticity, R_S, occurs always for $\phi=90°$, when $R_S=R_{S\ REDUCED}$.

15-16: a) The maximum ellipticity of a section through the ellipsoid of plane strain (and any other general strain ellipsoid) occurs when $R_S=L_1/L_3$. Oblique sections have ellipticities determined by:

$$R_{S\ OBLIQUE}=L_{1\ REDUCED}/L_{3\ REDUCED}=$$

$$[(L_1^2\cos^2\alpha+L_3^2\sin^2\alpha)/(L_1^2\cos^2\beta+L_3^2\sin^2\beta)]^{1/2}$$

and $R^2_{S\ OBLIQUE}=$

$$(R_S^2\cos^2\alpha+\sin^2\alpha)/(R_S^2\cos^2\beta+\sin^2\beta).$$

b) Plane strain is, partly, more complex than ideal oblate or ideal prolate strain, because the radial symmetry of the strain ellipse, that occurs in both prolate and oblate strain, does not exist for plane strain (see, also, exercise 14-20). However, if the plane of observation coincides with the S_1-S_3 plane, then the deformation analysis of plane strain is simple, because all the deformation occurs in the plane observed. But the strain ellipse in oblique sections may appear with misleading shapes. It may even expose as a perfect circle, when the section coincides with a plane of no-finite-strain.

Chapter sixteen

16-1: Write a good essay!

16-2: Most nations face problems in finding good repositories for radioactive waste. Commissions have been set up to assess the stability of salt domes. If the country possesses exposed granite plutons, these have, also, been studied as possible sites for the construction of storage cavities. Salt is easily excavated by selective dissolution. But one disadvantage of salt is that it represents, after ice glaciers, the most mobile rock mass on our planet, because its effective viscosity is relatively low, as compared to that of other rocks. Salt even flows under its own weight, as is evidenced by the salt glaciers in Iran and by the slow closure of ancient tunnels in salt mines through crystalline creep of the salt walls. On the other hand, cold, crystalline granite does not flow at near surface conditions and is basically immobile. However, one disadvantage of granite repositories is that the inability of crystalline creep in cold granite can lead to the accumulation of large stresses in granite plutons. This may lead them to fracture and fault, which can make underground cavities unsafe if levels of stress have not been examined prior to the cavity construction.

16-3: a) Obviously, length and strength (or time) are scaled down in analog experiments for practical purposes.

b) Yes, the velocity has the dimensions of [length.time^{-1}].

16-4: Make sure you do not repeat in your future career the errors listed here!

16-5: Write a sound research proposal.