Chapter 1: Introduction to Rock Mechanics

OUR KNOWLEDGE of rock deformation structures is based on field observations, laboratory measurements of their physical properties, and model simulations of their development. The principles of rock mechanics outlined here provide a basis for applications in geology, geophysics, and geotechnical engineering. All of these disciplines are dependent on theoretical knowledge of the mechanics of rocks. The domain of rock mechanics is a specialist branch of continuum mechanics, principally because of the special nature of rock texture and composition and the huge range of physical conditions under which they deform. Although much work remains to be done to improve our understanding of the mechanics of both rock fracturing and crystalline flow, major advances have been made during the 20th century. This book attempts to provide a sound introduction into the basic concepts of rock mechanics, covering both the brittle and ductile field.

Contents: The significance of continuum mechanics for the geosciences is outlined in section 1-1. The expansion of rock mechanics from strictly brittle studies, such as to include ductile creep, is outlined in section 1-2. The phenomenon of rock flow is briefly introduced with a historical perspective in section 1-3. The study of rheology and its application to rocks is defined in section 1-4. The principal applications of rock mechanics are sketched in section 1-5. The special character of the mechanics of rocks as compared to classical continuum mechanics, is briefly emphasized in section 1-6. A guide to the chapters of this book is given in section 1-7, followed by a section with annotated references.

Practical hint: **The concepts outlined in this book are accompanied by exercises designed to improve the reader's understanding of the subject matter. Complementary suggestions for special activities and laboratory tests are made in these introductory boxes of each subsequent chapter for your consideration.**

1-1 Rock mechanics and geoscience

We live on the regolith of our planet. Rock is, and has been, exploited by man in many ways. Ancient man of the Stone Age chipped away flints of chert to make spearheads and dagger-blades for hunting tools. Two chert pebbles were hit against one another in a skillful fashion to produce cracks, such that the desired shape for the artifact would be obtained. While working chert, ancient humans must have developed an intuitive understanding of the brittle strength of chert pebbles and the orientation of stress patterns required to control the fracture propagation. Knowledge of the mechanical strength of rock was, also, important for the construction of ancient temples, pyramids, fortresses, and bridges. For example, the Egyptian masterbuilders selected a scenic location for the mortuary temple of Queen Hatshepsut, built approximately 3,500 years ago at Deir el-Bahari near the Valley of the Kings (Fig. 1-1). The temple foundations are cleverly sunk into the easily excavated Nesma shales, overlain by more resistant limestones. However, the temple had almost entirely disappeared underneath rockfall from the heavily jointed nummulitic limestone cliffs above; the rock debris was not cleaned from the site until a century ago.

The technological revolution of the past century has dramatically improved our methods for exploiting earth materials. A general philosophy of modern science is to rationalize and understand the physical processes and physical appearance of natural phenomena. Physical quantities have been defined and then applied to measure the intensity, rate, and size of natural processes and the properties of the matter involved in them. *Rock mechanics* is the theoretical and applied science of the mechanical behaviour of rock; it is that branch of mechanics concerned with the response of rock to the force fields of its physical environment. This

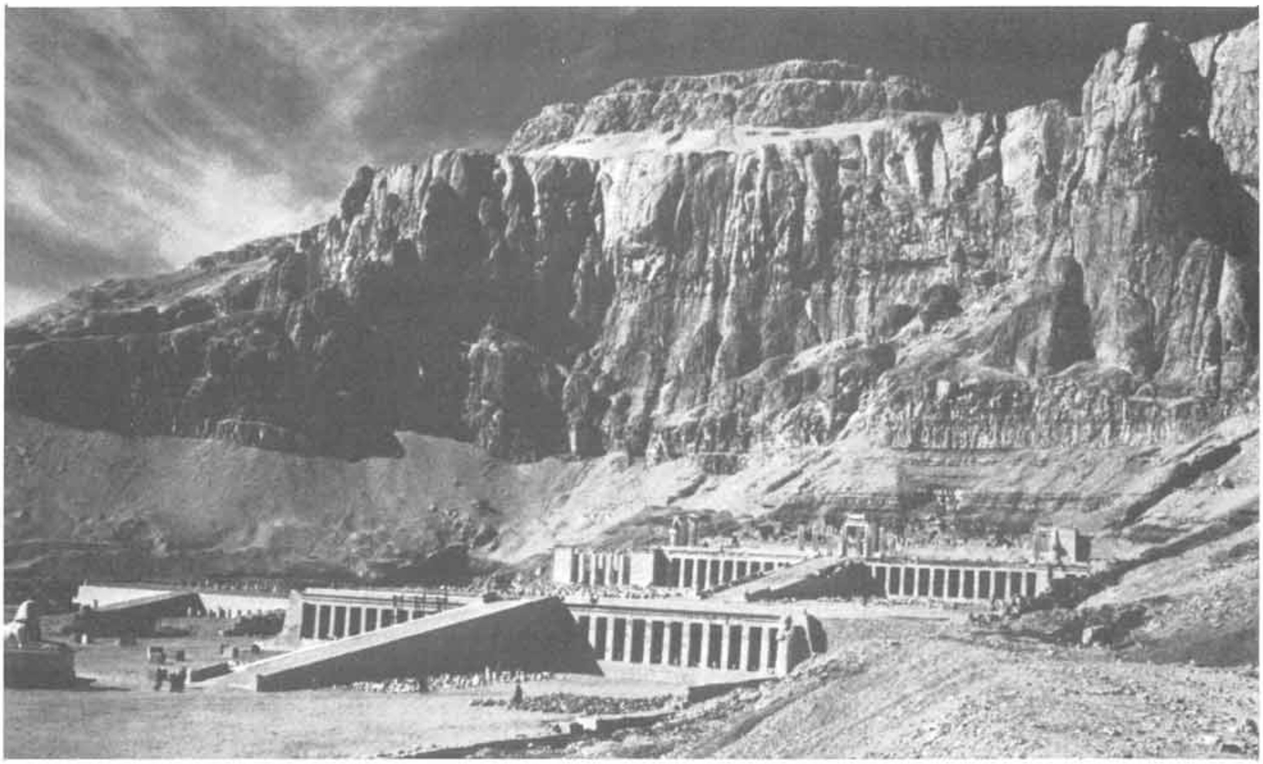

Figure 1-1: *Mortuary temple of Queen Hatshepsut, built against an imposing cliff at Deir el-Bahari near the Valley of the Kings, Western Thebes, Egypt. Rockfall continually threatens to damage the temple.*

definition was adopted by the 1964 *Committee on Rock Mechanics of the Geological Society of America*. Importantly, all deformation of rocks is ultimately due to forces and not the other way around, as is sometimes asserted in geoscience literature. Deformation consumes energy, rather than producing it. All matter in the Universe, including that of the Solar System and our mother planet, Earth, is subjected to forces. Deformation patterns form in the planetary matter when stress concentrations, natural or induced by humans, are sufficiently large to cause permanent distortion by brittle faulting, ductile creep, or both.

□Exercise 1-1: Explain why knowledge of rock mechanics may be useful: (a) in engineering operations, and (b) for geoscience in general. Give examples.

1-2 Ductile trends in rock mechanics

A quantitative approach to the mechanical behavior of rock is, and always has been, of great importance in civil engineering and mining projects. All construction works and mining

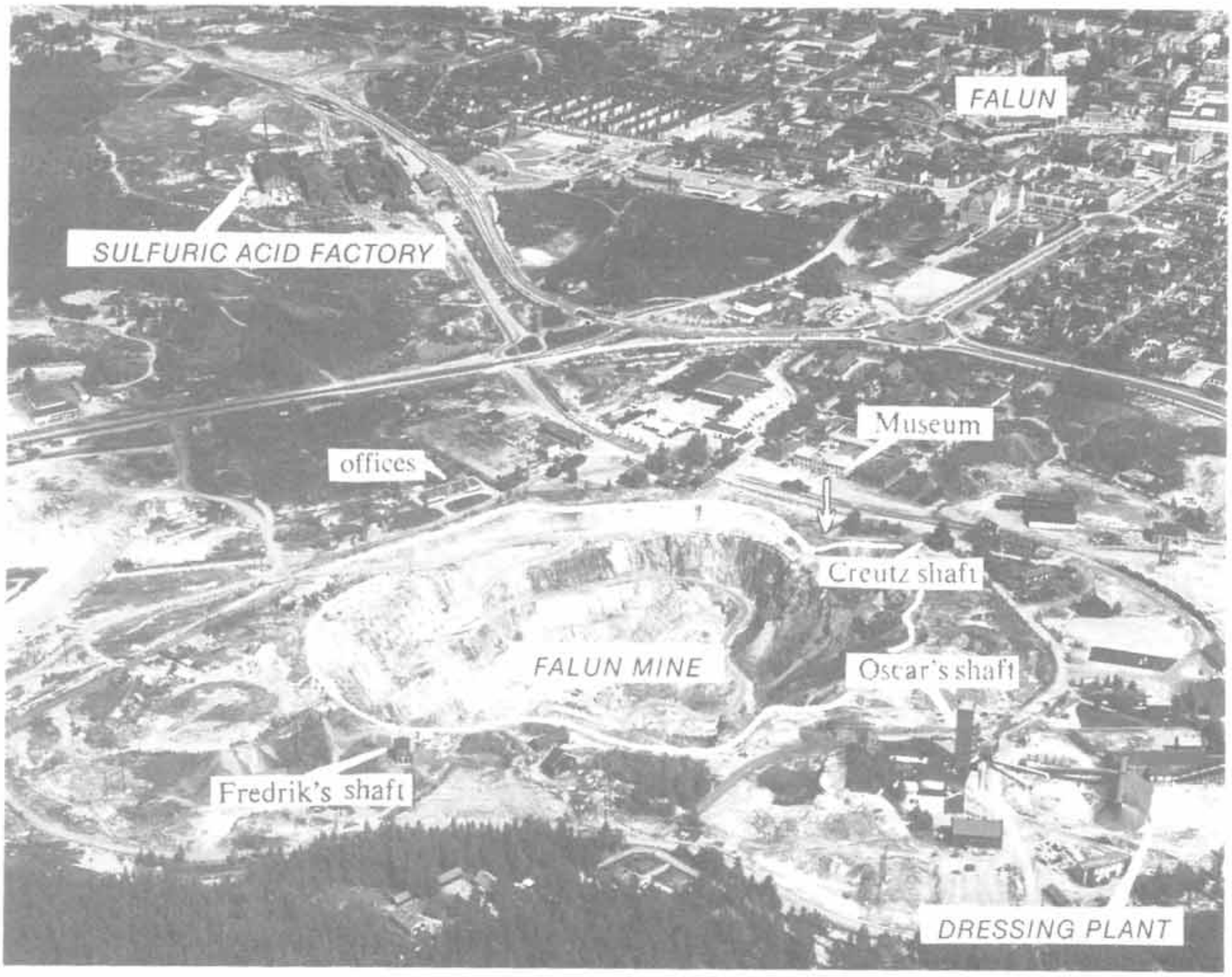

***Figure 1-2:** The Falun coppermine, Sweden, collapsed in the 17th century, due to the construction of ambitious tunneling systems, destabilizing the floor of the open pit, now one kilometer across.*

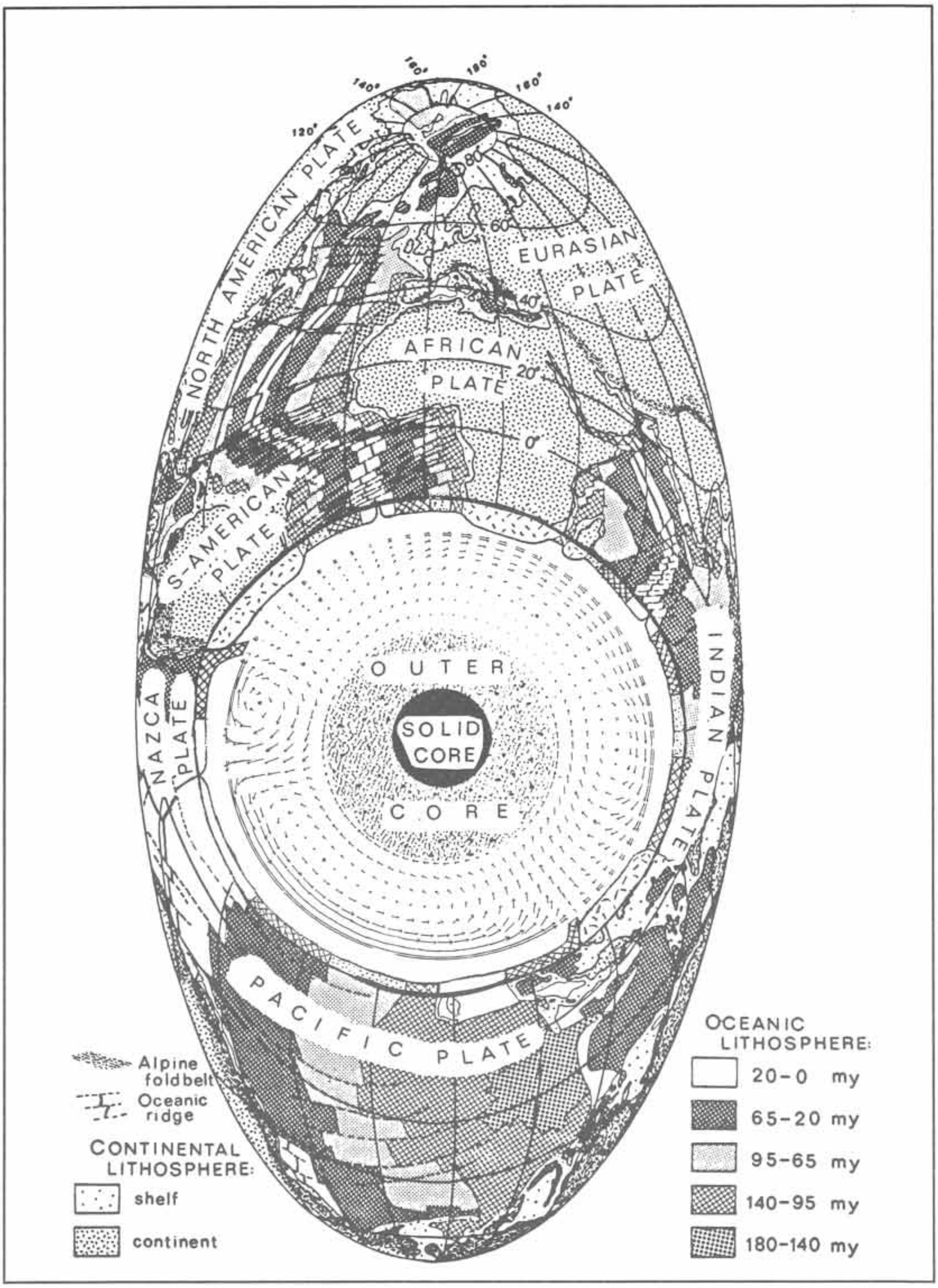

Figure 1-3: *The movement of tectonic plates at the Earth's surface is maintained by crystalline creep of mantle rock, circulating due to convective cooling of the core. The age of the ocean floor is indicated in millions of years (my). The map is a Hammer-Aitoff projection, cut open along the small-circle of 20° southern latitude. The flow pattern in the interior is conjectural.*

operations are confined to the solid upper and shallow region of the crust. The response of shallow rocks to engineering operations, involving excavation and loading of bed rock, other than rock salt, is exclusively elastic, sometimes accompanied by failure. Once failure has occurred, the movement of fault blocks is controlled by frictional boundary conditions of the fractures. Figure 1-2 illustrates a bird's-eye view of the Great Pit of Falun, Sweden. This coppermine had been operational for five hundred years, when the floor of the open pit collapsed with massive rumbling sounds on Midsummer Day of 1676. The network of subsurface tunnels had become so densely spaced that the unexcavated bedrock could no longer bear the weight of the overlying floor of the pit. Only one man, "Fat Matthew," was reported missing and his body was discovered in 1719 when a new gallery opened up one of the collapsed tunnels. "Fat Matthew" appeared to be mummified by the acidic fluids of the mine and was put on display for a fee, but he quickly lost luster and has long since been buried. The Falun mine was closed down in the early 1990's after nearly a millenium of ore extraction.

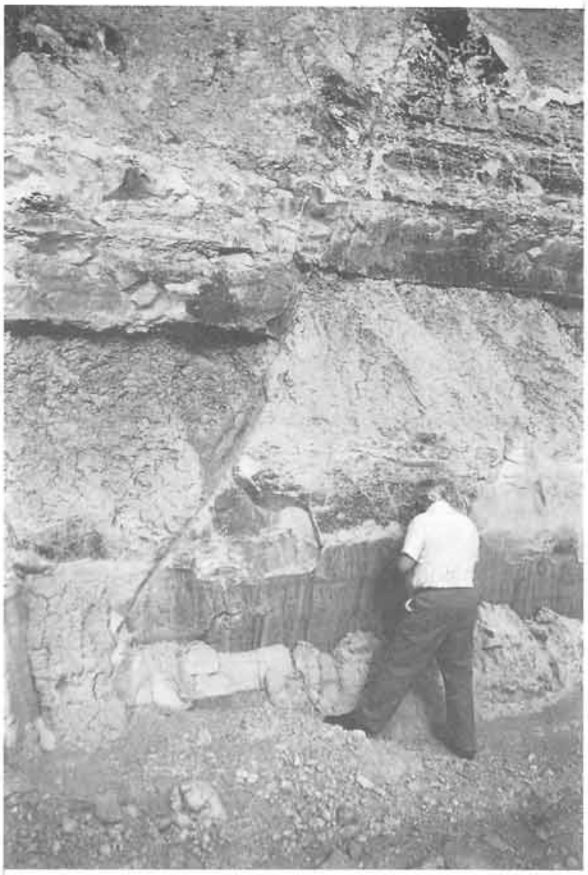

Figure 1-4: *Normal fault in limestone bed of the central Arabian graben system, SW of Riyadh, Saudi Arabia. Paul Hancock measures joints (short course for the International Union of Geosciences, 1994).*

Inspired by tunneling and excavation works, the study of rock mechanics has traditionally concentrated on laboratory tests, determining the mechanical constants and properties of rock specimens in the elastic and brittle fields. More recently, the processes of solid-state creep in crystalline rocks have come to play a fundamental role in structural geology, geophysics, and tectonophysics. Research on the crystalline creep of rocks has encouraged the application of basic concepts of *fluid dynamics* or *fluid mechanics* to explain the formation of ductile deformation patterns in rocks. Traditionally, fluid mechanics is that branch of continuum mechanics which studies the deformation and flow of fluids. Classical fluid mechanics concentrates on the movement of liquids and gases, materials without a preferred shape. However, the distinction between solids and fluids is poorly defined if considered in the framework of long time scales. Rocks deform by solid-state flow without a threshold stress if sufficiently hot to activate diffusion processes that modify the internal crystalline arrangement of the rock over vast periods of geological time. For example, the displacement of continents occurs by convective creep in the Earth's mantle (Fig. 1-3). The flow of rock and the associated deformation patterns can be described using the basic tools of fluid mechanics and adopting a continuum approach (see section 2-7). This development has not made the continuum mechanics of rocks in the brittle field less important for geoscience research. Joints and faults represent some of the most widespread geological structures exposed in rocks (Fig. 1-4). Describing and understanding the formation of these structures remains important for a range of practical applications.

□Exercise 1-2: Explain why the physics of fluids is important for understanding the deformation of rocks.

□Exercise 1-3: Candle wax and stearine lights consist of microcrystalline paraffin. They both deform in solid-state at body temperature if differential stress is applied. Warm some stearine in the palm of your hand until it is hot enough for crystalline creep upon squeezing. Attempt to explain how the crystalline paraffin can flow without breakage.

1-3 Rock flow

The notion that solid-state crystalline rocks may flow in fluid-like fashion over long periods of geological history took some time to become generally accepted. Field geologists, in the late 19th century, were puzzled by the mechanisms that could have allowed the fluid-like internal structures that developed with the formation of recumbent folds in major mountain belts (Fig. 1-5). It proved hard to convince the scientific community that crystalline rocks could accommodate the large deformations required to isoclinally fold these rocks without fracturing them at the same time.

Figure 1-5: *Recumbent folds in quartzofeldpsathic sandstone of a foldbelt at the back-arc islands of Japan, Kii peninsula.*

A scientific basis for flow of crystalline rock was not yet available when Alfred Wegener was to defend his hypothesis of continental drift for an international committee of geoscientists in Tulsa, Oklahoma, in 1928. The British geophysicist Sir Harold Jeffreys strengthened the opposition to Wegener's concept with the argument that continents could not move through solid rock like ships in the ocean. The Tulsa committee declared the theory of drifting continents invalid and implausible. As a result, the theory was no longer considered of interest to the mainstream of the earth science community. Renewed interest was raised by new and overwhelming evidence for the horizontal movement of continental and oceanic plates in the late 1950's.

When the plate tectonic theory re-emerged in the mainstream of the geoscience community, Jeffreys' principal argument against lateral continental movements had already become untenable. Theories on the flow of crystalline solids had become highly sophisticated. They were principally developed in metallurgy in response to the practical need to improve techniques of molding and rolling in the metal industry. In fact, theories on the creep of crystalline materials first found their way into the realm of geoscience via glaciology. The practical need to understand glacier surges and the spreading of ice tongues had urged the transfer of knowledge from one discipline to another, that is, from metallurgy to glaciology. Ice sheets can flow at our planetary surface fast enough to allow the monitoring of their displacements. These displacements usually are accompanied by the formation of internal deformation patterns (Fig. 1-6). As ice is a crystalline aggregate of minerals, it can be considered a rock. The crystalline creep of ice received major attention after the Second World War period and has been investigated rigorously ever since.

Early studies of the crystalline creep in monomineralic rocks, composed of quartz, calcite,

Figure 1-6: Folding of medial moraines in the distal (frontal) part of a glacier, Alaska.

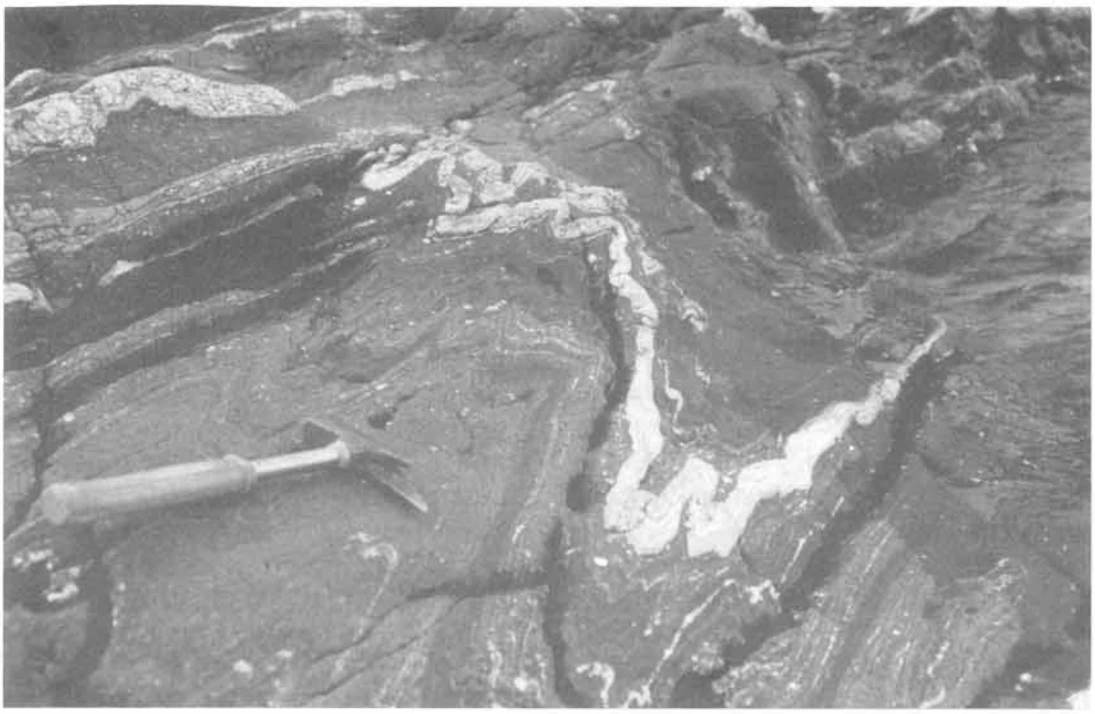

Figure 1-7a: Similar folds in one-billion-year-old Precambrian gneiss of the Mylonite zone, central Sweden. Fieldwork sponsored by the Natural Science Research Council of Sweden, 1992.

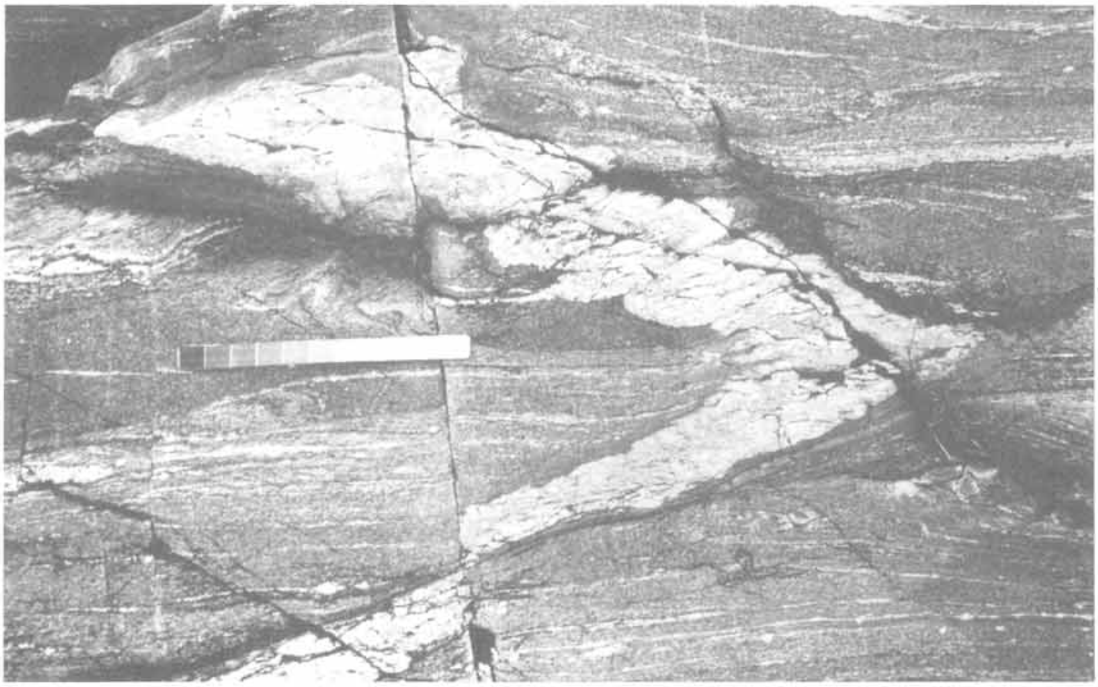

Figure 1-7b: Pegmatite vein folded under greenschist facies metamorphism. The fault was formed after cooling of the rock, following exhumation by successive erosion and uplift.

olivine, or halite, were carried out in the laboratories of metallurgists and glaciologists. Hot creep tests on rock specimens in the ductile field are now conducted in dedicated geoscience laboratories specializing in rock mechanics. The results of such tests have provided us with empirical equations that describe the ductile behavior of rock at the elevated pressures and temperatures prevailing in the deeper crust. This facilitates the quantitative analysis of large deformation structures in rocks.

□**Exercise 1-4: Discuss whether the decision taken at the 1928 Tulsa *Symposium on the Theory of Continental Drift* was justifiable considering the state of knowledge at the time.**

1-4 Rheology of rocks

The rheological behavior of rocks determines, together with the prevailing deviatoric stress and boundary conditions, which deformation structures may be formed. A wide range of rheological behavior has been observed in natural rocks. The term *rheology*, invented by Bingham, means the study of the deformation and flow of matter and was accepted when the *American Society of Rheology* was founded in 1929. The science of rheology describes the macroscopical mechanical response of a material to test forces or loads. The term *rheology* refers not only to *"the study of,"* but also has become synonymous with the *rheological behavior* itself, as implied when we write such sentences as *"the rheology of the crust is controlled by ... "*.

Most deformation structures in surface rocks have essentially been created during ductile flow at depths where high temperature and pressure conditions prevail. Subsequently, the rocks have become static and can no longer flow, once brought to the planetary surface by geodynamic processes (Fig. 1-7a). But ductile deformation patterns may be modified by brittle failure and

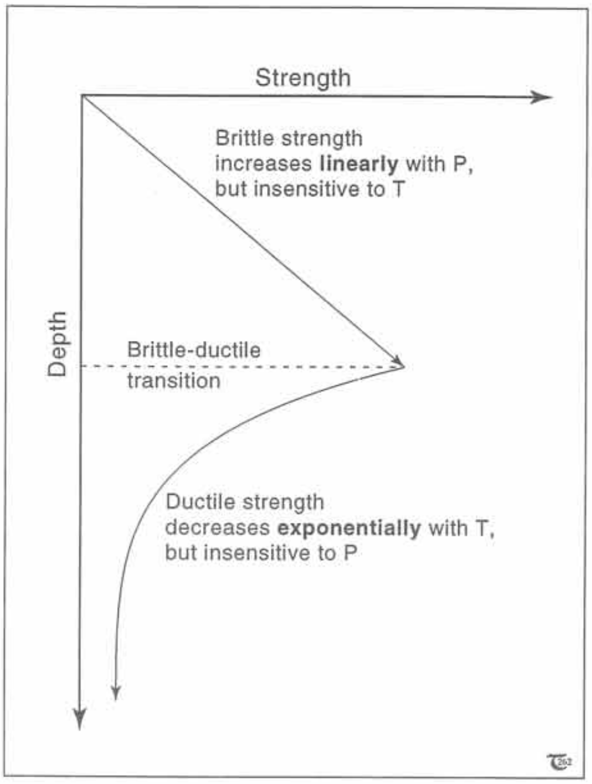

Figure 1-8: *Qualitative graph of crustal strength versus depth, showing the relative importance of temperature and pressure changes in the brittle and ductile regimes of rock deformation.*

fault movements at shallow depths (Fig. 1-7b). The physical parameters that control the basic features of complex geological deformation patterns are studied by rock mechanics. For example, the mechanical properties of rocks deforming in the brittle regime are not very sensitive to temperature, but they are quite sensitive to pressure. Frictional strength, therefore, increases linearly with depth. In contrast, the creep strength of rocks in ductile deformation is largely insensitive to pressure variations but decreases exponentially with depth, due to thermal softening (Fig. 1-8). The details of rock rheology and crustal strength profiles, such as that of Figure 1-8 are systematically discussed in chapters five to eight of this book. Detailed knowledge of rock rheology is fundamental for understanding the mechanics of rock deformation.

Figure 1-9a: John Ramsay explaining the development of transected folds in the Halifax Formation at Ecum Secum. Penrose Conference field trip, Nova Scotia, 1992.

Figure 1-9b: Gunnar Sigvaldason explains the crustal movements which led to the formation of the Hekla volcano in Iceland. Field trip sponsored by the Nordic Council of Ministers, 1990.

□ Exercise 1-5: Not unlike ice, rock salt may creep downhill. This is observed in the salt-glaciers or namakiers of the Zagros Mountains (see cover image). There, infra-Cambrian salt extrudes from the subsurface through the apex of doubly-plunging folds in the Asmari limestone. Attempt to explain how salt-glaciers maintain their flow.

1-5 Applications of rock mechanics

If one observes geologists in their field behavior, their attempts to interpret the mechanisms that may have caused the observed structures become immediately apparent. Figures 1-9a & b show two distinguished field geologists, using airspace to gesticulate movement patterns, while verbally explaining their kinematic models of rock deformation. It goes without saying that such interpretations must be based on a combination of careful field observations and sound understanding of the mechanical constraints of rock behavior.

Reconstruction of the geotectonic history of geological structures is useful in order to understand better the potential of further economic prospects hosted in the rocks of the area under study. One of the problems is that the structural data available may be limited, revealing only parts of the geometry of the structure. But knowledge of the tectonic history of a particular terrain can provide valuable arguments to reconstruct a geometrical structure only partially exposed to our vision. The geometry that best fits the observed structural data may eventually be completed using background knowledge of rock rheology, deformation rates, gradients, and deformation histories of well-studied regions. The study of deformation patterns is important further to develop a better understanding of the way in which continents break apart, grow, and remold.

Geotechnical engineers are, also, concerned with the mechanical behavior of rocks but from a point of view slightly different from that of their fellow geologists. Rather than seeking to explain the formation of geological structures, engineers are principally interested in the mechanical response of rock to excavation and loading operations at construction sites. Understanding the evolution of geological structures is essential for assessing the stability of future construction sites. Engineering geologists have to combine detailed field observations of the construction site and surroundings with theoretical knowledge of rock mechanics. Their site-investigation is commonly complemented with laboratory measurements of the mechanical properties of rock samples collected from the construction site proper. This is necessary to define the local rock properties more accurately than would be possible by consulting published data on properties of rocks sampled elsewhere. Their assessment must, also, include consideration of the threat of potential hazards, such as earthquakes, landslides, sinkholes, and volcanic eruptions (Fig. 1-10).

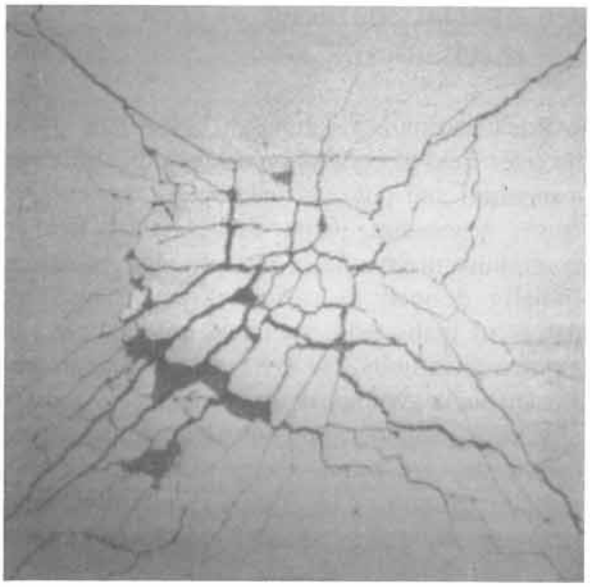

Figure 1-10: *Structural damage to a vertical support wall of a house near Agadir, Morocco, caused by the 1961 earthquake.*

□Exercise 1-6: The general statement by the Greek philosopher Heraclitus (500 BC), "Everything flows," (Panta rei), is often quoted in the geoscience literature, sometimes suggesting that *all* solid rocks are continually flowing. Do you think this is unconditionally true?

Figure 1-11: *Six-hundred-million-year-old rocks intruded by late aplite and pegmatite dikes, Khamis Mushayt, Saudi Arabia. Field work sponsored by King Fahd University of Petroleum and Minerals, 1994.*

1-6 Special character of rock mechanics

Rock mechanics, although being part of the broader field of continuum mechanics, deals with a medium and physical parameters, which set it apart, somewhat, from the classical field of continuum mechanics. Most physical quantities, initially defined to calculate the strength and forces of man-made, technical contraptions, are expressed in units that are rather small for measurements in geological time and space. Additionally, rock properties, measured in simple laboratory tests, are by necessity assumed either to remain constant or to obey a simple function of time, space, and physical conditions. However carefully defined, if such rocks are deforming over long periods of geological time, the properties of these rocks will still be poorly constrained. This is because rock composition and texture commonly vary in both space and time in a rather unpredictable fashion. For example, the intrusion of igneous dikes into a unit volume of rock instantaneously changes the composition, texture, and mechanical strength of the initial rock volume (Fig. 1-11). Likewise, metamorphic processes may, also, cause important changes in both the texture and mineralogical composition of entire tectonic provinces. Consequently, precise laboratory measurements of present-day rock properties may still lead to large inaccuracies if extrapolated over vast geological space and time.

Similarly, the fluid mechanics we now increasingly apply to geological instabilities was originally designed to describe the flow of low-viscosity liquids and gases. Many aspects of the creeping flow of crystalline rocks, indeed, can be described in terms of fluid mechanical principles. However, the motion of the traditional low viscosity fluids involves such large deformations that the internal deformations are rarely considered of interest, with the exception of studies on the mixing of fluids (Fig. 1-12). The principal focus of classical fluid mechanics therefore is on a flow description in terms of flow rates and streamline patterns. But the very large viscosity of solid-state rocks leads to flows so slow that finite strains remain relatively small, as compared to those in the traditional, low-viscosity fluids (e.g., water and air). Consequently, the principal focus in studies of rock flow is on the determination of finite strains and the description of deformation structures. The second part of this book demonstrates that streamline patterns can pro-

Figure 1-12: Iso-viscous mixing of a leucocrate blob, suspended in a mesocratic fluid in a container. The folds are due to shearing motion of the top and bottom walls of the box. Courtesy Julio Ottino.

☐Exercise 1-7: The effective viscosity of water is 0.001 Pa s. The average viscosity of mantle rocks is 10^{18} Pa s. How many orders of magnitude larger is the resistance to flow in mantle rocks as compared to that in water?

vide concise descriptions of the progressive deformation of crystalline rocks (chapter thirteen).

1-7 Guide to the chapters

This book is aimed at postgraduate students, already familiar with the geometrical variety of deformation structures in rocks and acquainted with the jargon to describe them. The mechanical behavior of rock and some basic principles of continuum mechanics are discussed in a systematic fashion. The text is divided into two parts. *Part one* introduces the fundamentals of rock mechanics and provides a state-of-the-art account on the rheology of rocks. *Part two* outlines elementary tensor calculus, some fluid dynamics, and practical methods for strain and deformation analyses. For the benefit of the reader, a list of symbols used in the main text is given in Table 1-1. Each of the sixteen chapters is briefly outlined below.

Part 1: Mechanics and Rheology

Chapter one outlines the significance of rock mechanics, rock rheology, and fluid dynamics for the understanding of deformation structures.

Chapter two carefully introduces fundamental physical quantities and the units in which these quantities are measured. The important distinction between scalar, vector, and tensor quantities is essential in any attempt to understand rock deformation.

Chapter three outlines the physical meaning and the mathematical dimension of the quantities of force, pressure, and stress. The introduction of tensor calculus is avoided at this introductory stage; it is not introduced until chapter ten. But examples of fluid pressure and lithostatic pressure are included here.

Chapter four further elaborates the detailed nature of stress and distinguishes the deviatoric and the total stress. The stress ellipsoid concept is introduced. The separation of normal and shear stress is outlined, followed by the Mohr circle of stress. The directional relationship between principal stress and force vectors is established. Stress trajectories are defined and applied to geological problems. Some methods of stress measurement are explained.

Chapter five is dedicated to elasticity, the elastic moduli, and mechanical models used for representing elastic rheologies. The effect of elasticity on lithostatic pressure and joint formation and anelasticity are, also, discussed.

Chapter six outlines crack propagation and load tests, using uniaxial and triaxial rigs. The formation of tension joints and shear joints is discussed. Fault movement is discussed in terms of the Coulomb criterion, Amonton's law, and Byerlee's law. Brittle strength envelopes are constructed for normal faults, reverse faults, and strike-slip faults.

Chapter seven moves on to the crystalline creep of rocks. The concepts of steady-state foliation and dynamic recrystallization are introduced. The principal creep mechanisms are outlined. These include pressure solution, diffusion creep, dislocation creep, and crystal plasticity. The use of deformation maps is explained. The implications of the terms ductile and plastic are briefly considered.

Chapter eight discusses crystalline creep in terms of an effective viscosity and creep laws. The difference between Newtonian and power-law creep is outlined. The effects of strain softening, strain-rate softening, thermal softening, and hydro-softening are explained. The rheological behavior of crustal and lithospheric slabs is evaluated, using so-called strength profiles. These can, also, be used to determine the depth of brittle-ductile transitions.

Part 2: Tensors and Deformation Analysis

Chapter nine provides a concise mathematical review to facilitate the application of mathematical tools and concepts to describe the deformation

Table 1-1: List of Symbols

Symbol	Quantity	SI-units	Introduced in equation
F	Force	N	3-1
m	Mass	kg	3-1
a	Acceleration	$m\ s^{-2}$	3-1
V	Volume	m^3	3-2
ρ	Density	$kg\ m^{-3}$	3-2
α	Angle of slope	-	3-3a
F_N	Normal force	N	3-3a
F_S	Shear force	N	3-3b
ϕ	Friction angle	-	3-4
μ	Friction coefficient	-	3-5
g	Gravity acceleration	$m^2\ s^{-1}$	3-6
P_{fluid}	Fluid pressure	Pa	3-7a
z	Length scale	m	3-7a
P_{lith}	Lithostatic pressure	Pa	3-7b
λ	Fluid pressure coefficient	-	3-8
$\sigma_{1,2,3}$	Total principal stress	Pa	4-1
P	Pressure	Pa	4-2a
$\tau_{1,2,3}$	Deviatoric principal stress	Pa	4-3
τ_S	Deviatoric shear stress	Pa	4-4
σ_N	Total normal stress	Pa	4-5a
σ_S	Total shear stress	Pa	4-5b
τ_N	Deviatoric normal stress	Pa	4-6a
τ_S	Deviatoric shear stress	Pa	4-6b
ξ	Angle of stress orientation	-	4-8
e	Elongation	-	5-1
L	Dimensional length	m	5-1
$e_{1,2,3}$	Principal elongations	-	5-2
γ	Angular shear strain	-	5-3
ψ	Angle of shear	-	5-3
ν	Poisson ratio	-	5-4
β	Compressibility	Pa^{-1}	5-6a
κ	Bulk modulus	Pa	5-6b
α	Thermal expansivity	K^{-1}	5-7
E	Young modulus	Pa	5-8
G	Shear modulus or rigidity	Pa	5-9
λ	Lamé constant	Pa	5-13
c_0	Cohesion	Pa	6-1
T_H	Homologous temperature	C or K	section 7-3
η	Dynamic viscosity	Pa s	8-1
A	Constant	varying	8-2
ν	Kinematic viscosity	$m^2\ s^{-1}$	8-3
t_m	Maxwell relaxation time	s	8-4
De	Deborah number	-	8-5
n	Power factor	-	8-6
η_{eff}	Effective viscosity	Pa s	8-7
Q	Activation energy	$J\ mol^{-1}$	8-8a
R	Boltzmann gas constant	$J\ K^{-1}\ mol^{-1}$	8-8b
dx	Infinitesimal length	m	9-2
ΔX	Finite length	m	9-2
d/dx	Derivative	m^{-1}	9-2
f'(z)	Derivative	m^{-1}	9-2
∇	Del	-	9-5
ψ	Stream function	$m^2\ s^{-1}$	excercise 9-2
v	Velocity	$m\ s^{-1}$	9-6

Symbol	Quantity	SI-units	Introduced in equation
τ_{ij}	Deviatoric stress tensor	Pa	9-11
F_{ij}	Deformation matrix	-	9-14
W_{ij}	Vorticity tensor	s^{-1}	9-16a
$\dot{\omega}$	Vorticity vector	s^{-1}	9-16a
D_{ij}	Strain rate tensor	s^{-1}	9-16b
$\dot{\gamma}$	Angular shear strain rate	s^{-1}	9-16b
$I_{1,2,3}$	Invariant of stress tensor	-	9-22a,b,c
i	Complex number (-1)	-	section 9-8
W	Complex function	varying	section 9-8
α,β,γ	Direction cosine angles	-	10-1a,b,c
l,m,n	Direction cosines	-	10-3
$\sigma_{x,y,z}$	Effective stress component	Pa	10-5a,b,c
σ_{eff}	Effective stress	Pa	10-6a
σ_{xx}	Stress tensor components	Pa	10-7
σ_{ij}	Stress tensor	Pa	10-9
$p_{1,2,3}$	Effective stress component	Pa	10-11a,b,c
θ	Inclination angle of plane	-	10-19a
S	Stretch	-	11-1
λ_e	Quadratic elongation	-	11-4
$S_{1,2,3}$	Principal stretches		11-5
e_{xx}	Elongations in strain tensor	-	11-8
$J_{1,2,3}$	Invariants of strain	-	11-14a,b,c
$\dot{e}_{ij}$	Strain rate tensor	s^{-1}	11-18a
λ^*	Viscous Lamé constant	Pa s	11-19
k	Flinn shape parameter	-	12-1a
K	Ramsay shape parameter	-	12-1b
D	Ramsay 3D strain parameter		12-2
L	Lode 3D strain parameter	-	12-3
N	Nadai strain parameter	-	12-4
ϵ	Natural, logarithmic strain	-	12-5a
θ	Inclination of stretch axis	-	12-8
J_{ij}	Displacement gradient tensor		12-9
β	Angle of distortion	-	12-11
E_{ij}	Strain tensor	-	12-15a
Ω_{ij}	Rotation tensor	-	12-15b
Φ	Potential function	$m^2\ s^{-1}$	13-7a
ζ	Complex variable	varying	13-8a
α	Flow asymptote angle	-	13-14
L_{ij}	Velocity gradient tensor	s^{-1}	13-16
$k_{1,2}$	Constants	-	13-21a,b
S_ϕ,S_α	Stretch of arbitrary lines	-	14-1
α_{A,A^*,B,B^*}	Orientation lines A,A*,B,B*		Chapter 14
$\theta_{1,2,3}$	Angles of plane	-	14-12
S_{OP}	Stretch of line OP	-	14-13
R	Ellipticity	-	15-4
A	Normalized sectional area	-	15-5
R_s	Final ellipticity of object	-	15-10a
R_f	Ellipticity of finite strain	-	15-10a
R_i	Initial ellipticity of object	-	15-10b
$S_{\phi,\alpha,\beta}$	Stretch of arbitrary lines	-	15-11
θ,ψ	Angles for angular distortion		15-13
$L_{1,2,3}$	Dimensional lengths of principal strain axes	m	15-13

of rocks. An introductory essay, for moral support of geoscientists hesitant to embrace mathematics, is included. Some attention is given to ordinary differentials, partial differentials, equations made up of differentials, integrals, tensors, matrices, complex variables, and complex functions.

Chapter ten moves on to apply tensor notation in stress studies. The components of the stress tensor are defined. The importance of choosing a suitable reference frame is emphasized. Outlined are the use of the Cauchy formula, summation convention, cubic equation, tensor transformation formula, and the invariants of the stress tensor.

Chapter eleven takes a thorough look at the concept of strain, and ways to quantify it. These include the strain ellipsoid, principal stretches, and finite strain tensor. The distinction between infinitesimal and finitesimal deformation is outlined. Strain trajectories are illustrated. The invariants of the strain tensor are defined, both in terms of principal strains and tensor strains. The relationship between finite strain and stress is explained for elastic deformations, as well as for ductile deformations.

Chapter twelve introduces the general distinction between coaxial and non-coaxial deformation. Measures for quantifying the magnitude of 3-D strain in a single scalar are outlined. The classical cases of plane deformation, that is, pure and simple shear, are systematically discussed. The components of deformation and the meaning of the deformation tensor are outlined.

Chapter thirteen provides an introduction to some of the basic concepts of fluid mechanics. The implications of streamlines, stream functions, potential functions, and the stream function summation rule are outlined. The velocity gradient and ways to model progressive deformation are, also, discussed.

Chapter fourteen outlines the behavior of competent and incompetent single layers during deformation. The incremental and finite strain ellipses are introduced, together with many analytical expressions for calculating the changes in length and orientation of lines during deformation.

Chapter fifteen introduces the concept of theoretical strain analysis and outlines the terminology of tectonites. Important formulae are given for determining plane strain, volume change, and strain estimates in sections oblique to the principal axes of strain. The techniques available for practical strain analysis are outlined. These techniques are the (R_f,ϕ)-method for elliptical objects, the stretched line method for linear objects, the Wellman and Breddin methods for angular objects, and the Fry method for centers of spaced objects.

Chapter sixteen concludes with emphasizing the continued importance of geological field studies and site-investigations is discussed. The purpose of model studies is outlined. The limitations of strain analysis for deformation studies are described. In particular, the distortional rotation component of deformation, neglected in strain analysis, needs to be taken into account in a complete deformation analysis. Finally, some topics for further study and research are suggested.

Each chapter includes suggestions for auxiliary reading in the annotated references. The general philosophy adopted here is that the number of references in the main text should be kept to an absolute minimum. In many instances, it is difficult to trace the original source of a particular expression or theory. All too often such equations and concepts have become common domain, and their attribution to a particular author would probably be misleading. This evolutionary view of scientific ideas pays *tribute* to the original innovators by adding a name to a concept, e.g., Burger model, Byerlee's law, Flinn diagram, Griffith cracks, Grigg's apparatus, Hsu plot, Kelvin-Voigt model, Maxwell model, Mohr circle, Poisson ratio, Young's modulus, etc.

The concepts outlined in this book are accompanied by mostly theoretical exercises, designed to improve the reader's understanding of the subject matter. Additional improvement of comprehension can be achieved by conducting or monitoring some practical laboratory tests and field tests. Complementary suggestions for special activities and laboratory tests are made in the introduction of each chapter for your consideration. Such tests are time-consuming and require the availability of, and access to, special apparatus and materials. These facilities may not always be available to you, but - if they are - may greatly help to cushion and consolidate the theory with practical activities.

References

Books

Previous authors have contributed textbooks to update the training of Earth Science students in the area of continuum mechanics. Some of these books concentrate on the broader field of geodynamics and tectonics. Other texts are dedicated to the application of mechanical principles to smaller scale structures seen in rocks. All of the textbooks listed below make excellent auxiliary reading. Classical textbooks on brittle rock mechanics and continuum mechanics of elastic media are referred to in chapters five and six. Literature on crystalline creep and general fluid mechanics is suggested in chapters seven and thirteen, respectively.

Theory of Continental Drift (1928, AAPG, Tulsa), edited by Willem Waterschoot van der Gracht, provides an interesting collection of views by the past Earth Science establishment on the movement of continents.

Folding and Fracturing of Rocks (1967, McGraw-Hill, 568 pages), by John Ramsay. A classical text for structural geologists, interested in the detailed study of deformation patterns in rocks.

The Earth (1970, 5th edition, Cambridge University Press), by Sir Harold Jeffreys, was first published in the 1920's and has been a standard text in geodynamics for half a century. Later editions of his book have appeared, and further updates of his book are now prepared by his heirs.

Stress and Strain (1976, Springer), by Win Means, is a milestone in advocating a more quantitative approach to rock deformation and was written especially for structural geologists. It summarizes much of the basic concepts of continuum mechanics for a geological readership. Professor Means has no plans for an update of his book, but he recently emphasized the importance of a quantitative approach to the study of rock deformation structures in a review article (Means, 1990, *Journal of Structural Geology*, volume 12, pages 953 to 971).

Geodynamics (1981, Wiley), by Gerald Schubert and Donald Turcotte, provides a clear exposition of geological applications of continuum mechanics. This book is intended for the advanced undergraduate and postgraduate student and assumes a basic understanding of differential equations and basic laws of physics. It further focuses on large scale tectonics, providing a wealth of explanations for a variety of geophysical phenomena, but necessarily leaving many smaller deformation phenomena beyond discussion.

Analysis of Geological Structures (1990, Cambridge University Press), by Nick Price and John Cosgrove, comprises a combination of descriptive geology and mechanical analyses.

Mechanics in Structural Geology (1991, Springer), by Brian Bayly, summarizes a lifetime experience and applies mechanical principles to deformation structures in rocks.

Structural Geology: Fundamentals and Modern Developments (1993, Pergamon Press), by S. Ghosh, includes fine discussions on the importance of stress and strain for understanding the formation of geological structures.

Mechanics in the Earth and Environmental Sciences (1994, Cambridge University Press), by Gerald Middleton and Peter Wilcock, concentrates on the physical processes within and at the surface of the Earth. Included are examples of flow in porous media, turbulence, and thermal convection.