Chapter 11: Strain and Strain Tensors

ROCKS DEFORMING by ductile creep may evolve large strains accompanied by rotational distortions. A variety of measures of strain has been introduced to enable quantitative descriptions of a range of deformation patterns. Strain is not unlike stress, in the sense that both are tensor quantities. Simple strain patterns can be described in terms of principal stretches or equivalent measures of strain. However, the strain component at each point of *complex* deformation patterns can be described most appropriately by a particular strain *tensor*.

Contents: This chapter aims to provide a careful introduction to the various measures of strain (section 11-1), principal stretches (section 11-2), and the finite strain tensor (section 11-3). Deformation theory is introduced (section 11-4), followed by the concept of strain trajectories, which visualize the variation in the orientation of the principal strain axes in heterogeneous deformations (section 11-5). The invariants of the strain tensor are summarized in section 11-6. The relationship between the stress tensor and the elastic strain tensor is outlined in section 11-7. The more general deformation tensor is discussed in section 11-8.

Practical hint: **Geological strain rates can be estimated in a number of ways. Examples are given in a comprehensive review by Kukal (1989, *Earth-Science Reviews*, volume 28, pages 1 to 284). Prepare a seminar, presenting your own estimates of geological strain rates for local examples.**

11-1 Strain ellipsoid

The state of strain in three dimensions can be *graphically* represented by a strain ellipsoid (Figs. 11-1a to d). The three principal diameters of the ellipsoid are termed the *principal axes* of strain. These axes always remain mutually perpendicular and do not need to maintain unit lengths. The length of the semi-axes can be expressed best by a *stretch*, S, defined as the ratio of a deformed line length, L, and the initial line-length, L_0:

$$S = L/L_0 \quad (11\text{-}1)$$

A stretch of unity means no strain at all, a stretch smaller than unity means shortening, and physical extension has a stretch larger than unity.

The difference between the stretched and unstretched lengths of the principal strain axes, if normalized, defines the *elongation*, e (Figs. 11-2a & b):

$$e = (L - L_0)/L_0 \quad (11\text{-}2)$$

It can be easily demonstrated that:

$$S = 1 + e \quad (11\text{-}3)$$

The elongation itself is sometimes referred to as the *extension*. The latter term may be misleading because negative elongation leads to shortening and no physical extension is involved.

The stretch and elongation are the two measures of strain used most frequently in geological applications. The stretch is most practical for representing the magnitude of finite strain, because it defines the normalized lengths of the principal axes of the finite strain ellipsoid (sections 12-4 & 12-7). The elongation features in the strain tensor (section 11-3), which is related to the principal stretches in a simple fashion only for coaxial deformations (sections 12-1 and onwards). Coaxial deformations have vanishing rotation tensors so that the deformation tensor is identical to the strain tensor. In such cases, the principal stretches follow directly from the strain tensor elements (eqs. 12-12a & b). In all other cases, the vorticity tensor needs to be taken into account to calculate the principal stretches from the deformation tensor by integrating incremental elongations and rotations over time (sections 12-8, 13-5, and 13-6).

Another measure, sometimes introduced for describing strains, is the *quadratic elongation*, λ_e:

$$\lambda_e = (1 + e)^2 \quad (11\text{-}4)$$

However, the term quadratic elongation, now firmly established in the literature, is unfortunate, because λ_e is, in fact, a quadratic stretch and not the square of the elongation. The quadratic elongation features in many equations where the square of the stretch is involved. One example is the equation for the strain ellipsoid, which can be expressed, using the quadratic elongation, rather than the square of the stretch (eq. 11-5). However, in this textbook use of the quadratic elongation is avoided for simplicity, rather the square of

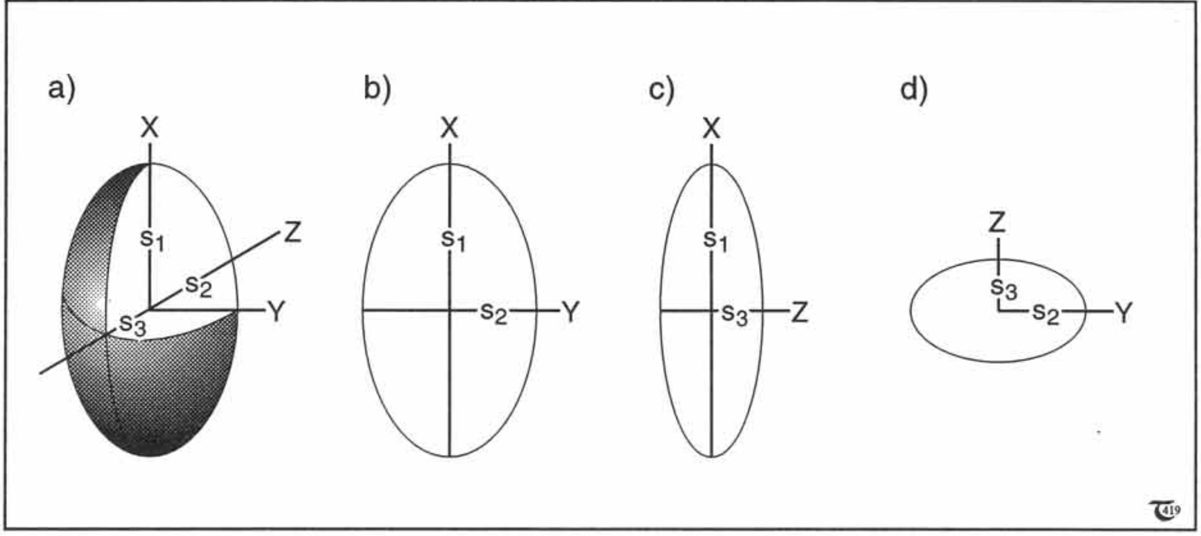

Figure 11-1: *a) Strain ellipsoid with principal stretches or normalized semi-axes S_1, S_2, and S_3. b) to d) Ellipse sections in the the three principal planes of the strain ellipsoid.*

the stretch is used instead where needed. A complementary measure of finite strain is the natural strain, defined as the natural logarithm of the stretch (see section 12-3).

None of the above strain measures can explicitly account for distortion of shape due to rotations. The changes of angles between lines in a deforming medium can be expressed in terms of the angular shear strain, γ, which is introduced in section 12-4. However, the angular shear strain is mathematically identical to the double shear elongations in the strain tensor, as explained in section 11-3.

□Exercise 11-1: Consider the strain ellipsoid of Figure 11-1. Assume that the stretch of the intermediate strain axis is unity. Make a table, giving for each of the three semi-axes of the strain ellipsoid: the absolute initial length, the absolute present length, the stretch, the elongation, and the quadratic elongation.

□Exercise 11-2: Calculate both the elongation and quadratic elongation for a line with a stretch of 0.7.

11-2 Principal stretches

The three semi-axes of the triaxial strain ellipsoid, S_1,S_2,S_3, have lengths normalized by that of the initial radius of the undistorted strain sphere. The suffixes are conventionally chosen such that strain magnitudes are ranked according to $S_1>S_2>S_3$. The equation for the strain ellipsoid, with the coordinate axes (X,Y,Z) taken parallel to the *principal stretches*, S_1,S_2,S_3, is:

$$(x/S_1)^2+(y/S_2)^2+(z/S_3)^2=1 \qquad (11\text{-}5)$$

with ellipsoid coordinates (x,y,z). If volume change is excluded during the deformation, the product of three principal stretches remains unity:

$$S_1S_2S_3=1 \qquad (11\text{-}6)$$

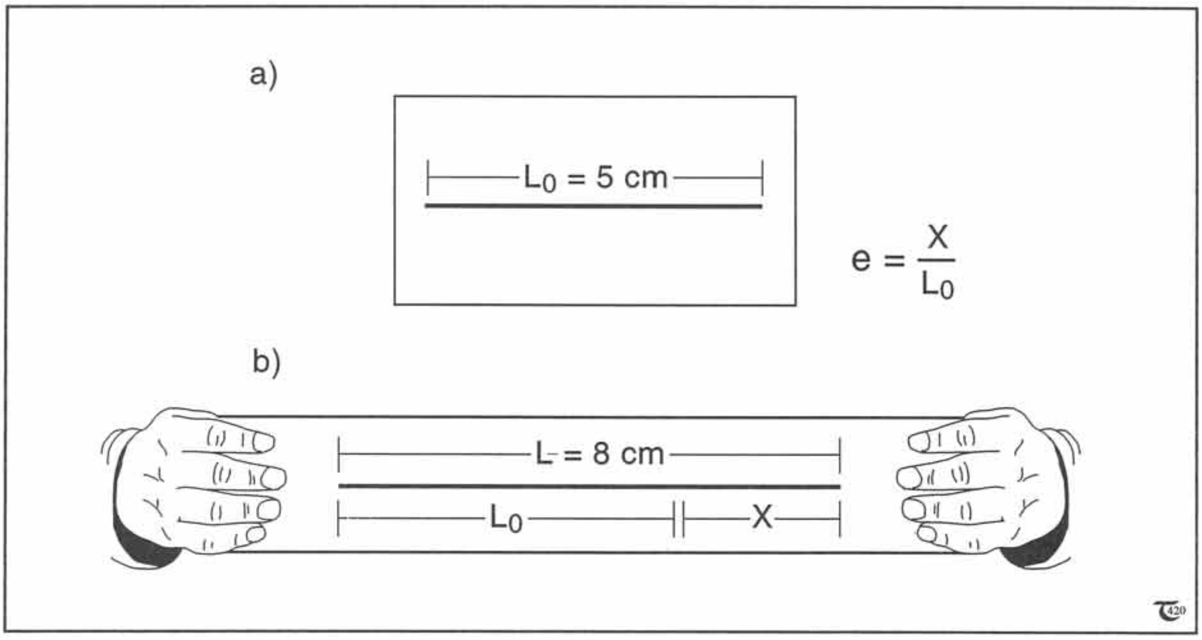

***Figure 11-2:** a) & b) Definition sketches of elongation, e, which compares the length of lines before and after deformation.*

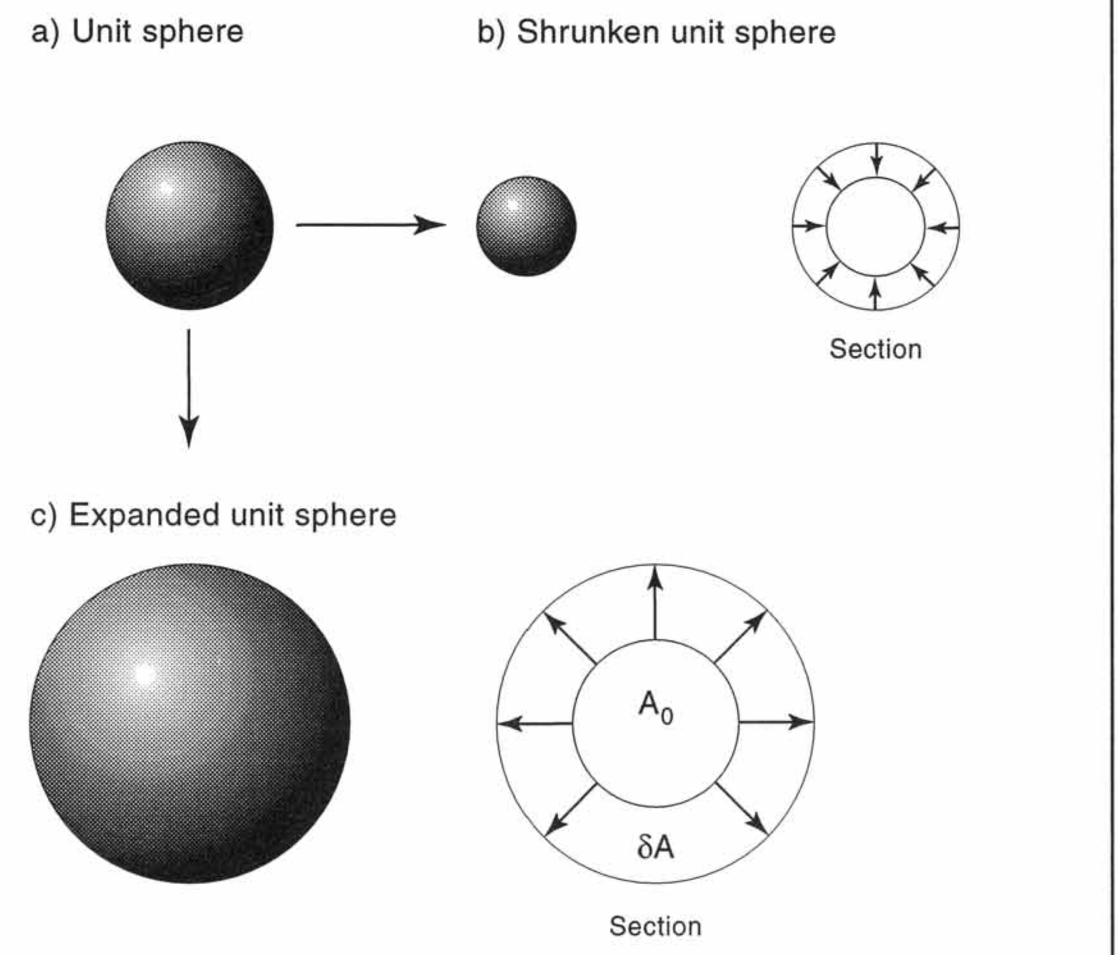

Figure 11-3: *a) to c) Dilation of a unit sphere: (a) undeformed, (b) shrinkage, and (c) expansion. See exercise 11-3.*

Volume change, however, may occur during deformation of rocks suffering from, among others, diagenesis, metamorphism, or hydrothermal solution. The strain ellipsoid may account for such a volume change if it can be demonstrated that the product of the stretches is no longer unity (see section 15-6). The volume, V_0, of the undistorted strain sphere will differ from that of the deformed ellipsoid, V. A measure for the volume change is the *dilation* $\delta V = (V - V_0)/V_0$:

$$\delta V = (S_1 S_2 S_3) - 1 \qquad (11\text{-}7)$$

A volume gain occurs if $\delta V > 0$, and a volume loss occurs if $\delta V < 0$. The effect of volume change is best illustrated for a sphere of unit radius (Fig. 11-3a), either uniformly shrinking, so that $(S_1 = S_2 = S_3) < 1$ (Fig. 11-3b), or uniformly expanding, so that $(S_1 = S_2 = S_3) > 1$ (Fig. 11-3c).

11-3 Strain tensor

The state of strain in a point can be represented by the three principal axes of the strain ellipsoid. The normalized length of the principal axes may be expressed in terms of the stretches, S_1, S_2, and

□Exercise 11-3: The unit sphere in Figure 11-3a has a volume, V_0, of (4/3)-πR_0^3 and a surface area, A_0, of $4\pi R_0^2$, with the undistorted radius R_0. Calculate the dilation and associated change in surface area for two different cases: (a) radially uniform stretch of 0.5 (Fig. 11-3b), and (b) radially uniform stretch of 2 (Fig. 11-3c).

S_3, or quadratic elongations, $\lambda_i=S_i^2$, or elongations or extensions, $e_i=S_i$-1. All of these three strain measures can be represented as tensor quantities. This becomes desirable if the orientation of the principal strains is spatially varying so that the strain trajectories become curved (Fig. 11-4). In that case, the principal strains in each location can be conveniently described by the ellipsoid equation (11-5) only if the coordinate axes rotate with the trajectory pattern so that the principal strains remain parallel everywhere to the coordinate axes. This is obviously impractical and, therefore, a spatially-fixed coordinate system is more useful in practical situations, requiring analysis at a level more advanced than possible with the ellipsoid description. The adoption of a fixed coordinate system means that principal elongations or strains are decomposed in their tensor components.

A common form of the strain tensor utilizes the elongations:

oriented parallel to:	X-axis	Y-axis	Z-axis	
acting on plane normal to X-axis:	e_{xx}	e_{xy}	e_{xz}	
acting on plane normal to Y-axis:	e_{yx}	e_{yy}	e_{yz}	(11-8)
acting on plane normal to Z-axis:	e_{zx}	e_{zy}	e_{zz}	

All the tensor elements, e_{ij}, are expressed in a particular coördinate system and act at an infinitesimal point on three mutually perpendicular planes (Fig. 11-5a), similar to that seen for the stress tensor. All tensor elements act parallel to the coordinate axes. If a different coordinate system is used, then the nine tensor elements will have numerical values different from those for any other coordinate orientation. The strain may be represented by its elements, e_{ij}, with subscript indices equal to either x, y, and z, or simply 1, 2, and 3 (Fig. 11-5b). The choice of alphabetical or numerical indices is, again, entirely arbitrary.

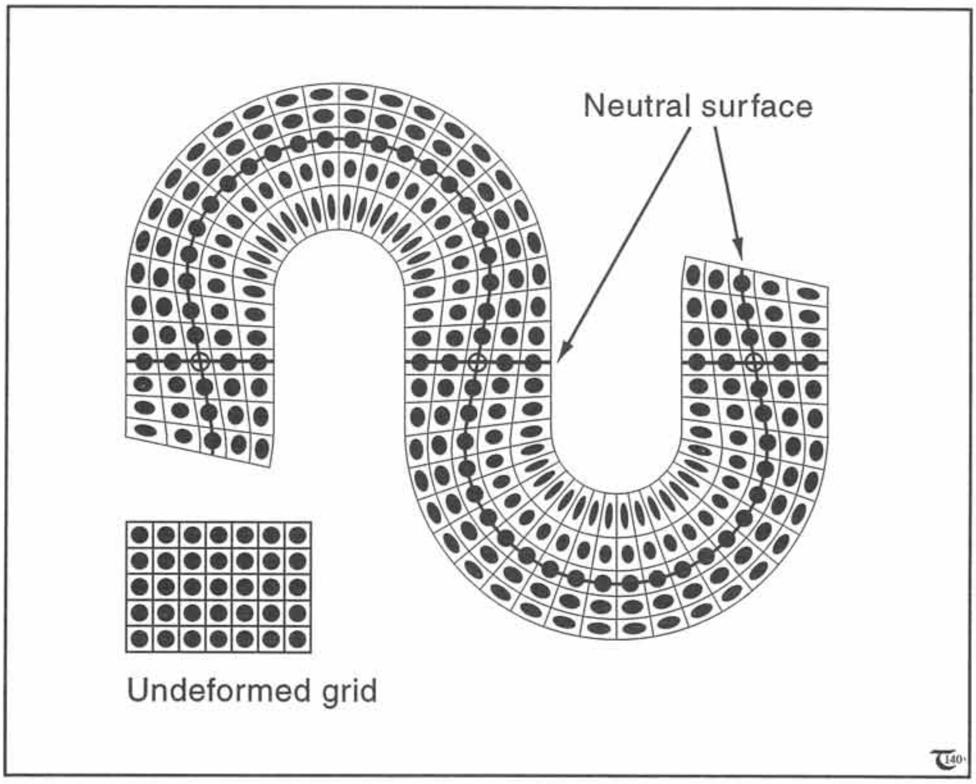

Figure 11-4: Example of spatial variations of finite principal strains in a section normal to the axial planes of buckle folds.

It is important to realize that the strain tensor is valid only for an infinitesimally small point. The state of strain at that point may be finite and, thus, quite large. If the deformation involves no gradients and is uniform throughout the rock volume studied, the infinitesimal strain tensor is, also, valid for finite volumes. However, even in such simple cases, to obtain the strain ellipsoid representation for the state of strain in that deformed rock volume is not straightforward. For example, the finite strain ellipsoid may result from deformation, involving not only the elongation of lines, but, also, distortional rotation of the angles between them. Without knowledge of the rotation tensor, no finite strain ellipsoid can be calculated from the strain tensor alone (sections 12-8, 13-5, and 13-6).

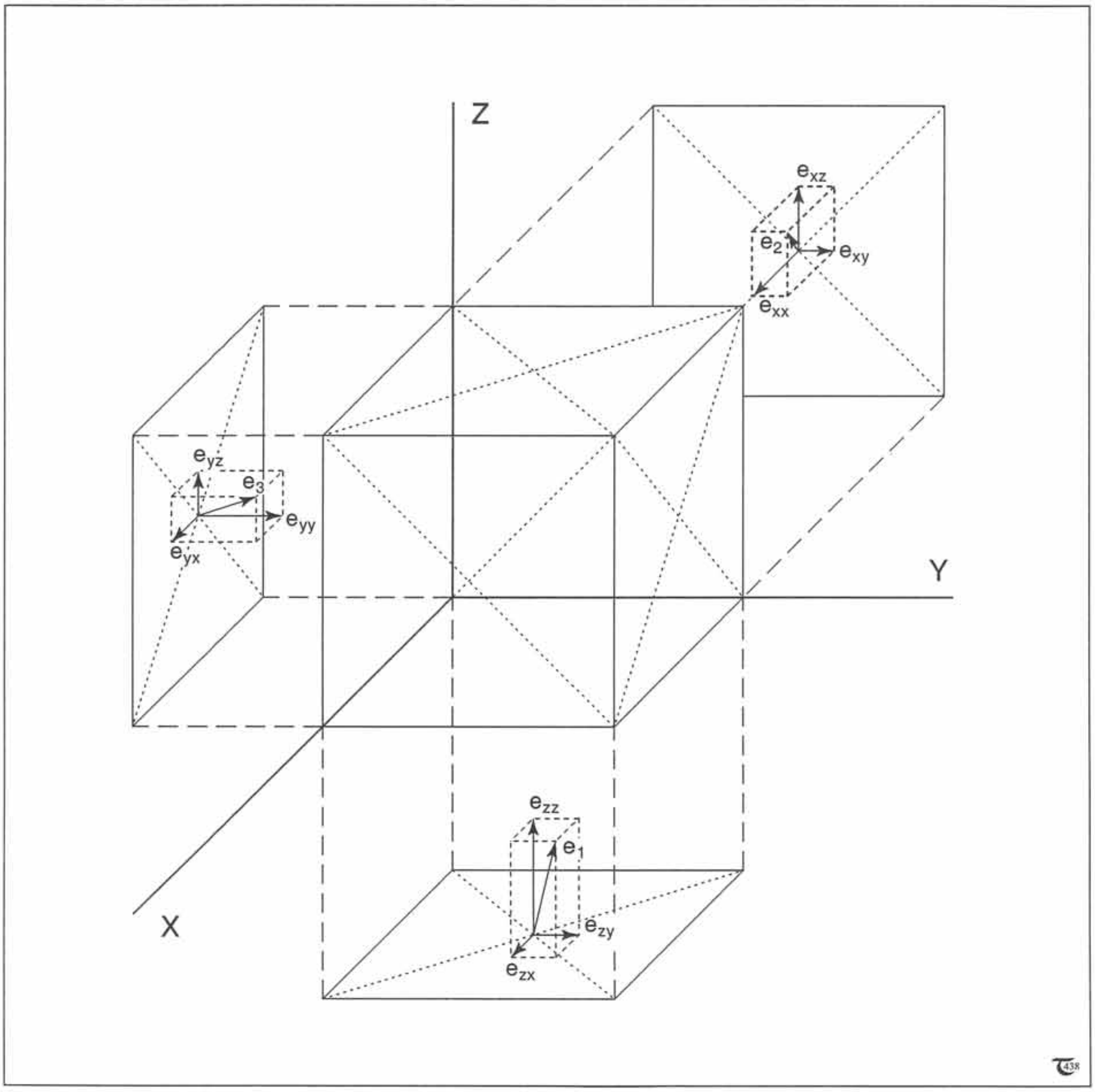

Figure 11-5a: *Projection of the three principal elongations (e_1, e_2, and e_3) into nine components, corresponding to the elements of the strain tensor, using alphabetical indices in XYZ-space.*

If the state of strain at only one point needs to be represented, it is advisable to choose the coordinate axes such that these coincide with the principal strain directions. If the orientation of the coordinate axes is further selected such that principal elongations, e_1, e_2, and e_3, are parallel to the X-, Y-, and Z-axes in this order, then the simplest possible form of the strain tensor is the following matrix (Fig. 11-6):

$$\begin{bmatrix} e_1 & 0 & 0 \\ 0 & e_2 & 0 \\ 0 & 0 & e_3 \end{bmatrix} \tag{11-9}$$

with the only non-zero elements $e_{xx}=e_1$, $e_{yy}=e_2$, and $e_{zz}=e_3$. For this particular case, the principal stretches of the finite strain ellipsoid follow directly from $S_1=1+e_1$, etc., provided the deformation is non-rotational (see section 12-1).

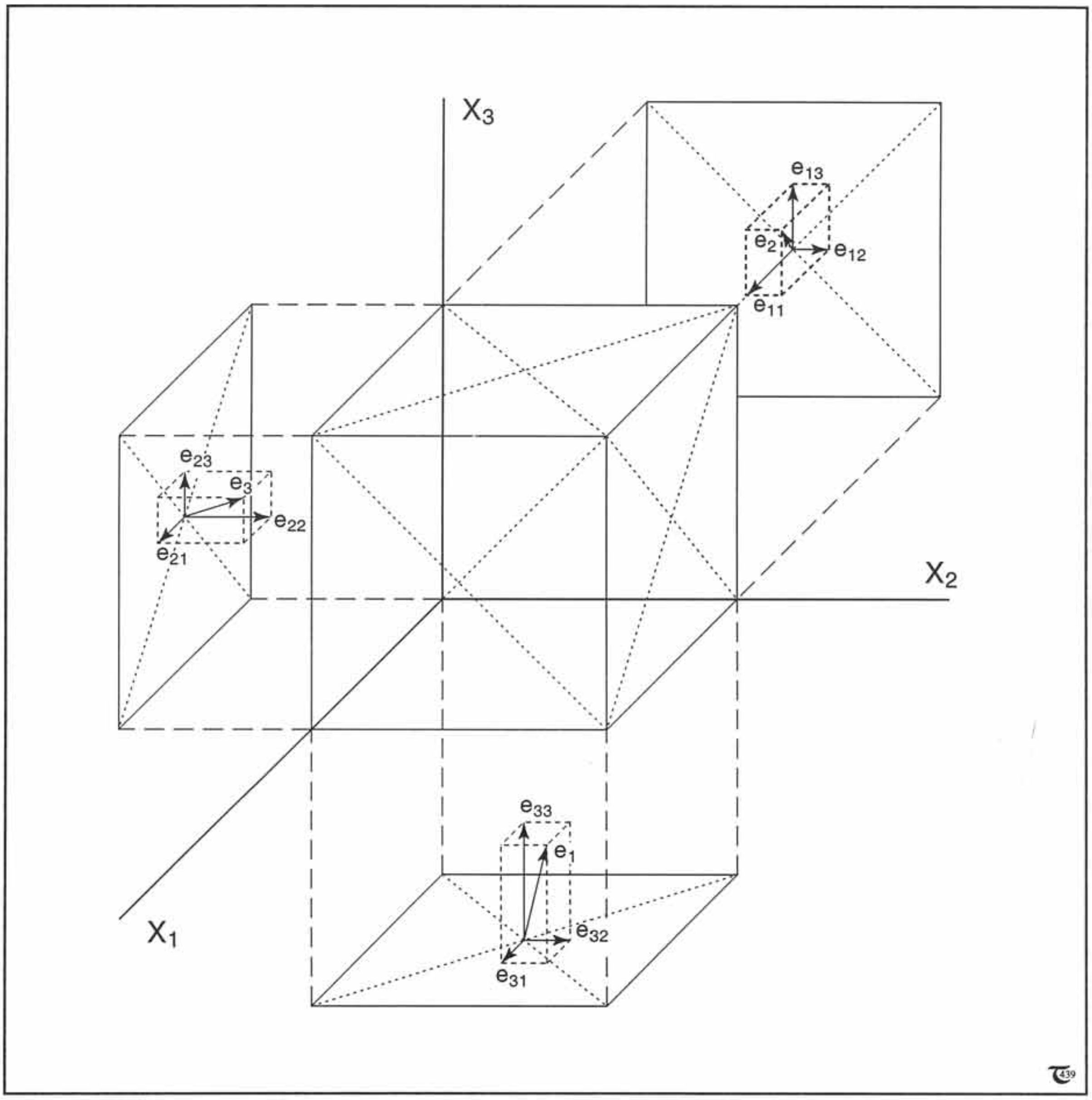

Figure 11-5b: *Projection of the principal elongations (e_1, e_2, and e_3) into nine components, each described by strain tensor elements in numerical indices of $X_1X_2X_3$-space.*

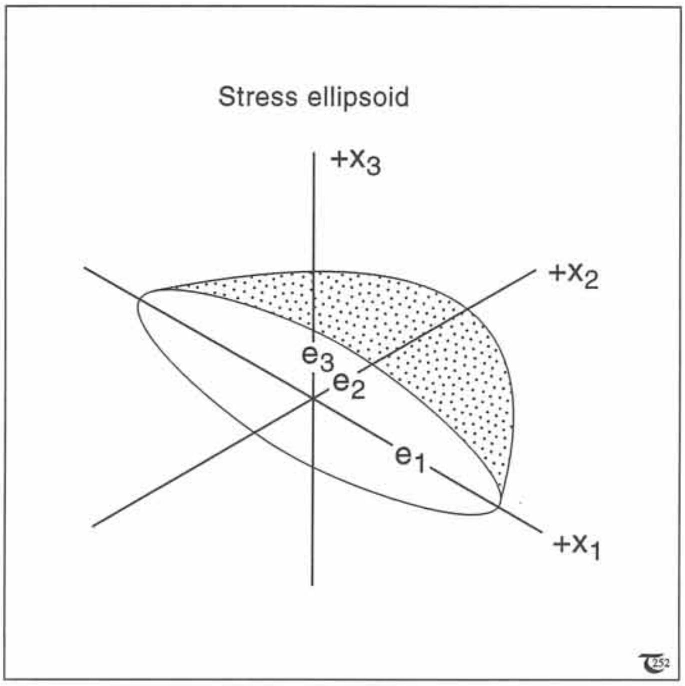

Figure 11-6: Coincidence of coordinate axes and principal elongations results in strain tensor of the simplest format.

For a strain state with the principal strain axes in an arbitrary orientation with respect to the coordinate axes, there generally are nine non-zero elements in the strain tensor. However, of these nine elements only six are independent because of certain equalities among the shear components, connected to the force balance requirement. It follows that $e_{xy}=e_{yx}$, $e_{xz}=e_{zx}$, and $e_{yz}=e_{zy}$ so that the strain tensor is *symmetric* about its trace elements e_{kk}. Also, the shear strains, e_{ij}, in the strain tensor can be expressed in terms of the engineering strain, γ (see section 12-4), rather than the elongation, using the following identity: $\gamma_{ij}=2e_{ij}$, which is valid only if i is unequal to j.

The formulae in sections 10-3, 10-7, and 10-10 for stress can be transcribed to elongation units for calulating the components of shear and normal strain in any arbitrary plane. The elongation, e_L, of an arbitrary line with original direction cosines (l,m,n) is given by:

$$e_L = l^2 e_{11} + m^2 e_{22} + n^2 e_{33} + (lm/2)e_{12} + (mn/2)e_{23} + (nl/2)e_{31} \qquad (11\text{-}10)$$

The transformation formula of section 10-9 applies, also, to strain tensors and all other expressions pertaining to stress in three dimensions can be transcribed to obtain similar expressions for strain in two dimensions.

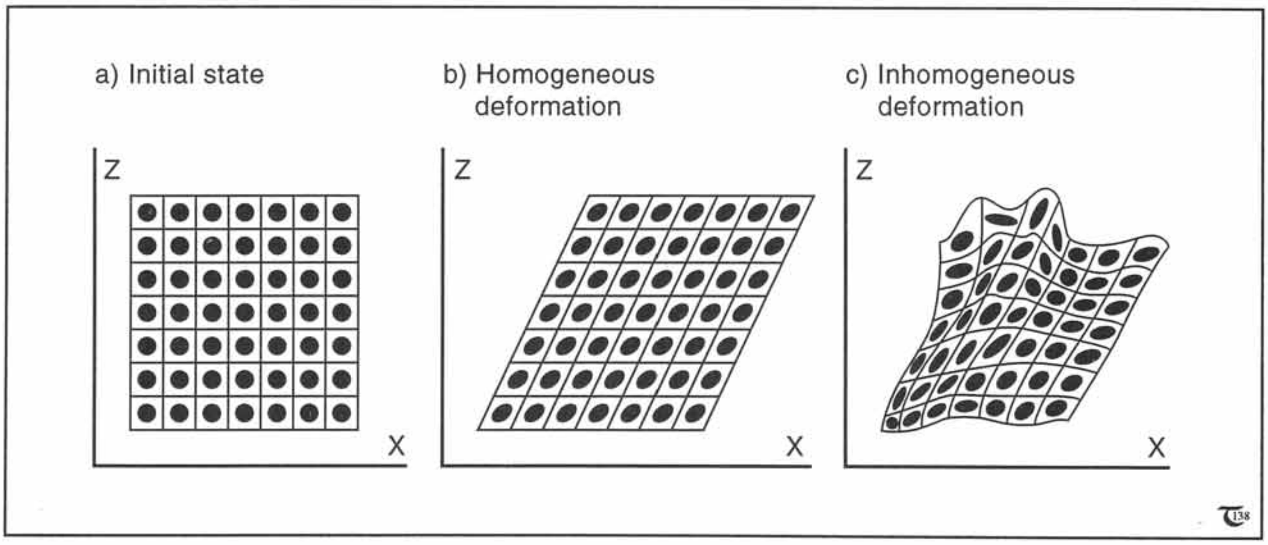

Figure 11-7: a) Initial state of an orthogonal grid. b) Homogeneous deformation of the grid. c) Inhomogeneous deformation of the grid.

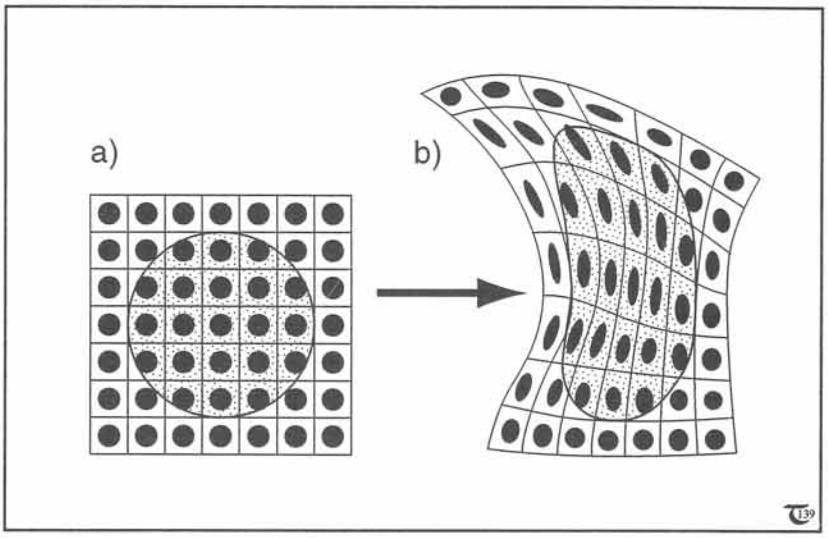

Figure 11-8: *a) & b) Deformation gradients vanish if the scale of observation chosen is sufficiently small. The large circle in (a) does not fit any strain ellipse in (b), due to heterogeneous deformation. But the smaller dots between the grid lines deform into ellipses of approximately homogeneous deformation.*

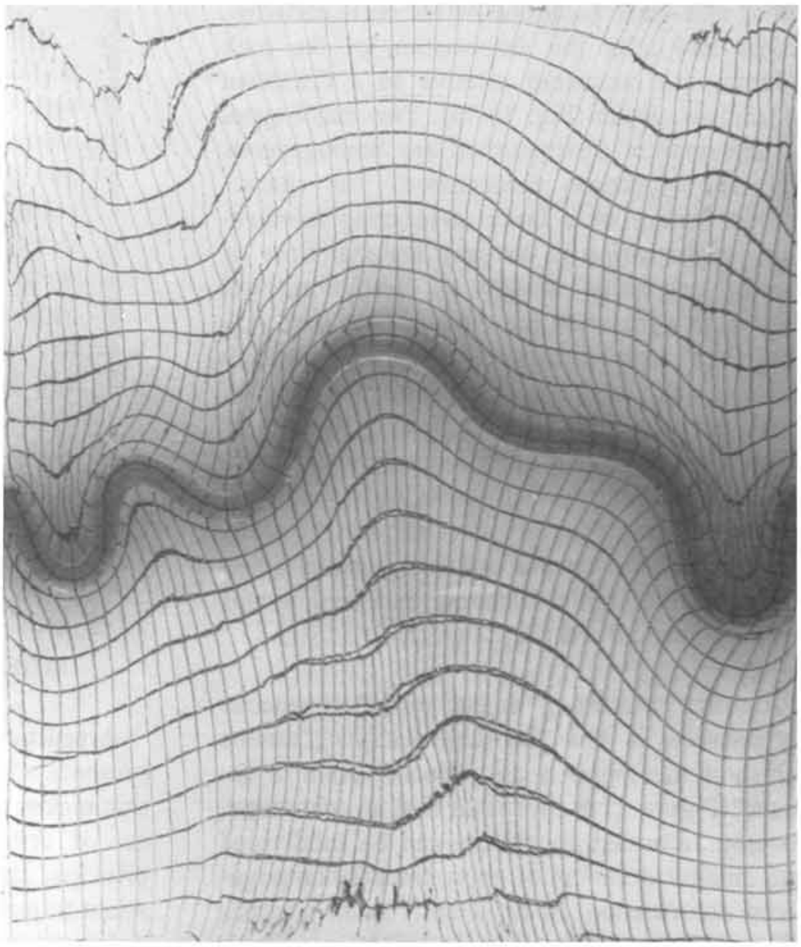

Figure 11-9: *Single layer of high viscosity wax, embedded in lower viscosity wax folds upon layer-parallel-shortening in pure shear box experiment. Large deformation gradients at the scale of the box vanish at the scale of the finite elements, outlined by the grid lines.*

☐Exercise 11-4: Show that the maximum shear strain is given by $(e_1-e_3)/2$. What is the direction of the material planes in which the shear strain is maximum?

11-4 (In)homogeneous and (in)finitesimal deformation

The study of coherent, non-faulted deformation structures investigates the difference in geometry, position, and orientation of markers, visible in the *deformed* state and known in the *undeformed* state. The measures employed to quantify the degree of deformation can be defined purely on the basis of changes in the particle positions. The initial and final positions of all rock particles, before and after the deformation of the rock volume, are compared relative to a Cartesian coordinate system (Fig. 11-7a). Two major types of deformation distinguished are homogeneous and inhomogeneous deformation. The characteristic feature of a body undergoing *homogeneous deformation* is that any material lines or planes, originally straight by definition, will remain straight after the deformation (Fig. 11-7b). Deformation experienced by every unit volume, small or large, is the same, and the rock contains a constant deformation gradient. Of course, it is more natural to encounter deformed rocks, comprising spatial variations in the deformation gradient. One characteristic feature of the resulting *inhomogeneous or heterogeneous deformation* is that initial material lines and planes have become curved (Fig. 11-7c).

Another interesting aspect of heterogeneous deformation is that the deformation gradients vanish if the length scale of observation is reduced. Figure 11-8 illustrates that an inhomogeneously deformed continuum can be subdivided into smaller domains of approximately homogeneous deformation. In sufficiently small volumes of a continuum, inhomogeneous deformation is not possible, and so-called *infinitesimal* deformation (of infinitesimal or small volumes) is implicitly homogeneous. In nature, finite deformation (of finite, or by comparison, large defined volumes) is mostly inhomogeneous, but smaller domains of homogeneous deformation still allow a quantitative analysis. For example, the matrix around a concentrically folded competent layer can be subdivided into unit volumes of approximately homogeneous deformation (Fig. 11-9).

☐Exercise 11-5: Demonstrate by illustration that lines in the same orientation, suffering different elongations, must represent an inhomogeneous deformation.

☐Exercise 11-6: Figure 11-10a illustrates the undistorted shape of a brachiopod shell. The trace of the passive marker grid, initially square and straight in Figure 11-10a, is distorted I) homogeneously, as portrayed in Figure 11-10b, and II) inhomogeneously, as portrayed in Figure 11-10c. a) Sketch the shape of the brachiopod in the deformed states of Figures 11-10b and 11-10c. b) Also, sketch the approximate strain ellipse shape and orientation in the subareas of approximate homogeneous deformation.

11-5 Heterogeneous deformation and strain trajectories

A practical method to display graphically the presence and intensity of heterogeneous deformation makes use of *strain trajectories*. These are a set of orthogonal gridlines, representing the principal stretch orientation at each point. The strain trajectories for a homogeneous or uniform deformation are an orthogonal set of straight grid lines. In contrast, the strain trajectories for a

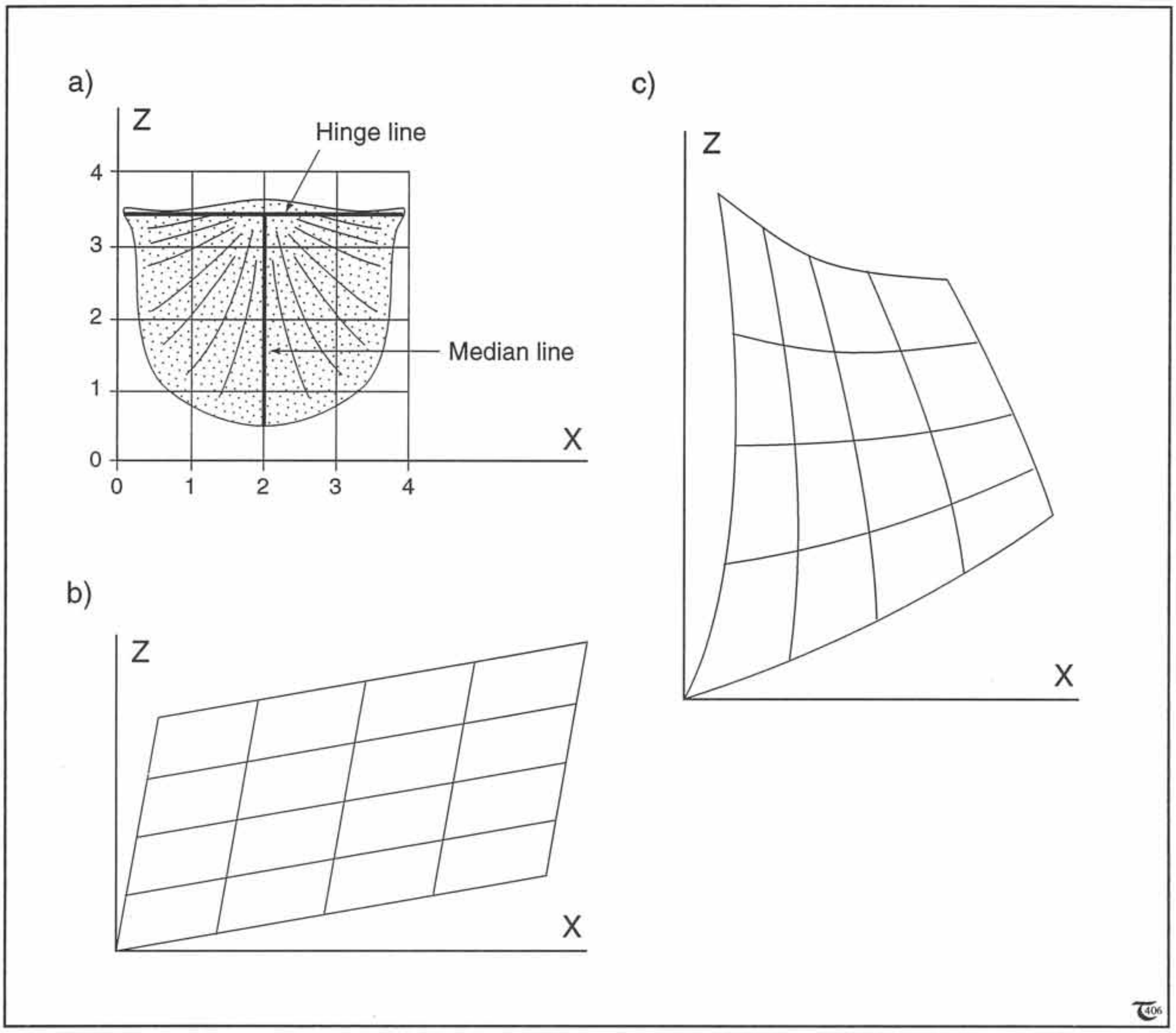

Figure 11-10: *a) to c) Deforming grid with brachiopod shell in initial state. See exercise 11-6.*

heterogeneous deformation is made up of an orthogonal set of curved gridlines. The lateral increase in the amount of angular shear strain is largest where the curvature of the gridlines is greatest. Figure 11-11a gives an example of the principal strain trajectory patterns for a differential simple shear. A particularly interesting trajectory pattern arises in the similar folding by differential simple shear of Figure 11-11b. The S_1 and S_3 trajectories are aligned and amalgamate at either side of *neutral zones*, in which $S_1=S_3=1$. The swap of S_1- and S_3-directions across the neutral zones is characteristic for similar folds.

The differential simple shear is easy to visualize in a model, using a stack of cards, held together in a box with open ends (Fig. 11-12a). Homogeneous or uniform simple shear occurs if all cards are displaced by the same angular shear strain (Fig. 11-12b). Differential or heterogeneous simple shear occurs if the angular shear strain varies across the width of the sheared stack of cards (Fig. 11-12c).

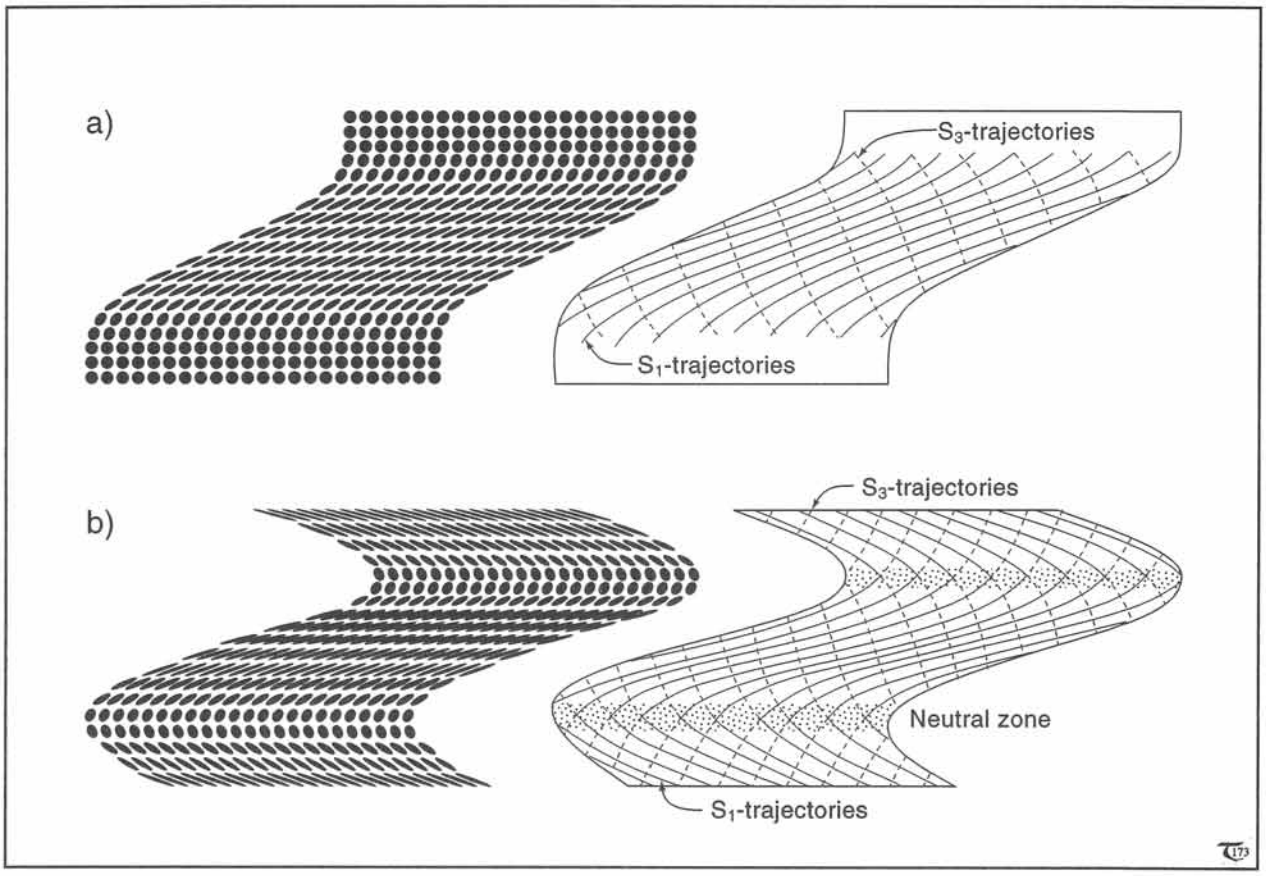

Figure 11-11: a) & b) Two examples of heterogeneous deformation patterns, resulting from differential simple shear. The corresponding strain trajectory patterns are outlined.

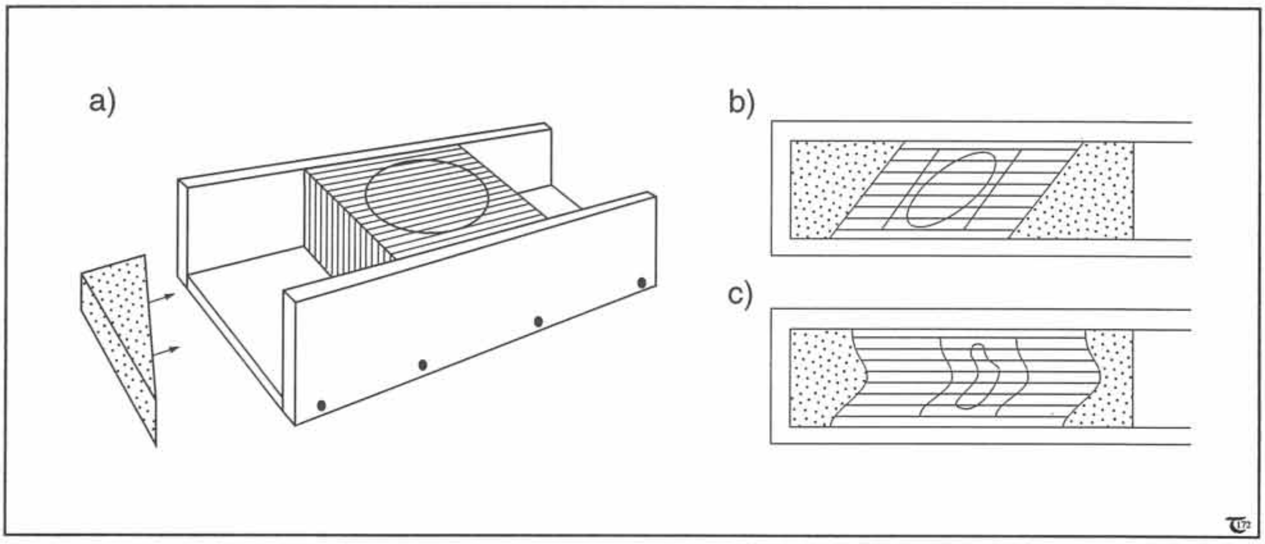

Figure 11-12: a) to c) Mechanical analog, made up of cards and box, for simulating deformation by uniform (b) and differential (c) simple shear.

Figure 11-13: *Left-lateral shear zone in mylonite schist of the upper Seve nappe, Swedish Caledonides, Handöl. Section is normal to the shear plane and near parallel to the movement direction. The brittle fractures postdate the ductile deformation. Courtesy Stefan Bergman.*

□ Exercise 11-7: Figure 11-13 illustrates a ductile shear zone in the upper Seve nappe, Swedish Caledonides. Discuss how the foliation may be related to the strain trajectories.

11-6 Strain tensor invariants

The strain equivalent of the cubic equation of stress (cf., eqs. 10-14 and 10-15a & b) is:

$$e_i^3 - e_i^2(e_{xx}+e_{yy}+e_{zz}) + e_i(e_{xx}e_{yy}+e_{yy}e_{zz}+e_{zz}e_{xx}-e_{xy}^2-e_{yz}^2-e_{zx}^2) - (e_{xx}e_{yy}e_{zz}+2e_{xy}e_{yz}e_{zx}-e_{xx}e_{yz}^2-e_{yy}e_{zx}^2-e_{zz}e_{xy}^2)=0 \qquad (11\text{-}11)$$

The cubic equation holds for any coordinate system, which means that for one particular orientation (i.e., when the coordinate axes coincide with the principal axes of strain) all shear strains vanish and the normal strains are equal to e_1, e_2, and e_3. Substitution of these conditions into the strain equivalent of equation (10-14) yields:

$$e_i^3 - e_i^2(e_1+e_2+e_3) + e_i(e_1e_2+e_2e_3+e_3e_1) - e_1e_2e_3 = 0 \qquad (11\text{-}12a)$$

or simply:

$$(e_i-e_1)(e_i-e_2)(e_i-e_3)=0 \qquad (11\text{-}12b)$$

The three invariants of strain, J_1, J_2, and J_3, are:

$$e_i^3 - e_i^2 J_1 + e_i J_2 - J_3 = 0 \qquad (11\text{-}13)$$

The individual invariants in terms of principal strains read:

$$J_1 = e_1 + e_2 + e_3 \quad (11\text{-}14a)$$
$$J_2 = e_1e_2 + e_2e_3 + e_3e_1 \quad (11\text{-}14b)$$
$$J_3 = e_1e_2e_3 \quad (11\text{-}14c)$$

In terms of the strain tensor elements:

$$J_1 = e_{xx} + e_{yy} + e_{zz} \quad (11\text{-}15a)$$
$$J_2 = e_{xx}e_{yy} + e_{yy}e_{zz} + e_{zz}e_{xx} - e_{xy}^2 - e_{yz}^2 - e_{zx}^2 \quad (11\text{-}15b)$$
$$J_3 = e_{xx}e_{yy}e_{zz} + 2e_{xy}e_{yz}e_{zx} - e_{xx}e_{yz}^2 - e_{yy}e_{zx}^2 - e_{zz}e_{xy}^2 \quad (11\text{-}15c)$$

An important aspect of the strain invariants is their physical meaning. For example, $J_1=0$ if there is no volume change involved in the deformation. The second strain invariant is a measure for shear strains. The third strain invariant vanishes if the strain is two-dimensional, i.e., for plane strain.

□Exercise 11-8: Before any ductile or elastic deformation occurs, any pore space is likely to be closed. The porosity of rock is defined as the void space per unit volume. If all the pore space could be closed, by subjecting the rock to a sufficiently large confining pressure, what would be the dilation or volume change (i.e., J_1) for the porosities shown in Figures 11-14a & b?

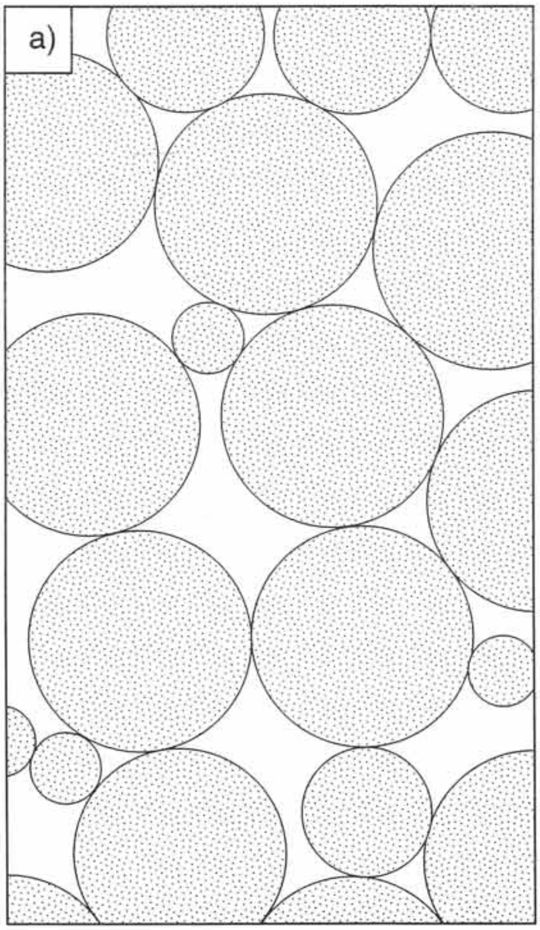

Figure 11-14a: *Section through a sandstone of 32 percent porosity. See exercise 11-8.*

11-7 Relationship between stress and elastic strain tensors

Although strain can be described purely on a phenomenological basis, it must be kept in mind that there are constitutive relationships between stress and strain. For Hookean elastic deformations the relationship is a straightforward, linear one. However, for deformations in ductile media, the stress is proportional to the strain-rate, and calculation of the resulting finite strains is somewhat tedious. The small increments of strain, implied in the strain-rate, must be integrated over time to establish the finite strain. This integration of strain increments can be upgraded into a constitutive expression, monitoring strain evolution over time, according to stress input and effective viscosity. Such advanced procedures are postponed for discussion in chapter thirteen.

The reason why the strain tensor is usually expressed in terms of the elongation, rather than stretches or quadratic elongations, is that a simple relationship exists between the deviatoric stress and strain tensors for an *elastic solid*:

$$\tau_{ij} = 2Ge_{ij} \quad (11\text{-}16a)$$

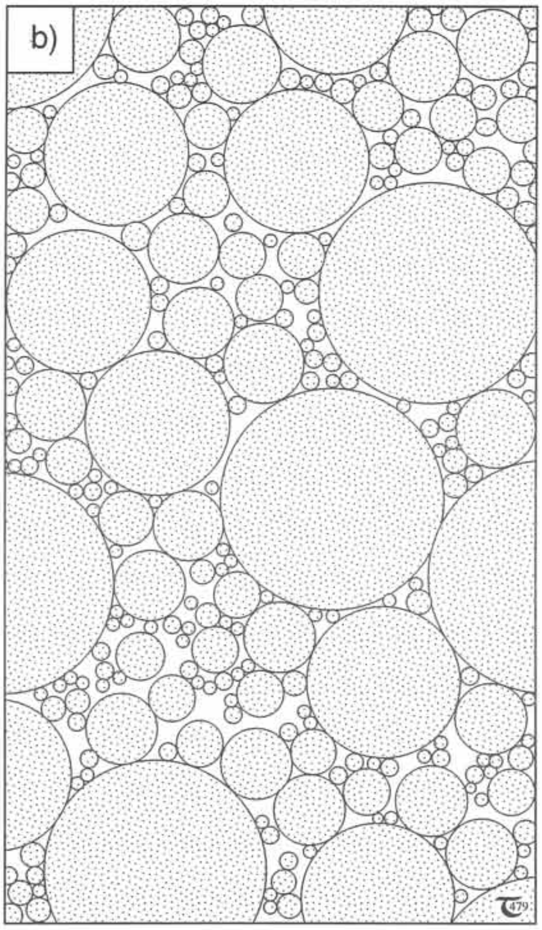

Figure 11-14b: Section through a sandstone of 17 percent porosity. See exercise 11-8.

with elastic shear modulus G. Equation (11-16a) is strictly valid only for incompressible solids; otherwise, the full form is:

$$\tau_{ij}=2Ge_{ij}+\delta_{ij}\lambda e_{kk} \qquad (11\text{-}16b)$$

with *Lamé's constant* defined in equation (5-13). The Lamé constant is infinitely large for *incompressible solids*. For any particular deviatoric stress, the normal strain contribution, e_{kk}, approaches zero if λ approaches infinity, so that for incompressible solids equation (11-16b) reduces to that of (11-16a). Most rocks have Poisson ratios of about 0.25 and, therefore, cannot be considered incompressible: expression (11-16b) is more appropriate than (11-16a).

□Exercise 11-9: Consider the hypothetical case of rock with an elastic elongation in a rock of $e_{11}=0.5$. a) If the shear modulus of the rock is 10 GPa, what would be the corresponding magnitude of the tensor element of deviatoric stress, τ_{11}? b) Why will such high elastic strain never occur in reality? c) Why will such high stress not occur either?

□Exercise 11-10: Write the tensor elements of Lamé's expression of equation (11-16b).

□Exercise 11-11: Consider again a typical shear modulus for rock of 10 GPa, and adopt a characteristic geological stress of 100 MPa. a) What is the maximum shear strain you can find? b) Explain why the contribution of elastic strains to the large deformation patterns, observed in some intensively folded lithologies, is insignificant. c) What is the explanation for the occurrence of non-faulted, high strain, deformation patterns?

11-8 Stress and ductile strain-rate

The relationship between stress and finite strain in viscous creep is not so straightforward as it is for elastic materials. Finite strain can be obtained by integration of the velocity gradient tensor over time. This yields a deformation tensor, which includes information on the finite strain and rotation, so that the principal stretches of a deformed unit sphere can be calculated (see section 12-8). However, somewhat analogous to the relationship between stress and elastic strain, the deviatoric stress in *viscous* materials relates to the strain-rate tensor:

$$\tau_{ij}=2\eta\dot{e}_{ij} \qquad (11\text{-}17a)$$

with the shear viscosity, η, and strain-rate, $\dot{e}$. This expression is strictly valid only for an *incompressible Newtonian body*. If the material behaves as a compressible visco-elastic body, then the full form is:

$$\tau_{ij}=2\eta\dot{e}_{ij}+\delta_{ij}\lambda^{*}\dot{e}_{kk} \qquad (11\text{-}17b)$$

with λ^* the viscous equivalent of Lamé's constant:

$$\lambda^{*}=(2\eta)/(1-2\nu) \qquad (11\text{-}18)$$

Although most rocks are compressible, the elastic strain-rates, $\dot{e}_{kk}$, may contribute to the total strain-rate for only a relatively short time. After elapse of the Maxwell relaxation time, the maximum compression has been achieved, and, the relationship between stress and strain-rate can be effectively described by equation (11-17a).

□Exercise 11-12: Calculate a characteristic or average geological strain-rate, given the fact that crustal stresses are on the order of 100 MPa and viscosities of crustal rocks range between 10^{20} and 10^{24} Pa s.

References

Books

Principles of Mechanics and Dynamics (1962, Dover), by W. Thompson and P.G. Tait (reprint of the 1879 edition). This is an often overlooked classic. Many of the equations have now become widely used through more accessible display in textbooks by J.C. Jaeger and J.G. Ramsay.

Deformation, Strain and Flow (1960, Lewis, London, 347 pages), by M. Reiner. This is an extremely instructive study book by the founder of the study of rheology.

Theory of Flow and Fracture of Solids (1963, McGraw-Hill, New York, 706 pages, two volumes), by A. Nadai. This is one of the most intelligent, erudite, and inspiring texts on the deformation of solids. It is rather mathematical and contains many concepts, which still have to find their way into the Earth Sciences. For example, detailed discussions on the energy balance in large deformations are included.

Mechanics of Incremental Deformations (1965, Wiley, New York, 504 pages), by M. Biot. Maurice Biot has contributed significantly to the understanding of folding in rocks, mainly drawing on the theory of elastic buckling. This book displays his profound understanding of deformation theory from the point of view of a mechanical engineer.

Folding and Fracturing of Rocks (1967, McGraw-Hill, 568 pages), by J.G. Ramsay. A classical text for structural geologists, interested in the detailed study of deformation patterns in rocks.

Elasticity, Fracture and Flow (1971, Methuen, 3rd edition), by J.C. Jaeger. This book contains many useful equations, primarily pertinent to small, elastic deformations and intended for mechanical engineers.

The Kinematics of Mixing: Stretching, Chaos, and Transport (1989, Cambridge University Press), by Julio Ottino. This is an inspiring account on mixing processes and fluid deformation. This text is aimed at chemical engineers, and only relatively few earth scientists will have the stamina to work their way through the mathematical equations. But it provides an excellent state-of-the-art account in the field and includes much experimental data.