Chapter 13: Particle Movements and Stream Functions

THE DEFORMATION of crystalline rocks by ductile creep involves particle movements, governed by the laws of laminar flow, according to the concepts of fluid mechanics. Consequently, stream functions may be applied to study the development of ductile deformation patterns in rocks. The continuum assumption is essential in this approach, as well as the exclusion of any inertia effects. Such inertia would lead to instability of the laminar flow patterns assumed. Basic examples discussed are rigid-body translation, pure shear, and composite flows, due to simultaneous pure and simple shear.

Contents: The physical meaning of streamlines, flownets, and streamtubes is explained in section 13-1. The stream function and the associated complex potential can be derived from the velocity field of any particular flow, as demonstrated in sections 13-2 and 13-3. The summation rule for stream functions is discussed in section 13-4. This yields a general stream function for flow within a ductile deformation zone, deforming by homogeneous plane strain, including superposed pure and simple shear components. This stream function is subsequently used to model finite deformation patterns analytically in section 13-5. The components of the velocity-gradient tensor are outlined in section 13-6.

Practical hint: **This chapter outlines how the deformation tensor can be obtained from the stream function. Prepare a seminar discussing the application of this approach to the specific examples of thermal convection (McKenzie, 1979, *GJRAS*, volume 58, pages 689 to 715) and Stokes flow (Schmeling and others, 1988, *Tectonophysics*, volume 149, pages 17 to 34).**

13-1 Particle movement paths and streamlines

The deformation history of a rock volume is fully known if the positions of any arbitrary rock particles can be traced before, during, and after the deformation. The paths, followed by such particles are known as *particle movement paths*, *streamlines*, or *flowlines*. In 2D projections, any flowline represents a surface normal to the plane of view, across which material transport does not occur. The flowline, thus, is part of an imaginary, impermeable surface.

Although the movement path of particles may differ from streamlines in turbulent flows of low-viscosity fluids, the situation remains simple in rock deformation. This is because any inertia effects are completely dampened by the high viscosity of rocks. This means that rock movement commonly ceases instantaneously when the driving force is taken away, in contrast to motions in low-viscosity fluids, where inertia is very important. Consequently, particle movement paths and streamlines always coincide in the flow of rock matter (Fig. 13-1).

Figure 13-2 is a complete *flownet* for the simple case of a rigid-body translation, where all particles are displaced at a constant rate, v. Streamlines are everywhere tangential or parallel to the velocity vectors, in this case forming a set of parallel lines. The spacing between streamlines may be arbitrary; the area between two adjacent flowlines is known as a *stream tube* or *flow tube*. If the flowlines are equally spaced and scaled as in Figure 13-2, the flux of rock matter, moving through each stream tube, is the same.

Figure 13-1: *Creeping flow in Alaskan glacier, visualized by dark moraine debris, which is carried downstream along the flowlines.*

The flownet of Figure 13-2 is made up of streamlines and a perpendicular set of so-called *equipotential lines*. Each potential line or surface represents an imaginary membrane, across which matter moves. Physically, a single potential line is a surface, for which the mechanical energy required to pass any unit volume of rock is everywhere constant.

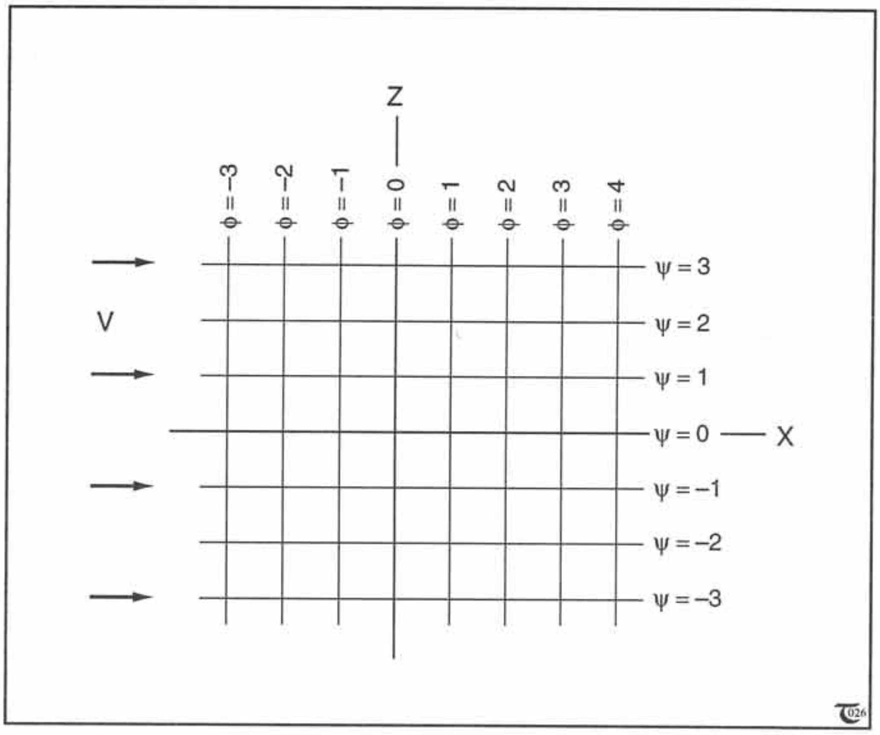

Figure 13-2: *Flownet for rigid body translation at uniform velocity, v. Indicated are streamlines, ψ, and (equi)potential lines, ϕ. See exercise 13-1.*

13-2 Stream functions

The streamlines for a particular flow or deformation can be visualized experimentally by tracing the movement of marked particles. The pattern of streamlines can, also, be described mathematically by a so-called stream function, ψ, which then defines the spatial velocity field:

$$v_x = \partial\psi/\partial z \qquad (13\text{-}1a)$$
$$v_z = -\partial\psi/\partial x \qquad (13\text{-}1b)$$

Each particular flow needs to be described by a different stream function and is valid only for the specific coordinate system, used to describe that flow. Of course, the coordinate system should be chosen such that the resulting stream function is the simplest one possible for the particular flow studied. For example, the uniform translation within the XZ-plane of Figure 13-2 may be described by the following stream function:

$$\psi = vz \qquad (13\text{-}2)$$

It follows from expressions (13-1a & b) that, for this rigid-body translation, $v_x=v$ and $v_z=0$.

The stream function has a number of interesting properties. For example, for low-inertia or laminar flows any stream function must satisfy the biharmonic equation:

$$\nabla^4\psi=0 \qquad (13\text{-}3)$$

Without further proof here, it is emphasized that solutions of the biharmonic equation automatically satisfy the requirement of balance of forces. In other words, any solution of the biharmonic equation represents some type of flow. However, in practice a stream function is determined from a known flow field, rather than vice versa, and can be recovered by integrating the velocity components, according to:

$$\psi=\int(\partial\psi/\partial x)dx+\int(\partial\psi/\partial z)dz+c= \int v_x dz+\int v_z dx+c \qquad (13\text{-}4)$$

□Exercise 13-1: The velocity field for a pure shear flow is given in Figure 13-3a. The velocity field must not be confused with the flowlines, shown in Figure 13-3b. Study the two diagrams, and answer the following questions. a) How do the streamlines and the velocity vectors relate? b) Identify, in Figure 13-3b, two flow tubes with the fastest and the slowest flux rate, respectively. c) Note that the velocity gradient in Figure 13-2 is zero - no deformation occurs. Draw the progressive deformation, caused by the flow of Figure 13-3b, starting with a unit cube. d) Complete the flownet of Figure 13-3b by adding the equipotential lines.

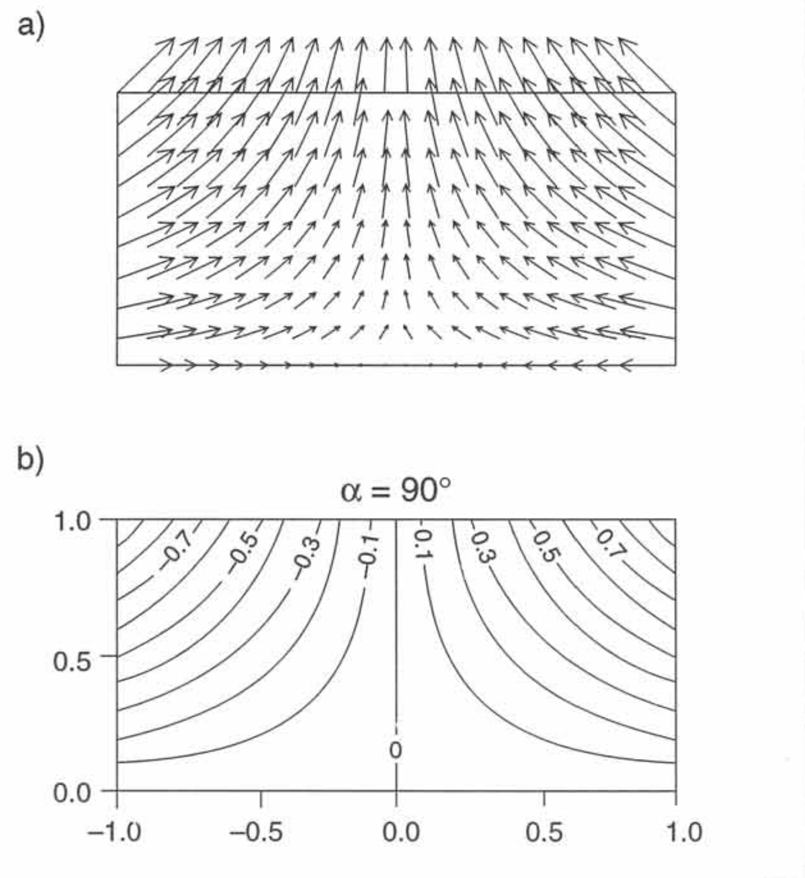

Figure 13-3: *a) & b) Velocity field and flowlines for a pure shear deformation, involving horizontal shortening and vertical extension. The non-dimensional flux of streamlines in (b) is with respect to the vertical reference streamline of zero flux. The box dimensions are normalized to unity. See exercise 13-2. (The streamlines are described by the stream function of equation (13-15), using α=90°.)*

For example, the progressive deformation by simple shear of Figure 13-4a is governed by the velocity field and streamlines, shown in Figures 13-4b & c. The velocity profile is given by $v_x = \dot{e}_{xz} z$ and $v_z = 0$, with tensor shear strain-rate $\dot{e}_{xz}$. The stream function, according to equation (13-4), for the simple shear is:

$$\psi = \dot{e}_{xz} z^2 \qquad (13\text{-}5)$$

using the boundary condition that $\psi_1 = 0$ at $z = 0$ to eliminate c. The stream function is expressed in units of flux [m^2s^{-1}] for fluid flow or, more practically, in units of [m^2Ma^{-1}] for rock transport. Remember that a stream tube constrains

□Exercise 13-2: The pure shear flow of Figures 13-3a & b can be described by the stream function, $\psi=\dot{e}_1 xz$, scaled by a principal strain rate, $\dot{e}_1=\dot{e}_{xx}$. a) Determine the expressions describing the velocity vector components in the X- and Z-directions. b) Draw the streamline(s), $\psi=0$. c) Determine the value of the shear strain-rate elements of the strain-rate tensor, using equation (13-6b), adopting a coordinate system, as implied by the above stream function. d) The streamlines of Figure 13-3b are labelled by fractional numbers with the top corners of the box of unit length coinciding with the streamlines of unity flux. Explain what information exactly is conveyed by the other numbers shown.

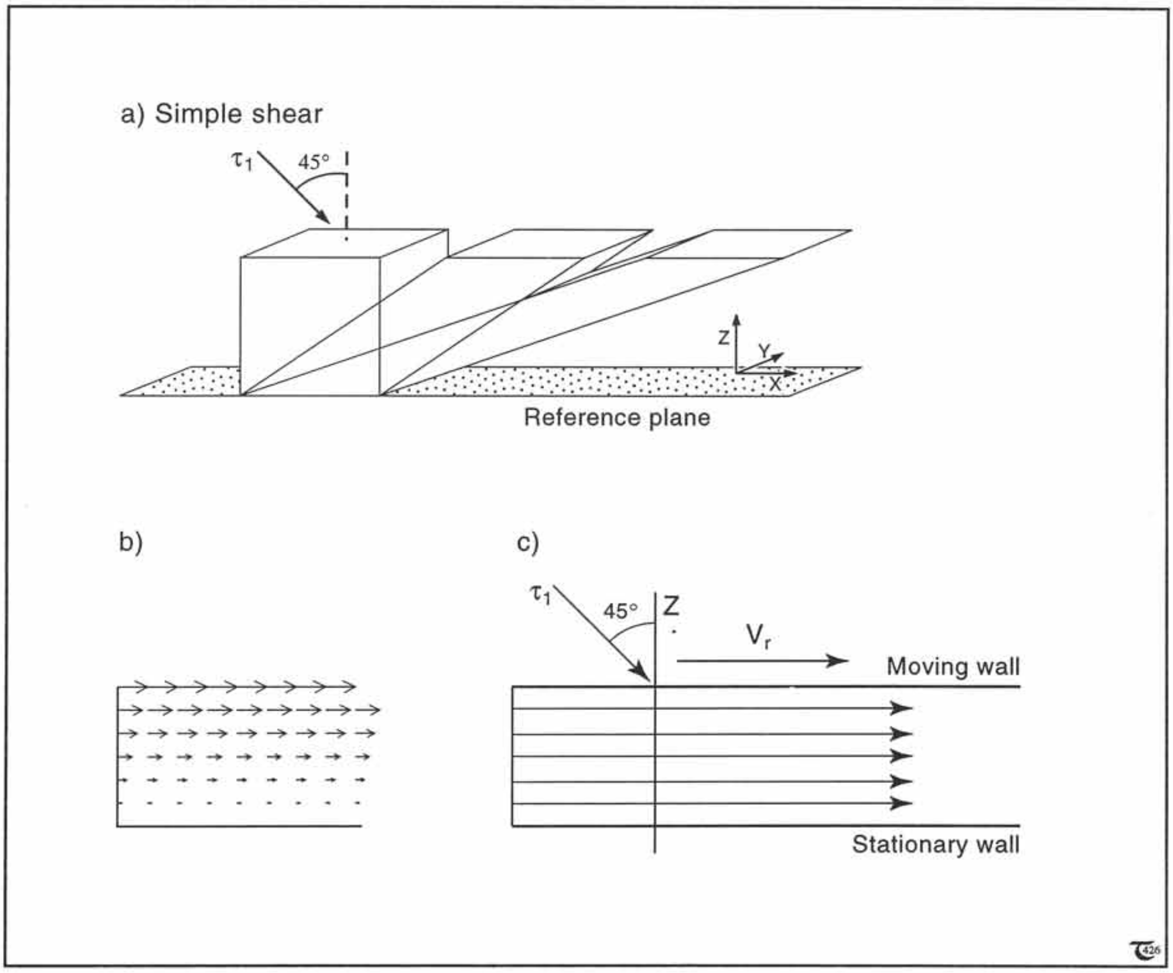

Figure 13-4: a) Progressive simple shear deformation, b) velocity field, and c) streamlines for simple shear. The major principal stress, τ_1, is consistently at 45° to the direction of flow.

material flow at a constant flux, hence the unit of $[m^2s^{-1}]$, for the flow is analyzed in 2D. Units would be $[m^3s^{-1}]$ if the unit length in the third dimension, perpendicular to the plane of flow, were to be included, but this is impractical for the two-dimensional streamline approach, adopted here.

The elements of the strain-rate tensor can also be obtained from a stream function according to:

$$\dot{e}_{xx} = \partial^2\psi/\partial x \partial z \quad (13\text{-}6a)$$
$$\dot{e}_{xz} = (1/2)[\partial^2\psi/\partial z^2 - \partial^2\psi/\partial x^2] \quad (13\text{-}6b)$$

Finally, ψ, equated to a constant flux, maps the trace of a particular streamline. Shown in Figure 13-2 are streamlines at constant intervals of 1 m^2 Ma^{-1}, assuming that the translation of the rock mass occurs at a uniform velocity of 1 m Ma^{-1}.

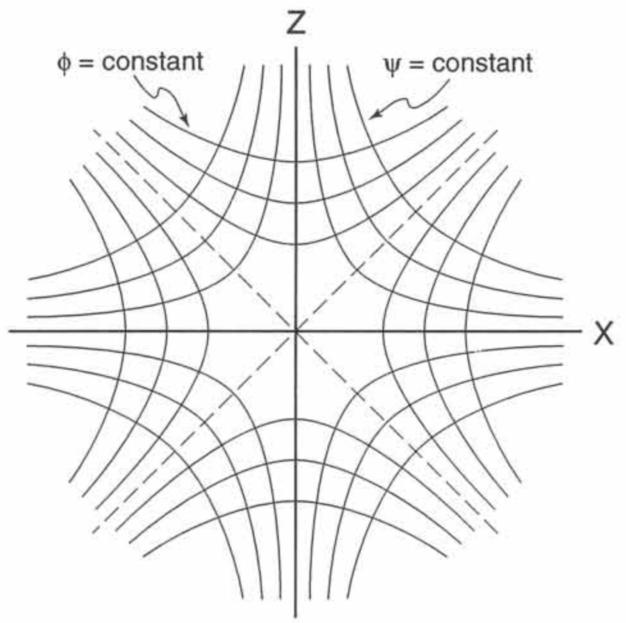

Figure 13-5: Flownet for pure shear deformation with streamlines, ψ, and potential lines, ϕ. See exercise 13-3.

13-3 Potential function and complex potentials

The flownet of Figure 13-2 represents a set of parallel lines - solutions of the stream function (ψ=constant) - and an orthogonal set of potential lines (Φ=constant). The stream function and the velocity potential are related by the Cauchy-Riemann equations:

$$\partial\Phi/\partial x = +\partial\psi/\partial z \quad (13\text{-}7a)$$
$$\partial\psi/\partial x = -\partial\Phi/\partial z \quad (13\text{-}7b)$$

The potential function, Φ, can, therefore, be derived by integration if ψ is known, and vice versa.

Because of the properties of streamlines and potential lines, flownet construction for geological deformations is subject to the following rules: (1) Flowlines and equipotential lines must remain orthogonal everywhere. (2) Equipotential lines must meet material boundaries, across which no flow occurs, at right angles. (3) Flownets form patterns of smooth curves in rocks of isotropic viscosity. (4) Refraction of flownets occurs across geological boundaries only if these represent a rheological boundary. These properties are very useful, because applying

☐Exercise 13-3: The flownet for a pure shear deformation is given in Figure 13-5. Use the rules of flownets to argue that the X and Z axes of the flownet in Figure 13-5 must coincide with, or be parallel to, material planes, across which no rock matter may move.

them aids in sketching the appropriate pattern of streamlines for a particular flow prior to any calculations or measurements.

It is common practice in fluid mechanics to represent a flow field by a so-called complex potential function, $W(\zeta)$. Two mathematical solutions of this complex function may be obtained by substituting the complex variable, $\zeta = x + iz$. The real part gives the potential function, and the imaginary part yields the stream function for that same flow. The usefulness of the complex potential function is that it allows concise formulations, which comprise all the information of a flow. Examples of complex potentials are:

a) Rigid-body translation:

$$W(\zeta) = -v\zeta \quad (13\text{-}8a)$$

$$W(x+iz) = -(vx+ivz) \quad (13\text{-}8b)$$

so that $\Phi = -vx$ and $\psi = vz$.

b) Pure shear deformation:

$$W(\zeta) = (\dot{e}_1/2)\zeta^2 \quad (13\text{-}9a)$$

$$W(x+iz) = (\dot{e}_1/2)(x+iz)^2 = (\dot{e}_1/2)(x^2-z^2) + i\dot{e}_1 xz \quad (13\text{-}9b)$$

so that $\Phi = (\dot{e}_1/2)(x^2-z^2)$ and $\psi = \dot{e}_1 xz$.

□Exercise 13-4: Demonstrate that the set of stippled lines in Figure 13-5 represent solutions of the potential function for $\Phi = 0$.

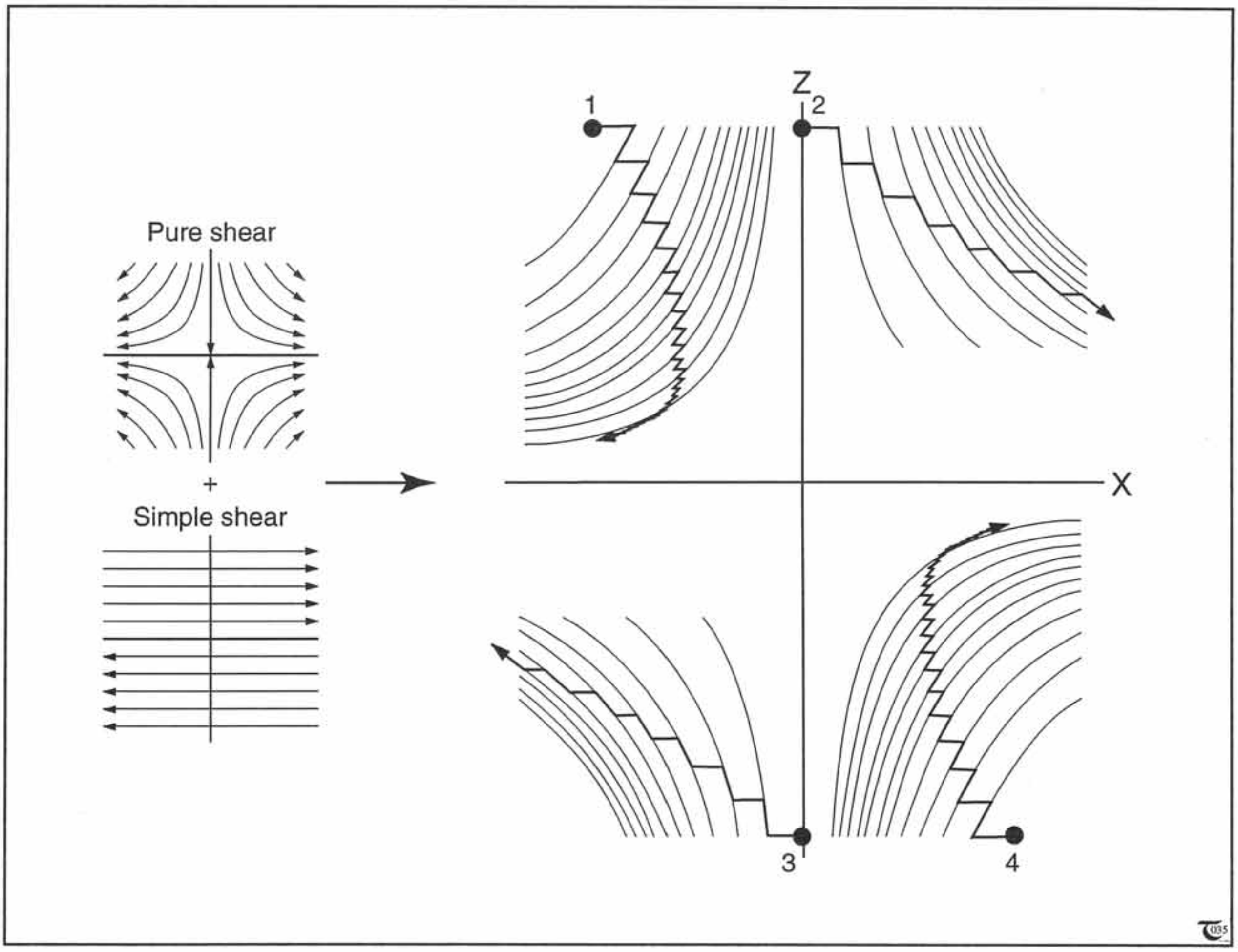

Figure 13-6: *The motion of particles, labelled 1 to 4, resulting from simultaneously operating pure and simple shear flows, is graphically integrated to show the effect of the stream function summation rule.*

13-4 Stream function summation rule

Many distortions, caused by the creeping movement of rock matter, are the result of so-called composite flows. That is, it is possible to distinguish two or more components of the velocity field that may be integrated separately to find the stream functions, ψ_a and ψ_b. Each of these stream functions then describes only one particular component of the flow. For example, one way of looking at progressive homogeneous deformations is to consider them as composite flows, resulting from the simultaneous superposition of pure and simple shear components of various strength. Figure 13-6 illustrates the particle movement paths for such a general composite flow. The movement of the particles, labelled 1 to 4, results from a flow, which comprises both pure and simple shear components, acting simultaneously. It should be emphasized that this separation of pure and simple shear components of the flow is purely artificial. The two flows are non-commutative, as they must occur in an integrated fashion, coevally, as graphically portrayed by the incremental superposition in Figure 13-6, to lead to the particle movement paths outlined.

Although the various flow components of a composite flow are not commutative, the stream functions, describing each of the flow contributions, are. For example, the respective stream functions, for the flow contributions in the reference frame of Figure 13-6, are:

$$\psi_{\text{pure shear}} = \dot{e}_{xx} xz \qquad (13\text{-}10a)$$
$$\psi_{\text{simple shear}} = \dot{e}_{xz} z^2 \qquad (13\text{-}10b) = (13\text{-}5)$$

Each stream function, for incompressible flow, may be added to another stream function to find the total stream function, governing the composite flow. This is valid, because the velocity components at any one point are vector quantities, which are commutative. Two components of a velocity in the X-direction, v_{x1} and v_{x2}, lead to a total velocity, $v_x = v_{x1} + v_{x2}$. The stream functions add up similarly: $\partial\psi_1/\partial z + \partial\psi_2/\partial z = \partial\psi/\partial z$. The same applies to the biharmonic function to ensure fulfillment of the force-balance equation, $\nabla^4\psi_1 + \nabla^4\psi_2 = \nabla^4\psi$.

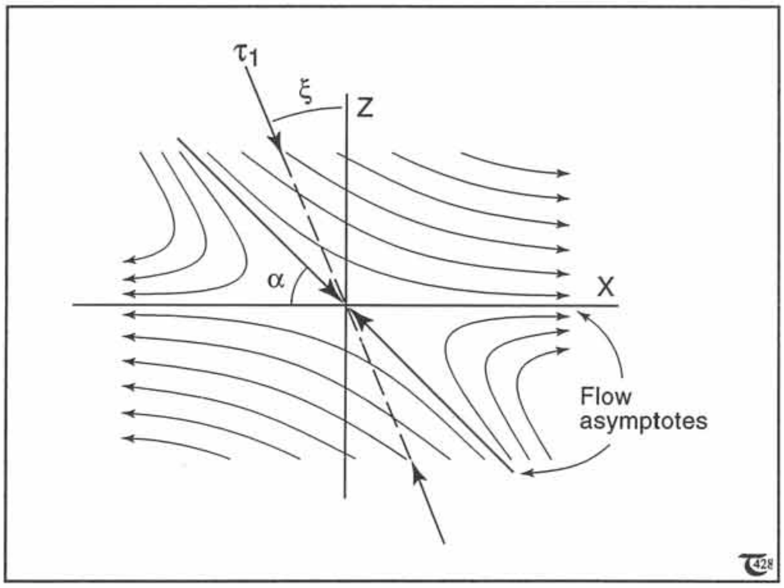

Figure 13-7: Arbitrary 2D flow pattern for plane strain, determined by the angle, α, measured between the two flow asymptotes. The orientation of the major principal strain-rate axis, $\dot{e}_1$, is related to α in equation (13-14).

The stream function for the composite flow of Figure 13-6 is:

$$\psi = \dot{e}_{xx} xz + \dot{e}_{xz} z^2 \qquad (13\text{-}11)$$

The relative importance of the pure and simple shear contributions is controlled by the relative magnitude of the tensor components for the normal strain-rate, $\dot{e}_{xx}$, and the shear strain-rate, $\dot{e}_{xz}$. Expression (13-11) can be reformulated in terms of a principal strain-rate, $\dot{e}_1$, by using the Mohr equations:

$$\dot{e}_{xx} = \dot{e}_1 \cos 2\xi \qquad (13\text{-}12a)$$
$$\dot{e}_{xz} = \dot{e}_1 \sin 2\xi \qquad (13\text{-}12b)$$

Substitution of expressions (13-12a & b) into (13-11) yields a general stream function:

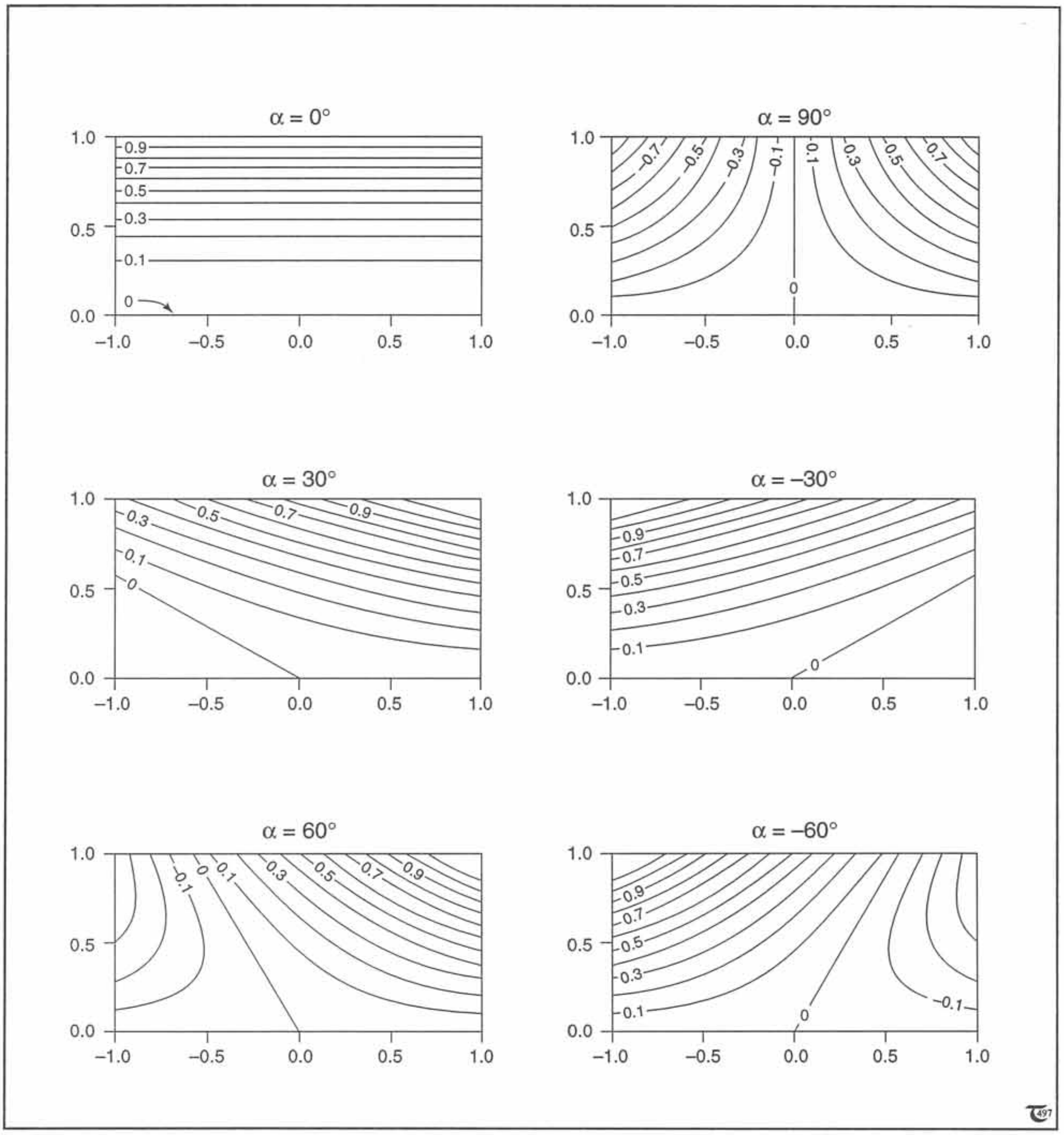

Figure 13-8: *Streamline patterns for a range of 2D-flows, restricted to deformations by plane strain. These flow patterns are governed by equation (13-15), using normalized strain rates and flow asymptote angles, α, as indicated.*

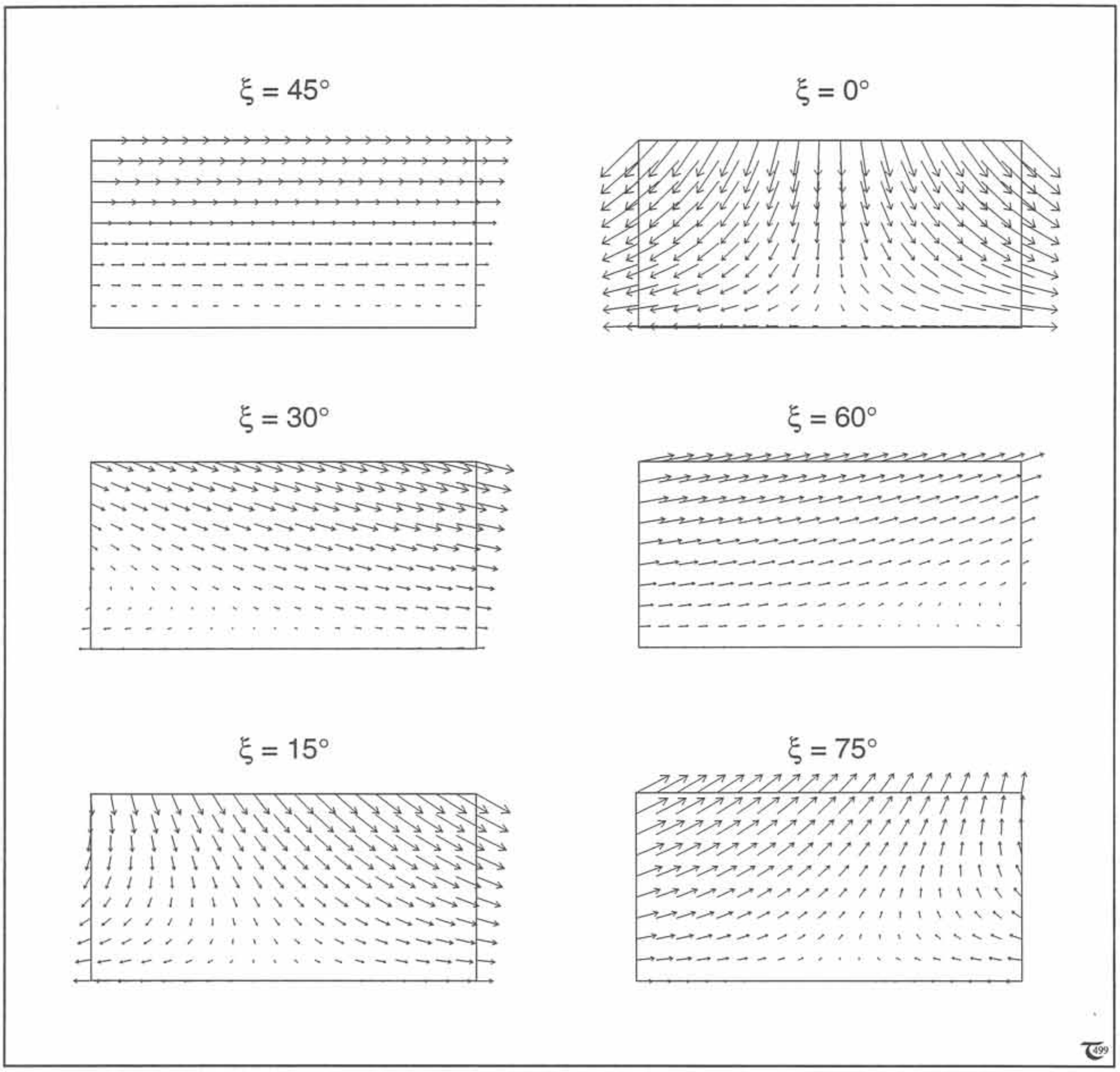

Figure 13-9: *Velocity fields, corresponding to the flows illustrated in Figure 13-8.*

$$\psi = \dot{e}_1(xz \cos 2\xi + z^2 \sin 2\xi) \qquad (13\text{-}13)$$

The spectrum of flow patterns, described by such a general stream function, all comprise so-called flow asymptotes. These comprise two straight lines, intersecting at an angle, α, depending on the orientation, ξ, of the principal strain-rate or stress (Fig. 13-7):

$$\alpha = 90°\text{-}2\xi \qquad (13\text{-}14)$$

Substitution of (13-14) into (13-13) yields:

$$\psi = \dot{e}_1(xz \sin \alpha + z^2 \cos \alpha) \qquad (13\text{-}15)$$

Equation (13-15), perhaps, is the single most powerful expression for describing composite flows, governing progressive deformation with monotonically increasing homogeneous strains. Figure 13-8 graphs solutions of equation (13-15) for a range of α-values, assuming the principal strain-rate is unity. The corresponding velocity fields are illustrated in Figure 13-9.

□Exercise 13-5: a) Derive a relationship between the angle, α, and the ratio of the normal and shear strain-rates, using expressions (13-12a & b) and (13-14). b) Determine for each of the cases, illustrated in Figure 13-8, the numerical value of the ratio, $\dot{e}_{xz}/\dot{e}_{xx}$.

13-5 Velocity gradients and progressive deformation

Stream functions, also, allow the determination of the finite deformation pattern, accounting for both the strain and rotation components, at any stage of the flow. This technique is particularly straightforward for flow fields, leading to homogeneous deformations, because the governing equations can be solved analytically. The first step is to find, by differentiation of the stream function, the elements of the velocity-gradient tensor, L_{ij}. The velocity-gradient tensor, **L**, ($L_{ij}=\partial v_i/\partial x_j$), describes the spatial variation in velocity, v_i, according to the tensor expression:

$$v_i = L_{ij}x_j \qquad (13\text{-}16)$$

The position of any particle at a particular time t can be found by integrating the set of differential equations (13-16) over time. For flows leading to heterogeneous deformations, this integration can be solved only by numerical methods, because the velocity gradients are non-linear. In contrast, homogeneous deformations have linear velocity gradients, and the solution for such flows can be achieved analytically.

The analytical solution of progressive deformation becomes particularly simple if the reference frame is chosen in a convenient orientation. Figure 13-10 shows a progressive deformation, resulting from a composite flow, described by the

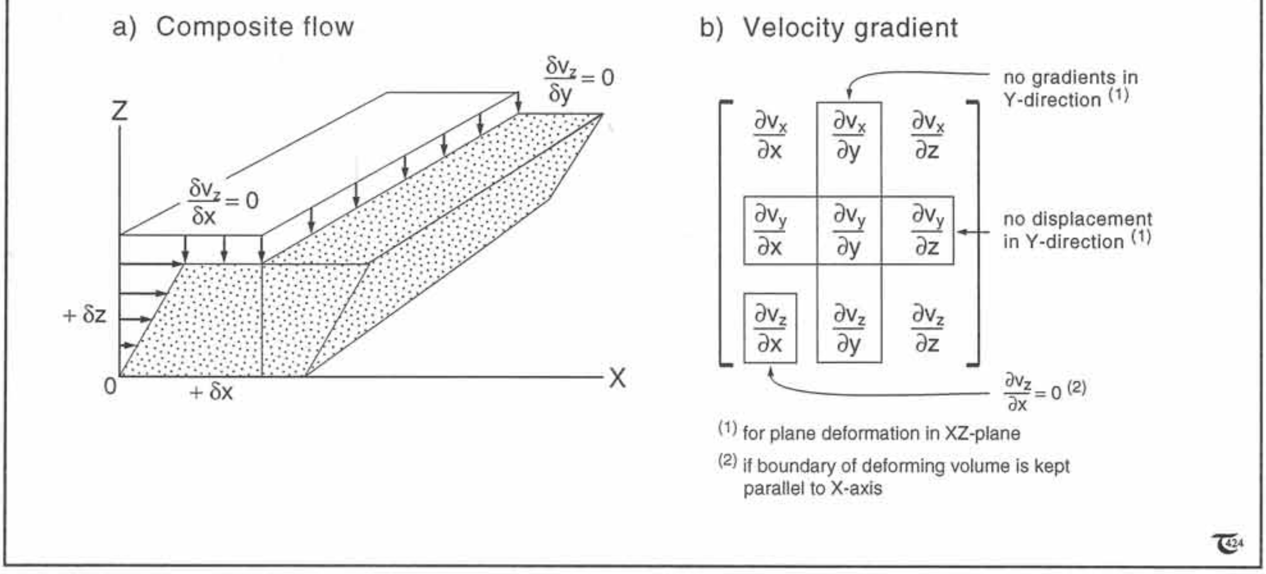

Figure 13-10: a) Composite flow in XZ-space, and b) corresponding velocity-gradient tensor.

stream function of equation (13-15). For this two-dimensional flow in the XZ-plane, only three of the velocity gradients need to be evaluated, as the other six gradients are zero. The velocity-gradient tensor for plane deformation can now be calculated from the stream function as follows:

$$L_{ij} = \begin{bmatrix} \partial v_x/\partial x & 0 & \partial v_x/\partial z \\ 0 & 0 & 0 \\ \partial v_z/\partial x & 0 & \partial v_z/\partial z \end{bmatrix} =$$

$$\begin{bmatrix} \partial^2\psi/\partial x\,\partial z & 0 & \partial^2\psi/\partial z^2 \\ 0 & 0 & 0 \\ -\partial^2\psi/\partial x^2 & 0 & -\partial^2\psi/\partial x\,\partial z \end{bmatrix} \quad (13\text{-}17)$$

It is worth noting that the term, L_{31}, also, vanishes for the situation in Figure 13-10, but the equation (13-17) is valid for a general plane deformation of arbitrary orientation, *but within* the XZ-plane.

The next step is to obtain, from the velocity-gradient tensor, **L**, the deformation tensor, **F**. These two tensors, for plane deformation in the XZ-plane, have the general format:

$$L_{ij} = \begin{bmatrix} L_{11} & 0 & L_{13} \\ 0 & 0 & 0 \\ L_{31} & 0 & L_{33} \end{bmatrix} \quad (13\text{-}18)$$

$$F_{ij} = \begin{bmatrix} F_{11} & 0 & F_{13} \\ 0 & 1 & 0 \\ F_{31} & 0 & F_{33} \end{bmatrix} \quad (13\text{-}19)$$

The integration of the velocity gradient equation yields the elements of the deformation matrix:

$$F_{11} = [(k_2-L_{11})/(k_2-k_1)]\exp(k_1t)-[(k_1-L_{11})/(k_2-k_1)]\exp(k_2t) \quad (13\text{-}20a)$$
$$F_{13} = [(-L_{13})/(k_2-k_1)][\exp(k_1t)-\exp(k_2t)] \quad (13\text{-}20b)$$
$$F_{31} = [(k_1-L_{11})(k_2-L_{11})/(L_{13}k_2-L_{13}k_1)][\exp(k_1t)-\exp(k_2t)] \quad (13\text{-}20c)$$
$$F_{33} = [(k_2-L_{11})/(k_2-k_1)]\exp(k_2t)-[(k_1-L_{11})/(k_2-k_1)]\exp(k_1t) \quad (13\text{-}20d)$$

The dummy constants, k_1 and k_2, are:

$$k_1 = (1/2)[(L_{11}+L_{33})+\surd[(L_{11}-L_{33})^2+4L_{13}L_{31}]] \quad (13\text{-}21a)$$
$$k_2 = (1/2)[(L_{11}+L_{33})-\surd[(L_{11}-L_{33})^2+4L_{13}L_{31}]] \quad (13\text{-}21b)$$

This solution of **F** is valid only if the square root contained in k_1 and k_2 is real, i.e., $[(L_{11}-L_{33})^2+4L_{13}L_{31}] \geq 0$, which corresponds to cases of non-oscillatory deformation. If $[(L_{11}-L_{33})^2+4L_{13}L_{31}] < 0$, this negative square root gives rise to complex variables, which occur when the streamlines form closed loops. This causes pulsating strains or oscillatory deformation (for details see Ramberg, 1975, and Weijermars, 1993, 1997).

The non-zero elements, of the velocity-gradient tensor for the composite homogeneous flow of Figure 13-6, are:

$$L_{11} = \dot{e}_1\sin\alpha \quad (13\text{-}22a)$$
$$L_{13} = 2\dot{e}_1\cos\alpha \quad (13\text{-}22b)$$
$$L_{33} = -\dot{e}_1\cos\alpha \quad (13\text{-}22c)$$

The corresponding non-vanishing deformation tensor elements are:

$$F_{11} = \exp(t\dot{e}_1\sin\alpha) \quad (13\text{-}23a)$$
$$F_{33} = 1/F_{11} \quad (13\text{-}23b)$$
$$F_{13} = (F_{11}-F_{33})\cot\alpha \quad (13\text{-}23c)$$

Recall that $F_{22}=1$ for plane deformations. Figure 13-11 plots the stages for the progressive deformation of a rock layer, using the deformation tensor of equations (13-23a to c), assuming a characteristic geological strain-rate of 0.315 Ma^{-1} or $10^{-14}\ s^{-1}$, for various flow-asymptote angles, α.

□Exercise 13-6: Consider the stream function for a *pure shear* flow, $\psi=\dot{e}_1xz$. a) Determine the velocity gradient. b) Determine the deformation tensor. c) Calculate the time required to achieve a principal stretch of $S_1=2$ if the strain-rate is 0.315 Ma^{-1}. d) Plot a graph, showing the relationship between the time and S_1 for this strain-rate. e) Also, plot S_3 in the same graph.

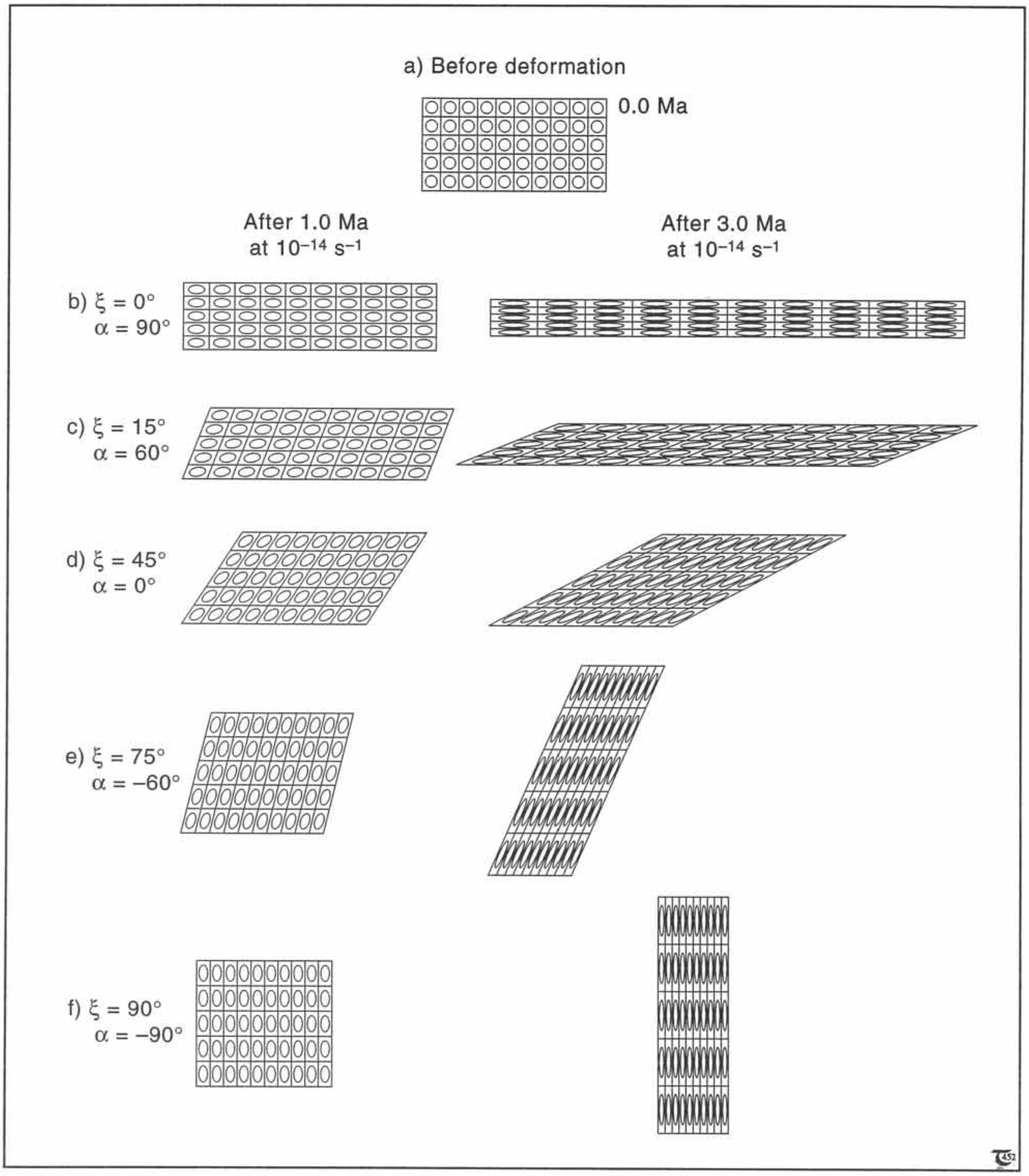

Figure 13-11: *a) to f) Progressive deformation sequences for a range of plane strain flows, generated by the deformation tensor of equations (13-23a) to (13-23c).*

□Exercise 13-7: Consider the stream function for a *simple shear* flow, $\psi = \dot{e}_{xz}z^2$. Determine the velocity gradient.

13-6 Components of the velocity gradient tensor

The velocity-gradient tensor is a square matrix, *L*, equal to ∇v:

$$\begin{bmatrix} \partial u/\partial x & \partial u/\partial y & \partial u/\partial z \\ \partial v/\partial x & \partial v/\partial y & \partial v/\partial z \\ \partial w/\partial x & \partial w/\partial y & \partial w/\partial z \end{bmatrix} \qquad (13\text{-}24)$$

The velocity-gradient tensor can be written as the sum of the symmetric strain-rate tensor, D_{ij}, and the skew-symmetric vorticity tensor, W_{ij}:

$$D_{ij} = (1/2)[(\partial v_i/\partial x_j) + (\partial v_j/\partial x_i)] =$$

$$\begin{bmatrix} \dot{e}_{11} & \dot{\gamma}_{12}/2 & \dot{\gamma}_{13}/2 \\ \dot{\gamma}_{21}/2 & \dot{e}_{22} & \dot{\gamma}_{23}/2 \\ \dot{\gamma}_{31}/2 & \dot{\gamma}_{32}/2 & \dot{e}_{33} \end{bmatrix} \qquad (13\text{-}25a)$$

$$W_{ij} = (1/2)[(\partial v_i/\partial x_j) - (\partial v_j/\partial x_i)] =$$

$$\begin{bmatrix} 0 & \dot{\omega}_3/2 & -\dot{\omega}_2/2 \\ -\dot{\omega}_3/2 & 0 & \dot{\omega}_1/2 \\ \dot{\omega}_2/2 & -\dot{\omega}_1/2 & 0 \end{bmatrix} \qquad (13\text{-}25b)$$

In the case of homogeneous deformation, the strain-rate and vorticity tensors will comprise only linear terms. The vorticity accounts for the rotation rate of the principal axes of the finite strain ellipsoid. Clockwise rotation rates are here taken positive, and counter-clockwise rotation rates are negative, but the opposite convention is, also, commonly used.

□Exercise 13-8: The vorticity, $\dot{\omega}$, is equal to twice the angular velocity or rotation-rate vector, Ω: $\dot{\omega} = 2\Omega$. Express the vorticity tensor in terms of angular velocities.

□Exercise 13-9: Use the fact that the sum of the strain-rate and vorticity tensors is equal to the velocity gradient tensor to prove mathematically that, for the plane deformation of Figure 13-10a, the magnitude of $\dot{\omega}_y$ is equal to $2\dot{e}_{xz}$.

References

A. Books

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Fluid Mechanics (1967, McGraw-Hill), by W.F. Hughes and J.A. Brighton. This volume of Schaum's Outline Series in Engineering provides practical examples of fluid mechanics for engineers and scientists.

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