Chapter 2: Physical Quantities and Continua

FOR A skillful application of mechanical theories to rock deformation, it is important to become familiar with, and be at ease with, both the units and magnitude of physical quantities used. In rock mechanics, we have to deal with physical quantities of extreme magnitude. For example, the viscosity of upper mantle rock is 10^{20} Pascal second (Pa s), its elastic shear modulus is 10^{10} Pascal (Pa), and the characteristic strain-rate is 10^{-14} per second (s^{-1}), of either stretch or angular shear strain (both expressed in dimensionless units). The accuracy in estimation of such extreme powers of ten is difficult, and a good approximation of the order of magnitude of these quantities is a remarkable achievement in itself. This chapter guides the reader into the realm of the physical quantities on which our understanding of rock deformation is based. These quantities can be used successfully in mechanical analyses only after adopting a so-called continuum assumption.

Contents: Fundamental quantities and their physical units are outlined in sections 2-1 and 2-2. Standardized SI-units and practical metric number names are introduced in sections 2-3 and 2-4. The role of geological time in deformation processes is briefly emphasized in section 2-5. The important distinction between scalar, vector, and tensor quantities is explained in section 2-6. Finally, the objective and limitations of the continuum assumption are outlined in sections 2-7 and 2-8.

***Practical hint:* Professional societies are becoming more and more involved in keeping their membership informed about the conversion to, and use of, SI metric units. In the present work, the 1982 recommendations and guidelines of the *Society of Petroleum Engineers* on the use of SI metric units are followed. Consult a copy of their publication (see reference section).**

2-1 Fundamental quantities

Physical quantities require measures to express the variation in their magnitude. Once a measure has been defined, essentially arbitrarily, a technical instrument can be designed to determine the magnitude or number of units as a measure of a particular quantity. There are, in fact, only a small number of independent or fundamental quantities, so many physical quantities are expressed in composite units. Length, time, mass, and temperature all are examples of fundamental quantities. Their dimensional formulae can be denoted by capital letters raised to the appropriate powers, conventionally enclosed in square brackets. Table 2-1 lists the dimensional formulae for a number of fundamental and derived physical quantities.

Alternatively, the mass-length-time system of quantities *[MLT]* can be replaced by the completely equivalent force-length-time system *[FLT]*, also included in Table 2-1. It follows, upon deeper reflection, that fundamental quantities are fundamental only by definition and no intrinsic fundamental property is involved. Fundamental quantities are chosen to be consistent with a series of techniques of measurement. Two other sets of "fundamental" quantities, sometimes used in engineering, are the pressure-length-time system *[PLT]*, and the density-length-time system *[DLT]*. For problems involving heat transfer, absolute temperature is usually taken as a fundamental quantity, as, for example, in the mass-length-time-temperature system *[MLTθ]*. Other fundamental or base quantities are used to measure electrical current, luminous intensity, and amount of atomic substance.

□Exercise 2-1: Explain why the systems of quantities *[MLT]*, *[FLT]*, *[PLT]*, and *[DLT]* are all physically equivalent.

Table 2-1: *Fundamental and derived physical quantities, their SI units and dimensional formulae.*

A. Fundamental quantity	SI unit	Symbol	Dimension
Mass	kilogram	kg	*[M]*
Length	meter	m	*[L]*
Time	second	s	*[T]*
Temperature	Kelvin	K	*[θ]*
El. current	Ampère	A	*[I]*
Luminosity	candela	cd	*[-]*
Amount of substance	mole	mol	*[n]*

B. Derived quantity	SI unit	Symbol	Dimension
Force	Newton	N (kg m s^{-2})	$[MLT^{-2}]$
Pressure	Pascal	Pa (kg m^{-1} s^{-2})	$[ML^{-1}T^{-2}]$
Density	kg m^{-3}	kg m^{-3}	$[ML^{-3}]$
Velocity	m s^{-1}	m s^{-1}	$[LT^{-1}]$

2-2 Physical units

The actual definition of units used for physical quantities is based on arbitrary criteria. Man introduced weights and measures for trade and commerce, taxation, land administration, and, finally, for scientific investigations. Initially, simple natural objects were used to define measures. For example, the weight of gemstones is still expressed in carats, derived from the carod seed, one unit of which weights about 0.2 grams (Fig. 2-1). The gram itself is a measure of mass, defined some two centuries ago by the *French Academy of Sciences*, and is equivalent to the mass of one cubic centimeter of water at its temperature of maximum density.

During the past few centuries, a multitude of *length units* has been introduced, serving one purpose or another. Length, like many other quantities, was initially scaled so that it could be estimated without resorting to special instruments hence, the established unit of one foot, based on a human length scale. The now universally accepted unit of meter (named by the French after the Greek term "metron" for "measure") was originally related to the size of the Earth. One quadrant of the Earth's circumference was defined as exactly 10^7 meters or 10^4 kilometers. There are variations in the radius of the Earth, and the length of the meter as a precision term has been stabilized by a recent redefinition, including the wavelength of an atomic spectral line. The meter is the length equal to 1,650,763.73 wavelengths in vacuum of the radiation corresponding to the transition between the levels $2p_{10}$ and $5d_5$ of the Krypton-86 atom, according to the *Eleventh General Conference on Weights and Measures* of 1960.

Time is another quantity with a fascinating history and obviously is of paramount importance to geology. Aristotle defined it as a quantity referring to motion from the point of view of earlier or later. Indeed, time is a unique quantity because it cannot be measured instantaneously. It must measure changes of motion. We believe so strongly in this concept that a person will be acquitted from suspicion in a crime if it can be proven that the person was in another location B at the time of the crime in location A. This assumes that it is impossible to move from A to B within zero time (which is possibly detested by Einstein's relativity theory).

Astronomy played a major role in defining time units, and the measurement of time is, therefore, partly connected to the rotation of the Earth. Periods of light and darkness were divided into poorly defined temporal hours in ancient times. Even today's calendars, in many cultures - including the western Gregorian almanac, are based on the number of days required by the Earth to complete one revolution around the Sun.

Figure 2-1: *The weight of diamonds is expressed in carats, one unit of which weights about 0.2 grams.*

Pope Gregory XIII endorsed, in 1582, a seasonal calendar, based on tables that included calculations by Copernicus, although the Church strongly opposed Copernican cosmology. The division of a day and night into 24-hours stems from the fourteenth century when the first mechanical clocks were invented. The precise standardization of time measurements dates from the foundation, by navigational need, of the *Royal Observatory at Greenwich* in 1675. This, also, prompted division of the hour into sixty minutes and the minute into sixty seconds. The second has been adopted as the standard unit of time by the *International System of Units* (SI units). The *Thirteenth General Conference on Weights and Measures* adopted the following definition in 1967: The second is the duration of 9,192,631,770 periods of radiation, corresponding to the transition between the two hyperfine levels of the ground state of the Cesium-133 atom.

□Exercise 2-2: Discuss whether it would be scientifically sound to propose the introduction of any new physical quantity if no apparatus exists to measure units of the appropriate scale.

□Exercise 2-3: Some physical quantities are dimensionless; they are initially constituted of measurable units but are expressed in terms of either a fractional change or percentage. Explain why strain is an example of a dimensionless quantity.

2-3 SI metric units

One of the earliest metric systems was created by Gabriel Mouton, vicar of St. Paul at Lyons, in 1670. However, metric units became of recognized importance when, in 1790, the *French Academy of Sciences* was requested by the central government (*National Assembly*) to establish a uniform system of weights and measures. The system designed was made compulsory in France in 1840, followed by legal recognition of the system in the United States of America through an act of Congress in 1866. The subsequent implementation of the system has been slow. In 1875, seventeen countries signed a treaty on the use of metric units; by 1900, some 35 countries had accepted these units. However, not until 1971 was it recommended by the *U.S. Secretary of Commerce* that the metric system should be introduced nationally through a coordinated program. As of October 1995, Britain has been ordered by *European Union* officials to adopt the metric system for all packaged foods, switching to kilograms from the British imperial system of pounds and ounces.

***Table 2-2:** Some additional, derived quantities and their SI units.*

Quantity	SI-unit	Symbol	Equivalent
Frequency	Hertz	Hz	s^{-1}
Radiation	Bequerel	Bq	s^{-1}
Amount of el.	Coulomb	C	$A\ s^{-1}$
Energy	Joule	J	N m
Power	Watt	W	$N\ m\ s^{-1}$
Magnetic flux	Weber	Wb	$N\ m\ A^{-1}$
Voltage	Volt	V	$N\ m\ A^{-1}\ s^{-1}$
El. resistance	Ohm	Ω	$N\ m\ A^{-2}\ s^{-1}$
El. capacitance	Farad	F	$N^{-1}\ m^{-1}\ A^2\ s^2$
El. conductivity	Siemens	S	$N^{-1}\ m^{-1}\ A^2\ s$

The term "SI" originates from renewed attempts by the French to introduce a universal system of metric units according to "*Le Système International d'Unités*," at conferences in 1960, 1964, 1968 and 1971. Worldwide academic and industrial groups have incorporated and are progressively following the recommended *International System of Units*. The SI system is not identical to, but is closely related to, schemes of earlier attempts to standardization, such as cgs (centimeters, grams, seconds), mks (meters, kilograms, seconds), or mksA (meters, kilograms, seconds, Ampères), all of which are systems of metric units. Spelling and rules of punctuation of SI units are identical in all languages to aid the global communication of quantitative measurements. The seven fundamental SI quantities, together with the standard units, correspond to fundamental units used in Table 2-1. Some additional, derived quantities are listed in Table 2-2.

The standardization and associated conversion of units is a slow process, and many alternative units are still in use. Although the established length measure for scientific communications is the meter, the petroleum industry is reluctant to rescale their vast databases. Many of their older drilling records are measured in feet, which - multiplied by the size of concession areas in acres - gives rise to reservoir-volume estimates in terms of rather awkward acrefeet. Many other deviations from SI metric units are still encountered in the literature and the appropriate conversion to SI units can be made using Table 2-3.

□Exercise 2-4: Give the equivalent volume in SI units for one acre-foot.

Table 2-3: Conversion factors for non-metric units to metric units.

To convert	To	Multiply by
Inch	m	0.0254
Foot	m	0.3048
Yard	m	0.9144
Mile	m	1609.3
Acre	m^2	4046.9
Hectare	m^2	10,000
Pound	kg	0.4536
Dyne	N	10^{-5}
Dyn cm^{-2}	Pa	0.1
Pound in^{-2}	Pa	6895
Bar	MPa	0.1
Atm	MPa	0.1013
Poise	Pa s	0.1
Gallon (US)	L	3.7854
Gallon (UK)	L	4.5461
Barrel	L	158.98
Erg	J	10^{-7}
Calorie	J	4.187
Horsepower	W	745.70

2-4 Metric number names

Powers of the ground number *ten* feature frequently in the SI metric system, and prefixes are recommended for use if either large or small numbers recur. For example, the time scale in geology is in millions of years, so the use of the mega-annae or Ma (10^6 years) is a permitted practice. (Annae is plural for year in Latin.) Table 2-4 lists the prefixes used to distinguish orders of magnitude in metric units. It should be carefully noted that capital (upper case) Latin, lower case Latin, and lower case Greek letters are employed as prefix symbols. For example, the kilometer or km is 1,000 meters; the micrometer or μm is 10^{-6} m. The symbols for SI metric prefixes are included in Table 2-4. The radius of the Universe is over 10^{40} meters, but it may, also, be measured in parsecs, an astronomical measure (Fig. 2-2). The largest terrestrial deformation structures are principally confined to the approximately 1,000-kilometer-wide fold belts, which puts an upper limit on the size of deformation features. These belts comprise large recumbent fold nappes, covering hundreds of square kilometers. The other end of the scale bar is constrained by the size of the individual grains of which rocks are composed.

□Exercise 2-5: Only metric systems benefit from the practical fact that their units break down into multiples of ten. For example, 1 km=10^3 m=10^5 cm=10^6 mm=10^7 μm. Now consider the old English length unit which puts 12 inches (or "thumbs") into a foot, 3 feet into 1 yard, and 5,280 feet into the mile. a) Calculate how many yards are there in a mile. b) Do you prefer non-metric or metric units?

Table 2-4: Metric number names for multiples of ten.

Magnitude	SI-prefix	Symbol
10^{-18}	atto	a
10^{-15}	femto	f
10^{-12}	pico	p
10^{-9}	nano	n
10^{-6}	micro	μ
10^{-3}	milli	m
10^{-2}	centi	c
10^{-1}	deci	d
10	deka	da
10^2	hecto	h
10^3	kilo	k
10^6	mega	M
10^9	giga	G
10^{12}	tera	T
10^{15}	peta	P
10^{18}	exa	E

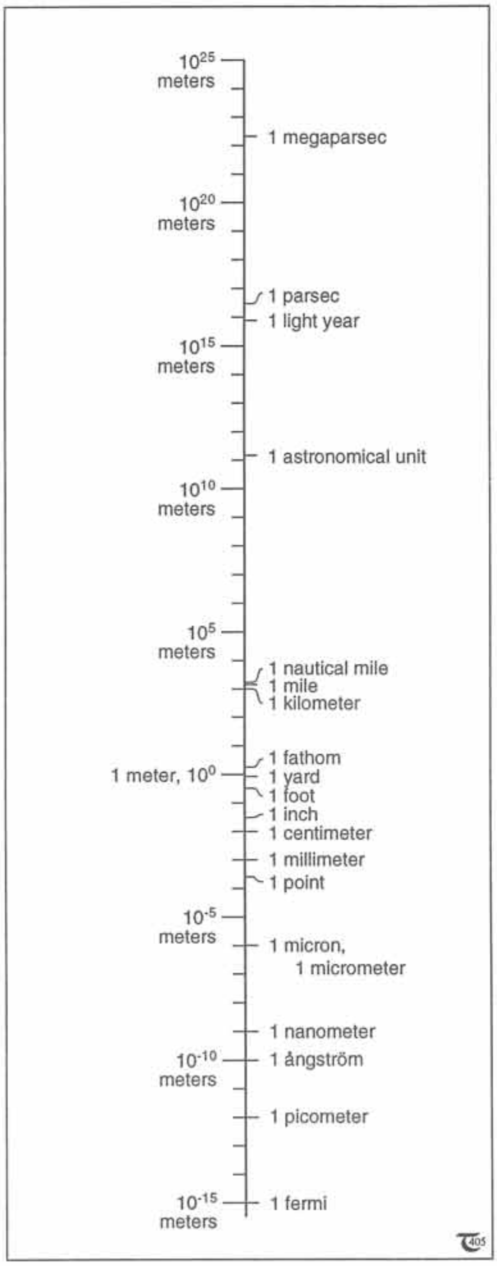

Figure 2-2: Logarithmic length scale of metric units (left) and corresponding practical units (right).

2-5 Geological time scale

The Big Bang theory places the formation of the known Universe some ten to fifteen billion years back in time. Earth was subsequently formed by gravitational accretion of a proto-planetary cloud about 4.6 billion years ago. The surfaces of our Moon and neighboring planets were battered by showers of meteorites some four billion years ago. The resulting impact craters observed on Mars, which has the same diameter as the Earth, suggest that the crust of our planet must have solidified before four billion years ago. Obviously, for geological processes, time is one of the most important quantities, and the recently developed knowledge of its immense span (Table 2-5) helps geoscientists in understanding better the slow formation of rock deformation patterns.

Because four billion years is such a long time as compared to the human life span, it is not easy for us to develop a good grasp of the geological time scale. It may help to improve your understanding by thinking of the following examples. The emergence of the shield volcano of Mauna Loa, Hawaii, and the strato-volcano of Etna, Sicily, each took about one million years. Early homonids appeared only one million years earlier. Many different phases of mountain building have been recognized to have occurred throughout geological time. The most recent Neogene and Alpine fold belts are due to the collision of active continental margins on tectonic plates, previously created by the breakup of Pangea. This breakup started about two hundred million years ago, but Pangea had been a stable supercontinent in the preceeding two hundred million years. The formation of mountain ranges at collisional plate boundaries occurs at time-averaged uplift rates in the order of one hundred meters per million years. We cannot observe such uplift rates by direct means, but they are accompanied by the formation of large deformation patterns in the interior of such mountain ranges. The world's highest mountain chain, the Himalayas, started to form after the closure of the Tethys ocean, due to the subsequent slow ploughing of the Indian subcontinent into mainland Asia during the past forty million years. In conclusion, studies of natural deformation patterns indicate that they commonly take millions of years to form.

□**Exercise 2-6:**
a) Obtain a rough measure of tectonic strain-rate by dividing plate velocity by the total width of an orogen.
b) Next, calculate the maximum strain that may have accumulated in crustal rocks over the four billion years of its solid-state history.

Table 2-5: *Simplified geological time scale showing the absolute age of divisions between the major periods and epochs. Up to ten digits notation is used to highlight the enormity of the time span covered.*

Eon/Era	Period/yrs	Epoch/yrs	Major events
Phanerozoic			
Cenozoic	Quaternary	Holocene	
		10,000	Extinction of large mammals
		Pleistocene	Ice ages
		1,600,000	
	Tertiary	Pliocene	Emergence of homonids
		5,300,000	
		Miocene	Increasing specialization of mammals
		23,700,000	
		Oligocene	
		36,600,000	
		Eocene	
		57,800,000	
		Paleocene	Spread of primitive mammals
		66,400,000	Extinction of dinosaurs
Mesozoic	Cretaceous		Spread of flowering plants
	144,000,000		
	Jurassic		First birds
	208,000,000		First mammals
	Triassic		First dinosaurs
	245,000,000		Extinction of many invertebrates
Paleozoic	Permian		
	286,000,000		
	Carboniferous (Eur)		First reptiles
	(360,000,000)		
	Pennsylvanian (US)		
	320,000,000		
	Mississippian (US)		
	360,000,000		First forests
	Devonian		First amphibians
	408,000,000		
	Silurian		First air-breathing animals
	438,000,000		First land-plants
	Ordovician		
	505,000,000		First vertebrates (fishes)
	Cambrian		Spread of marine invertebrates
	570,000,000		
Precambrian			
Proterozoic			
	2,500,000,000		
Archean			
	3,400,000,000		First organisms (blue-green algae)
	4,600,000,000		Formation of Earth

2-6 Mathematical properties of quantities

One of the most tedious aspects of physical quantities is their mathematical dimension. Quantities must be carefully distinguished as either *scalar*, *vector*, or *tensor quantities*. Their mathematical dimension greatly affects the way in which one may apply and manipulate such quantities. Scalar units are *zero-order tensor* quantities, represented by a 1x1 matrix, i.e., a single number. Vector quantities are *first-order tensors*, which can be represented in any arbitrary coordinate system as a 1x3 matrix. The common tensor quantity is a *second-order tensor,* which typically uses a 3x3 matrix notation.

For example, *scalar quantities*, such as temperature and pressure are simple to deal with because their values are valid for at least one particular location. Such quantities are not direction-dependent at that point. Scalar quantities, therefore, may be summed and subtracted in a straightforward fashion. However, in manipulations involving *vector quantities*, such as velocity and force, it is important to take into account not only their (1) spatial position, but, also, their (2) direction of operation. The summation of forces and velocities, therefore, requires vector addition.

Still more involved are quantities with *tensor properties*, such as stress and strain. These quantities are of paramount importance in geological applications of continuum mechanics, and their tensor formulations are outlined in Part 2 of this book. What makes tensor quantities sometimes awkward to deal with is that they can be manipulated only if three circumstances are fixed and specified: (1) the spatial location, (2) the direction of the operating quantity, and (3) the orientation of the material plane(s) on which the quantity is operating.

2-7 Continuum assumption

Measurements of any material properties would be rendered useless if made on volumes where these properties are either unrepresentative or intrinsically unstable. For example, the density of a porous sandstone may turn out, surprisingly, to be zero if measured within one of its pores at a length scale smaller than the pore size. It would, therefore, be meaningless, after having spent so much effort in establishing acceptible measures of physical quantities, if no constraints were placed on the nature of the materials to which such quantities may be applied. For this purpose, a fundamental principle of continuum mechanics is the *continuum assumption*. It assumes that the properties studied in a point of the continuum material are valid as a point average, irrespective of the detailed physical structure of the medium itself. In practice, this means that the property has been established at the macroscopic level, studying a representative, elementary volume (Fig 2-3).

For example, continuum mechanics of rocks is concerned with the macroscopic deformation in which the smallest characteristic length is much larger than the size of the grains. This approach allows modeling of the rock deformation by a mathematical idealization, commonly in the shape of some functional relationships among the constitutive variables. Creep laws of rock flow include the assumption that the various types of statistical averages of certain crystal motions give rise to the global flow (see chapter eight). The established flow laws suggest that the rocks internally flow as an isotropic viscous medium and thus allow geophysical modeling of the internal geometry of the rock deformation by continuum mechanics. The microscopic process of crystal plasticity is important for understanding and explaining why the flow can occur in the first

□Exercise 2-7: Group each of the following quantities in one of three categories of the appropriate mathematical dimension (scalars, vectors or tensors): velocity, stress, temperature, strain, time, strain rate, acceleration, force, length and vorticity.

place (see chapter seven), but it is essentially ignored in the continuum assumption.

□Exercise 2-8: Explain why spatial changes in the composition and texture of a rock unit (Fig. 2-4) may either complicate or jeopardize the continuum assumption.

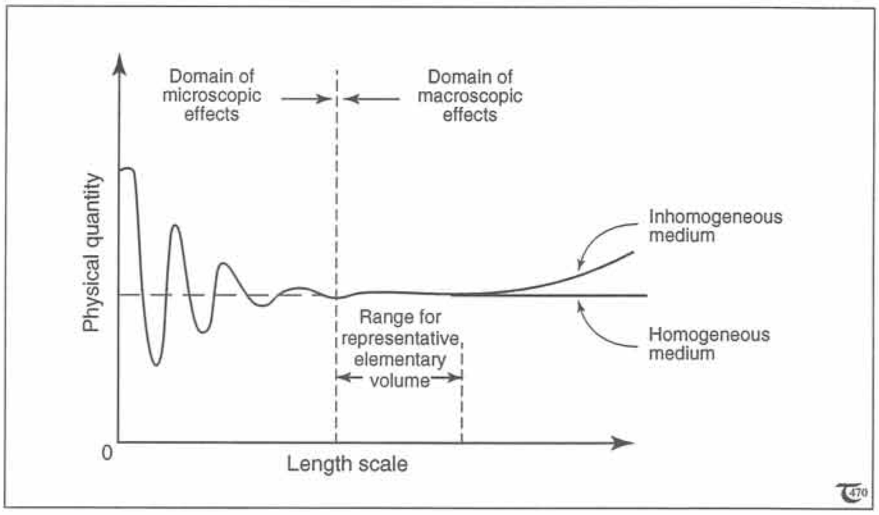

Figure 2-3: The continuum assumption considers macroscopic, physical properties at the scale of representative elementary volumes.

2-8 Limitations of continuum assumption

In order to be able to describe the rheological behavior of rocks, it is preferable to apply the description to volumes which remain texturally and compositionally unchanged during the deformation. *Time-dependent rheology*, accounting for compositional and textural changes, is rather complex, and our present knowledge of rock flow allows only a very simplistic approach. This implies that current constitutive equations cannot account for compositional changes, textural changes, or time-dependent volume changes. The limitations of this simplistic continuum model are severe for the creeping flow of rock, where time scales involved are much larger than those of instantaneous elastic distortion or brittle faulting. The stress field in rupturing rock is continuously modified by the growth of the new internal separation boundary, which, also, complicates the continuum approach once failure has occurred. If the length of the cracks and joints is smaller than the volume of the rock under study, the continuum assumption may still apply to the whole

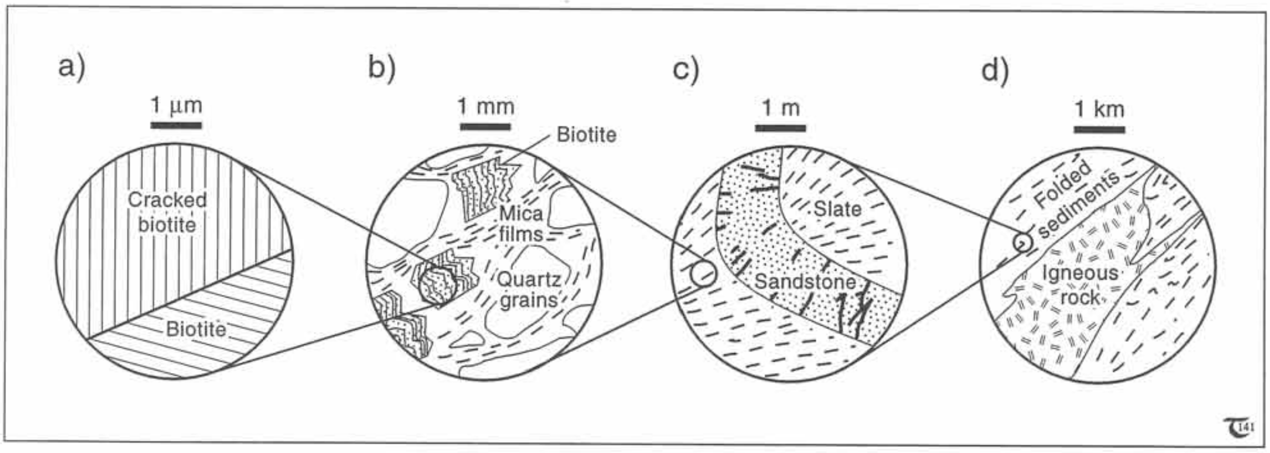

Figure 2-4: a) to d) Telescoping views of rock specimen at various length scales. See exercise 2-8.

volume. However, if the length of discontinuities is equal to or larger than the dimensions of the volume studied, these discontinuities must be distinguished as internal boundaries, separating discrete elements of the continuum.

References

Many books and articles are dedicated to physical quantities. Mentioned here are only a few texts, which have either a particularly practical or inspiring appeal to your author.

A. Books

Time and the Sciences (1979, Unesco), edited by Frank Greenaway, is an interesting collection of essays on the concept of time from the point of perspective of a variety of disciplines.

The SI Metric System of Units and SPE Metric Standard (1982, *Society of Petroleum Engineers*), outlines instructions on the use of the metric system as a voluntary standard endorsed by the SPE Board of Directors. It contains many useful conversion tables and explanations on the use of metric units.

Powers of Ten (1982, Scientific American Library), by Morrison and others, is a marvelously illustrated popular science book that creates a good sense for the dimension of length units, covering the entire range of scales from atoms to the Universe.

The Values of Precision (1995, Cambridge University Press), edited by Norton Wise, contains a collection of essays, reconstructing the role of social and bureaucratic factors in the development of physical measures.

B. Article

The detailed nature of the continuum assumption, required for the accurate modeling of rock deformation, has received very little attention. This is particularly so for ductile creep models. A discussion on possible verification methods of continuum models for crystalline creep is included in the following paper:

Weijermars, R. and Poliakov, A. (1993, *Tectonophysics*, volume 220, pages 33 to 50). Stream functions and complex potentials; implications for development of rock fabric and the continuum assumption.

☐Exercise 2-9: Discuss a possible continuum approach for an unconsolidated sedimentary rock, compacting during deformation.