Chapter 3: Force, Pressure, and Stress

DEFORMATION OF rocks is principally due to forces. Comprehensive understanding of the nature of forces, their dimension and components, is essential for developing a thorough grasp of the mechanics of deformation patterns. This chapter attempts to introduce the physical meaning and practical purpose of the quantities force, pressure, and stress. Calculation of forces may sometimes be impossible if the total rock volume, involved in the deformation, is unknown. This is unlike the quantities of stress and pressure, which can be used in most practical applications without needing to be concerned with the volume of rock involved.

Contents: The difference between body and surface forces is explained in section 3-1. Force units, force components, and the concept of force per unit area are outlined in sections 3-2 to 3-4. The important distinction between pressure and stress is explained in section 3-5. Applications demonstrating the role of pressure in geological problems are discussed in sections 3-6 and 3-7.

> ***Practical hint:***
> **Arrange a visit to an operational drill-site, and ask the Chief Engineer to explain the practical details of bore mud handling and pressure control. Please observe any safety precautions with utmost care!**

3-1 Body and surface forces

Any rock body is continually subjected to a variety of forces generated either internally or externally or both. An important distinction is, therefore, made between *body forces* and *surface forces*. Body forces act throughout the volume of the deforming rock and are proportional to the volume involved. Tidal forces are an example of body forces, because the force acts on any point inside the gravitationally attracted mass of the rock (Fig. 3-1a). Most forces in geological applications are generated in one way or another by gravity body forces. For example, tidal forces, caused by the combined effects of the Moon's and Sun's gravitational pull, may uplift the

Earth's surface semi-diurnally up to one meter. Magnetic forces, also, classify as body forces.

Surface forces act either on an external or an internal surface of a material volume and are not generated inside that volume. Glacial loading is an example of surface forces, because the force is transmitted at the contact surface between the ice and the solid rock (Fig. 3-1b). Long-term elastic deformation is caused by loading forces of glacial ice sheets reaching several kilometers' thickness. The resultant deflection of the Earth's surface may be on the order of one hundred meters.

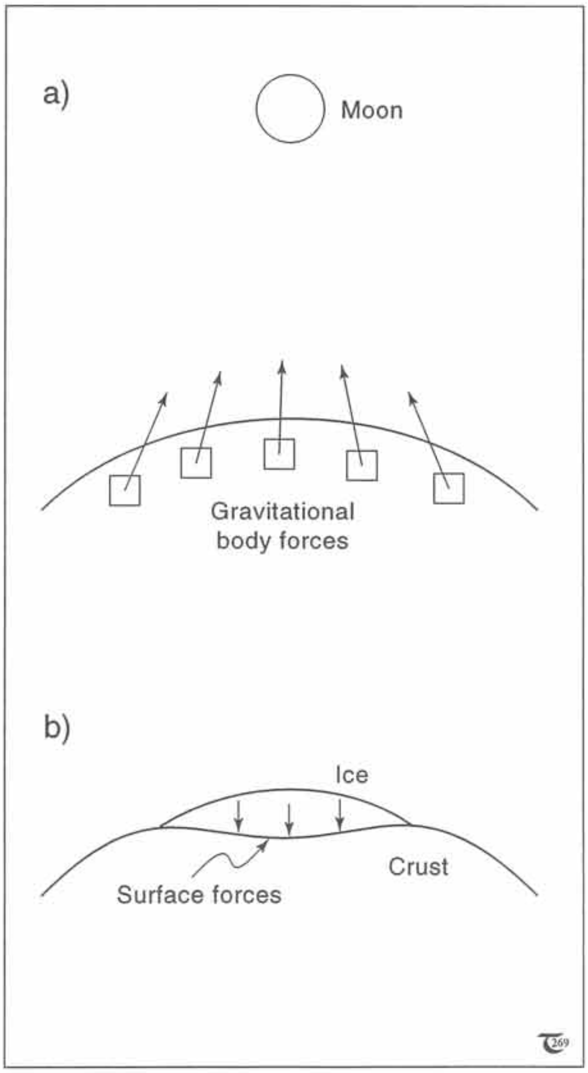

Figure 3-1: a) & b) Principle sketches of body forces and surface forces.

□Exercise 3-1: Gravity on the Moon is much less than on Earth. The same is true for the planets Mars and Mercury. Explain why any future construction on the Moon can be made of materials proportionally thinner than construction of similar materials and design on Earth.

3-2 Force units

Force, F, is a physical vector quantity, the magnitude of which is expressed in Newtons. The *Newton* (N) is a derived unit having the dimensions of [kg m s^{-2}], resulting from the product of mass, m (kg), and acceleration, a [m s^{-2}]:

$$F = ma \tag{3-1}$$

The direction of the vector F is inherited from the direction of the acceleration, a, which is a vector quantity. The most familiar force known to us is *weight*, which is per definition the force experienced by a mass, m, in the direction of gravity's acceleration and hence normal to the Earth's surface. The mass of a rock is the product of volume, V, and rock density, ρ:

$$m = V\rho \tag{3-2}$$

Figure 3-2 shows a unit cube of granite resting on a horizontal rock surface. The granitic block has an approximate density of 2,700 kg m^{-3}, and, because it has a unit volume, its mass is 2,700 kg. The weight of the block is 2,700 kg x 9.8 m s^{-2} =26,500 Newtons.

Humans tend to express their own body weight in the physically incomplete quantities of kg or lbs, which are quantities of mass. This is misleading, because a geologist with a mass of 100 kg really exerts a force of 980 Newton on the bedrock with the inherent risk of cracking the edges of a cliff this person is walking over. The term *force* may be employed in conjunction with a number of established adjectives which have well-defined physical implications. An example is the term *net force*, referring to the *vector* nature of forces, so that the resultant, effective, total or net force can be calculated by vector addition. Consequently, the total weight of two unit volumes of granite on top of one another is 53,000 Newtons.

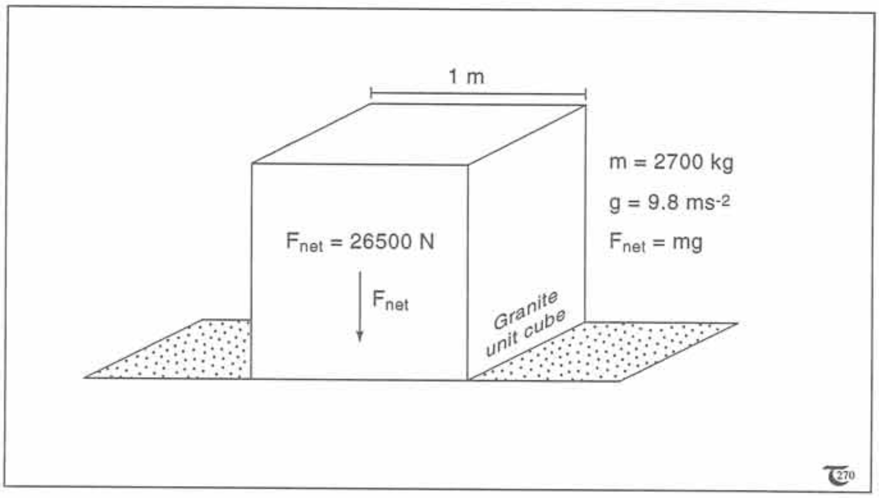

Figure 3-2: *Unit cube of granite resting on a horizontal surface.*

□Exercise 3-2: Estimate the force on the surface of one square meter of a horizontal limestone bed, overlaid by a one-kilometer-thick load of sandstone.

3-3 Force components

Figure 3-3 shows a unit cube of rock resting on a topographical slope. The frictional force at the base of the block prevents it from gliding further down slope. The block is, again, of granitic composition and thus has an approximate weight of 26,500 Newton. The granite block has a weight due to body forces, but this weight exerts a surface force at the top of the solid bed rock. The net force on the contact plane between the debris block and the solid bed rock can be decomposed into two vector components, a *normal force* perpendicular to the topographical slope, F_N, and a *shear force*, F_S, parallel to the slope, α:

$$F_N = F_{net} \cos \alpha \qquad (3\text{-}3a)$$
$$F_S = F_{net} \sin \alpha \qquad (3\text{-}3b)$$

The normal force is the ineffective component, whereas the shear force is acting in the direction opposite to the frictional force.

It appears from experiments that friction on straight contact planes between unwelded rocks is an intrinsic rock property; any rock volume will start to glide down slope if the angle α is steeper than 25° to 45°. The critical angle, ϕ, is called the *angle of internal friction* in rock mechanics and is exactly the same as the angle of repose used in sedimentology:

$$\phi = \tan^{-1}(F_S/F_N) \qquad (3\text{-}4)$$

The related *coefficient of internal friction*, μ, is defined as:

$$\mu = F_S/F_N = \tan \phi \qquad (3\text{-}5)$$

and varies between 0.5 and 1. The most characteristic friction coefficient for rocks, μ, is about 0.85, implying an angle of internal friction, ϕ, of 40°. These values have been established on the basis of sliding experiments, compiled by Byerlee (Fig. 3-4). The compiled data suggest that friction on existing faults, or other discrete planes of separation in rocks, is largely insensitive to, or independent of, the rock type involved.

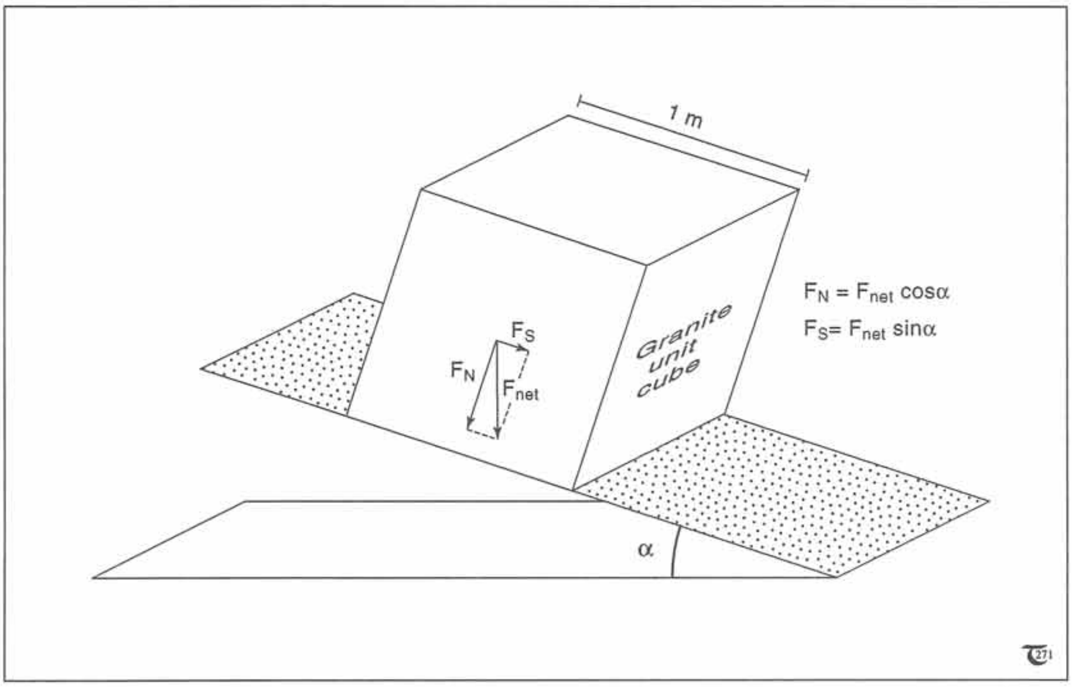

Figure 3-3: *Unit cube of granite, resting on a surface, dipping at angle, α.*

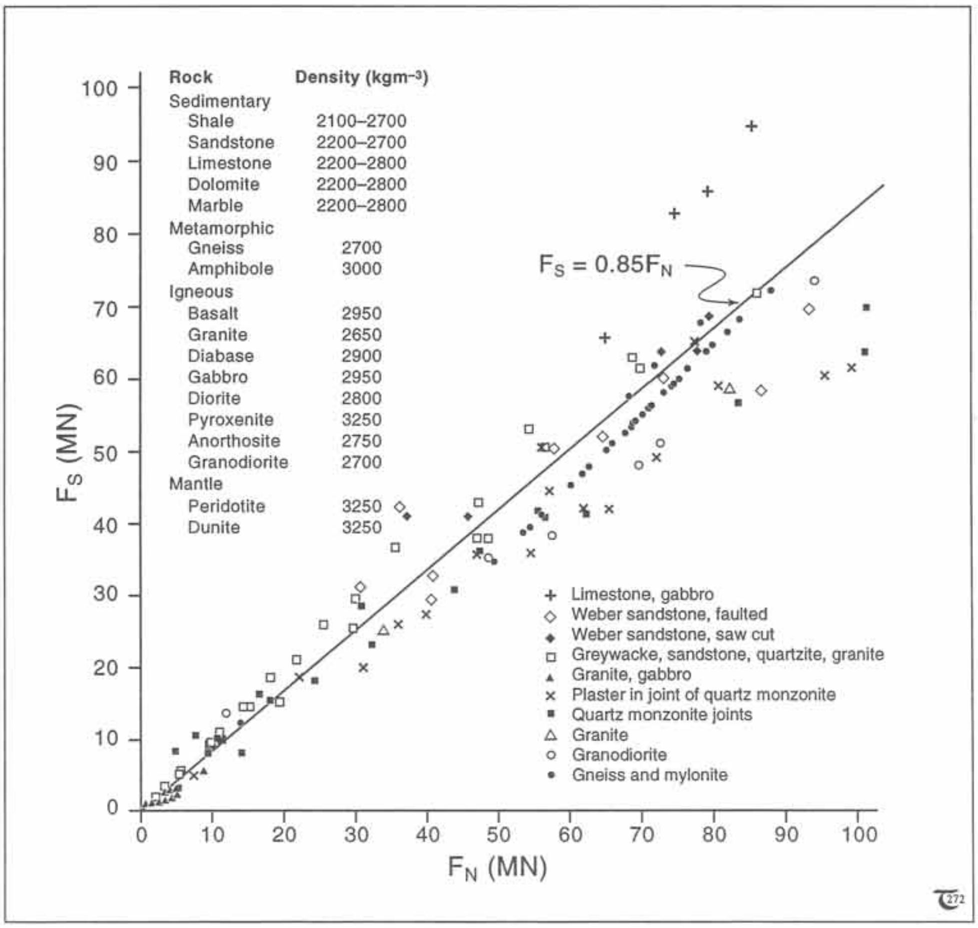

Figure 3-4: *Graph of coefficients of internal friction for a variety of rock types under a range of relative magnitudes for the normal and shear force. The regression curve suggests an average value of 0.85 is representative for all rocks tested.*

□**Exercise 3-3: Consider the (gravity) sliding of a loose block. The block is rectangular with a base area of 4 m^2 and height of 1 m. The rock density is 2,700 kg m^{-3}, and the friction coefficient is μ=0.7. a) What is the orientation and magnitude of the smallest force required to move the block if resting on a horizontal surface? b) If a man of 100 kg can push his own weight, how many men do you need to push the block forward? You will now realize why Egyptian pharaohs started constructing their burial pyramids as soon as they assumed power during their lifetimes. c) Graph the variation in magnitudes of the normal and shear forces (F_N, F_S) if the basal slope (α) ranges between 0° and 90°. d) What is the angle of internal friction (ϕ) for the movement plane between the block and the basal slope? e) Graph the acceleration (a) of the block for supercritical basal slopes, i.e., $\phi<\alpha<90°$. f) Graph the additional push force needed to move the block over subcritical basal slopes, i.e. $\phi>\alpha>0°$.**

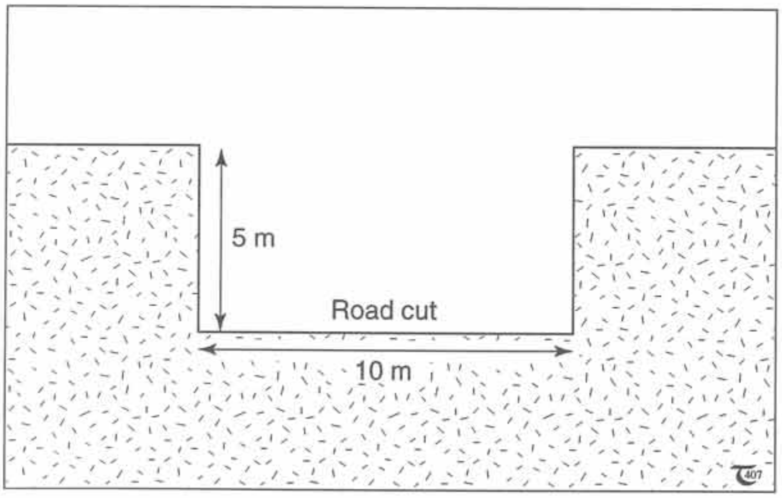

Figure 3-5: Vertical section through a hypothetical road cut. See exercise 3-4.

□**Exercise 3-4: Consider the stability of the walls of a road cut through solid granite as shown in Figure 3-5. Assume the locally established friction coefficient, μ, is 0.5. a) If there are any joint planes developed inside the granite, indicate which inclinations would pose the danger of rock slide. Consider only joint planes with a strike parallel to the road. b) The chief engineer considers building two concrete pylons, on the shoulders of the road cut, for a bridge suspended over the road. Would the construction affect the critical inclination or angle of internal friction of joint planes in the walls of the road cut? e) If pylon height is cheaper per unit length than bridge span, where would you alternatively put the foundation of the two pylons?**

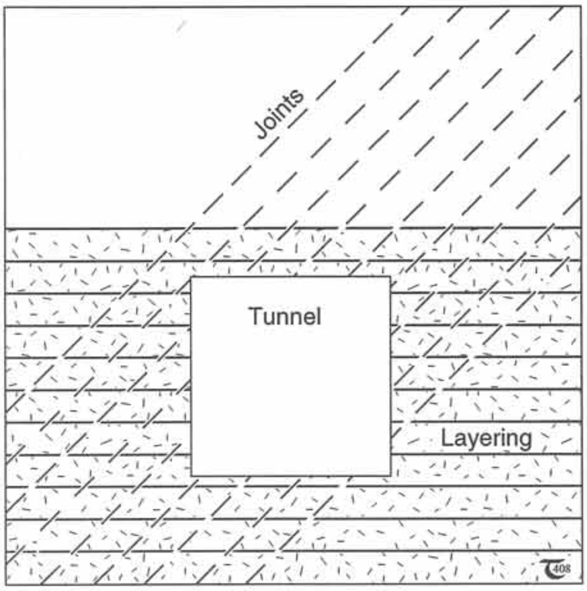

Figure 3-6: Vertical section of a hypothetical tunnel through a horizontal sedimentary sequence, transected by oblique joints as indicated. See exercise 3-6.

□Exercise 3-6: Consider the stability of the roof of a horizontal tunnel of square cross-section through a sequence of horizontal sedimentary rock beds (Fig. 3-6). The individual beds are consolidated sandstones, separated by thin layers of unconsolidated clay, causing the sandstone beds to separate easily. The rocks are transected by a set of joints, as indicated, striking parallel to the tunnel. The coefficient of friction on the joint planes is 0.5. a) Shade the area of unstable wall rock. b) Consider modification of the shape of the tunnel roof. Show how a highly arched roof increases the area of potential rock slide. c) Joints are commonly developed in shallow rocks. If you were planning a tunnel, would you prefer construction through sedimentary rocks or massive igneous rocks?

□Exercise 3-5: a) Consider the effect of the block shape and volume on gravity sliding by completing exercise 3-4a) to 3-4e) for a cube of rock with a base area of 2 m^2 and height of 2 m. The density and friction coefficient are similar to that used in exercise 3-3.
b) Is the angle of internal friction dependent on the *shape* of the block?
c) Consider a block of rock of 1 m^3 resting on a slope. Assume the rock density is 2,700 kg m^{-3}, and the friction coefficient is μ=0.7. What is the critical slope angle (ϕ) for which spontaneous sliding occurs? d) Is the angle of internal friction or critical angle for rock sliding dependent on the *volume* of the rock resting on a potential slide plane?

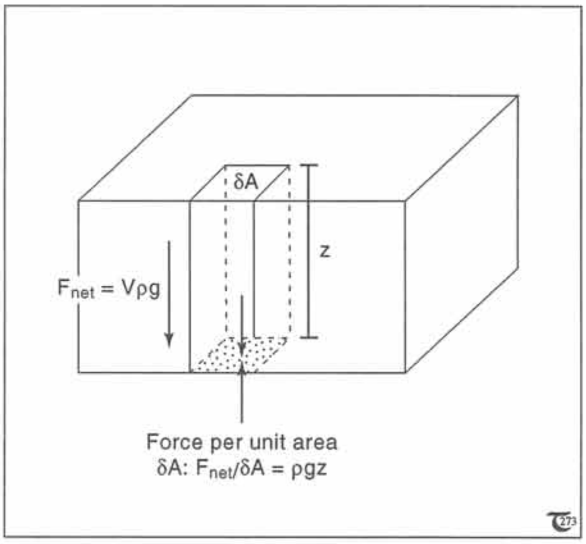

Figure 3-7: The surface force per unit area, due to gravity, at depth, z, within a crustal section, given by ρgz.

3-4 Force per unit area

Force is an important physical quantity, featuring in many geological calculations. It is a simple, straightforward vector quantity. However, there are serious limitations to calculation versatility with forces. For example, a gravitational load force is always specified by the total mass involved in the loading. In order to know the total mass, one needs to know not only the density, but, also, the total volume of the rocks involved [cf. eq. (3-2)]. There are many instances where knowledge of the total rock volume, V, is uninteresting, impractical, or not available. More freedom of calculation is achieved by defining another quantity, the force per *unit area* (Fig. 3-7):

$$F/\delta A = \rho g V/\delta A = \rho g z \quad (3\text{-}6)$$

with gravity acceleration, g, and unit area, δA. The obvious advantage is that computations with force per unit area require only a length scale, here z, and nullify the need to know the other two length scales normal to z. The magnitude of F/δA is expressed in *Pascal* (Pa), a derived unit having the dimensions of [kg m^{-1} s^{-2}], resulting from the division of force units by surface area.

□Exercise 3-7: Calculate the stress on a surface of one square meter of a horizontal limestone bed, overlaid by a one-kilometer-load of sandstone.

3-5 Pressure and stress

Two different kinds of quantities, resulting from the definition of force per unit area (F/δA), must be considered: *pressure* and *stress*. It is easy to confuse these two derived force quantities, but they should be strictly separated. The magnitudes of both pressure and stress are expressed in Pascal. But stress and pressure differ *physically* in the sense that pressure at a point is equally great in all directions, whereas the magnitude of a stress is valid only for a particular orientation. Pressure and stress magnitudes and orientations can be *graphically* represented by arrows of length, scaled for their magnitude in a particular orientation. The arrows for a pressure are always radially symmetric, whether positive and inflatory (Fig. 3-8a) or negative and confining (Fig. 3-8b). In contrast, the stresses in two non-parallel planes through the same material point will be different; hence, stress is sensitive to the orientation in space. In two dimensions, there is one direction of minimum stress and one of maximum stress. They are mutually perpendicular, and the stress gradients in between them can be represented by a stress ellipse (Fig. 3-8c).

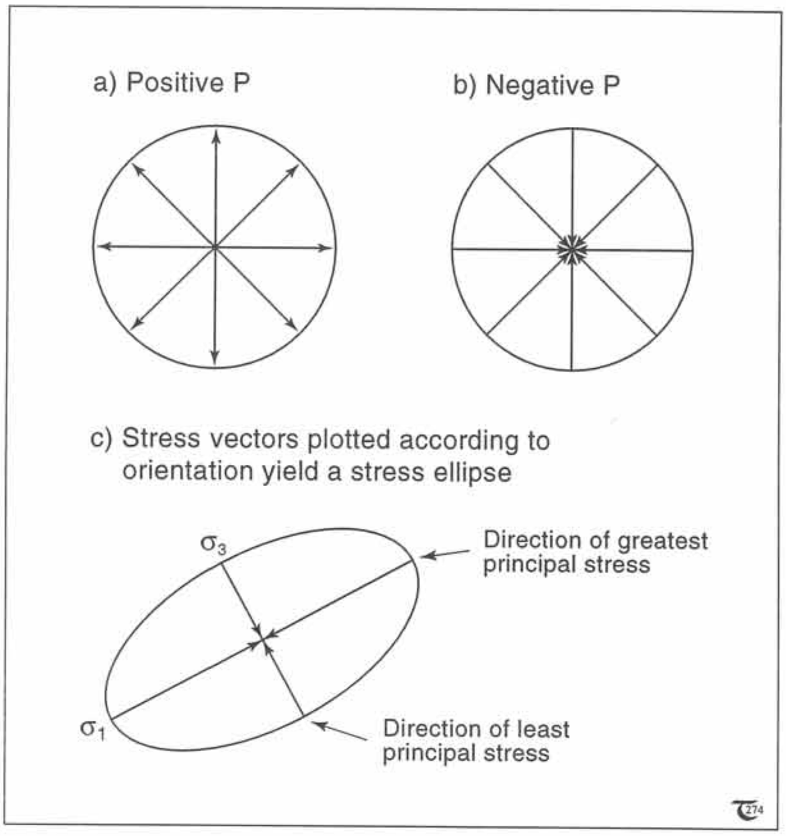

***Figure 3-8:** a) to c) Graphical representations by arrows for: (a) positive pressure, (b) negative pressure, and (c) negative stress, all in two dimensions.*

Pressure and stress further differ *mathematically* in the sense that pressure is a *scalar* and not a *tensor* quantity like stress. A pressure, therefore, may be represented by a simple scalar symbol, P. Scalars, sometimes referred to as a zero-order tensors, comprise one component only and have a magnitude that is independent of the coordinate system used. In contrast, any stress will need a suffix telling the user its orientation. For example, σ_1 is used to denote the major principal stress (see chapter four). The magnitude of a principal stress is valid only for its particular orientation, and a different suffix arises if the stress is specified for a different direction. If an actual coordinate system has been established, then the stress at a point can be best described by a tensor. The term tensor is used here principally to denote second-order tensors, which have nine elements with magnitudes that depend on the particular coordinate system used (see chapter ten). Force and velocity vectors are examples of so-called first-order tensors. These quantities are comprised of three components to which normal vector rules apply. This is unlike second-order tensor quantities, where simple vector rules no longer apply. For example, the total state of stress is represented by two arrows (2D) or three arrows (3D) and not just one arrow, as is the case for force vectors. The number of components in a tensor of order n is determined by 3^n.

□Exercise 3-8: Calculate the fluid pressure in a continuous, thin film of water, trapped at the base of: a) the horizontal block in Figure 3-2, and b) the tilted block of Figure 3-3, staying clear from the edge effects.

The physical effects of pressure and stress are as follows: Pressure without gradients inside a fluid cannot cause instantaneous motion. In contrast, any stress inside a fluid will cause motion, and, in fluids, deviatoric stresses are commonly associated with pressure gradients. Solids, such as rock, can sustain both stress and pressure without necessarily deforming. Some geoscientists use the terms stress and pressure in a fashion which distinguishes these forces per unit area less strictly as tensor and scalar quantities. In contrast, all forces per unit area in *fluids* are by them exclusively referred to as pressures. Conversely, any forces per unit area in solid *rocks* are by them categorically termed stresses. According to their school, the lithostatic pressure, P_{lith}, referred to in sections 3-6 and 3-7, is preferably termed σ_V or S_V, denoting lithostatic stress rather than pressure (σ_V is considered by them to act vertically only, hence the subscript, v). The concept of pressure in the stricter, mathematical sense (scalar), is outlined in the next section, and the more complex (tensor) properties of stress are elaborated in chapters four and ten.

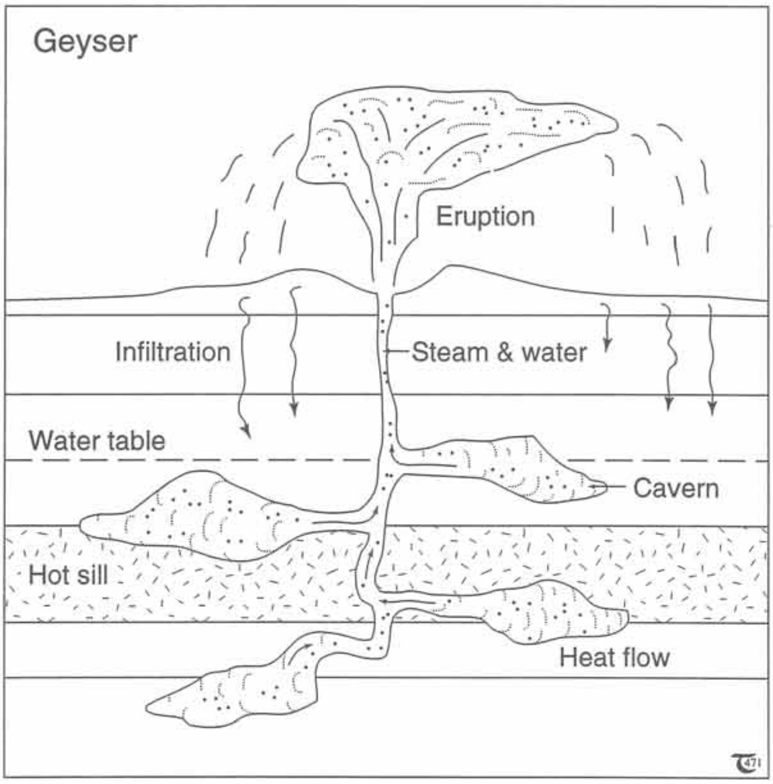

Figure 3-9: Principle sketch of a geyser, ejecting steam into the air for as long as the pressure in the subsurface reservoir is supercritical.

3-6 Fluid and lithostatic pressure

Pressure plays a dramatic role in many geological situations. *Volcanic eruptions* occur when the magma pressure increases above the equilibrium or hydrostatic value. This may occur, among other means, by thermal expansion of the magmatic melt, addition of magma to a confined magma chamber, or external loading by lunar and solar tides, which can elastically contract the wall rock so that the volume of the magma chamber decreases. Tidal loading is not the major mechanism of volcanism, but it may trigger an eruption. The eruption of *geysers*, where steam spouts periodically into the air, is similarly due to the sudden increase of pressure when a water column is reaching boiling temperature by geothermal heating (Fig. 3-9). The expanding water vapor increases the pressure in the geyser conduit above hydrostatic values, and subsequent eruption is inevitable.

In the *hydrocarbon industry*, it is extremely important for well control during exploration drilling to pump down mud of appropriate density into the well head. The bore fluid must have a density which matches that of the wall rock, so that blowouts are prevented. Consider a vertical bore hole filled with fluid. The *fluid pressure*, P_{fluid}, will increase linearly with depth, according to:

$$P_{fluid}=\rho_{fluid}gz \quad (3\text{-}7a)$$

The pressure gradient, dP/dz, is assumed to be in equilibrium, and no movement of the fluid is allowed.

Water inside the open pipe of a stable, cased bore hole will exert a fluid pressure, linearly increasing with depth (Fig. 3-10a). The pressure at the open, upper end is zero, and maximum pressure is reached at the bottom of the hole. There is no gradient of pressure in the horizontal direction (Fig. 3-10b). Similarly, if a space is closed, as in pores of impermeable rocks, fluid pressure is constant in all directions (Fig. 3-9c). Although the pressure is constant in all directions around the point for which the pressure arrow is shown, it is common to portray pressure arrows perpendicular to solid surfaces in order to emphasize the balancing of forces at either side of the surface.

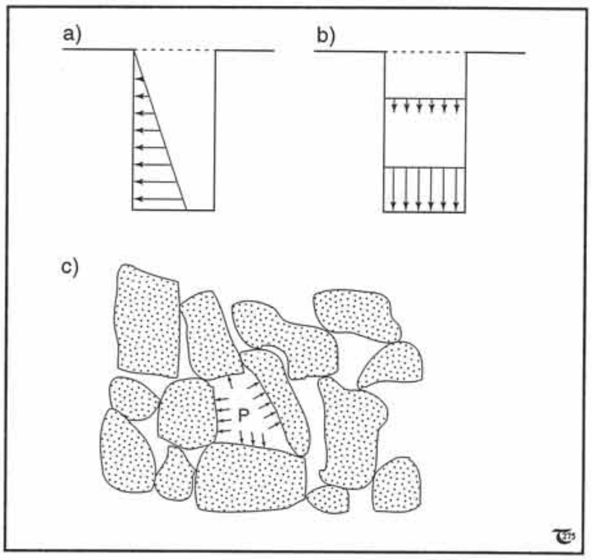

Figure 3-10: *a) to c) Fluid pressure in flooded bore hole increases linearly with depth (a), but there is no horizontal pressure gradient (b). The pressure in closed, fluid-filled pore-spaces is uniform (c).*

The *lithostatic* or *geostatic pressure* on the walls of the bore hole, if not involving any stress component, will be:

$$P_{lith}=\rho_{lith}gz \quad (3\text{-}7b)$$

This expression assumes that the rock density remains constant with depth. However, the lithostatic load may be larger than the fluid pressure if $\rho_{lith}>\rho_{fluid}$. This is commonly the case, except in overpressured portions of basins where fluid pressure is not dependent on the density of the fluid. The lithostatic load, therefore, does not involve pressure only, but it, also, will cause a so-called deviatoric stress (introduced in section 4-2), which will tend to collapse the walls of the bore hole. The convergence of the walls will lead to a kinematically maintained, and, therefore, temporary, increase of the fluid pressure, so that fluid may extrude.

□Exercise 3-9: Consider the pressure in an artesian well. A horizontal porous sandstone layer has been displaced by a dip-slip shear zone with reverse thrust of the hanging wall (Fig. 3-11). The sandstone layer is an aquifer and is tapped by a vertical well, as illustrated. Calculate the fluid pressure in locations A, B, C, and D.

3-7 Coefficient of fluid pressure

Geoscience commonly employs a *coefficient of fluid pressure*, λ, denoting the ratio of fluid pressure and lithostatic load:

$$\lambda = P_{fluid}/P_{lith} \qquad (3\text{-}8)$$

For well control, ideally λ should be kept close to unity. However, if P_{fluid} becomes too large, the casing or wall rock of the well may fracture and costly bore mud will be lost into the newly opened gashes at depth. On the other hand blowout and bore hole closure become more likely if P_{fluid} is much smaller than P_{lith}.

Another complication is that rocks generally compact during burial, and thus their density increases with depth. Generalized compaction curves show that the density of argillaceous sediments exponentially increases with depth (Fig. 3-12). In such cases, the lithostatic pressure no longer increases linearly with depth but must be calculated by integration:

$$P_{lith} = \int_z \rho_{lith}(z)\ g\ dz \qquad (3\text{-}9)$$

Bore mud of constant density must be pumped down so that the total weight of the mud column matches that of the rock column constituting the wall of the bore hole.

Fractured rock with interconnected fissures reaching the surface, if filled with water, will have a coefficient of fluid pressure, λ, of 0.42, assuming a bulk rock density of 2,400 kg m^{-3} and water density of 1,000 kg m^{-3}. This is the so-called *hydrostatic state*, where the pressure is maintained by an open water-column with zero pressure at ground level. If a water-saturated layer is buried and enclosed by aquicludes of impermeable rock, then the water pressure generally cannot remain hydrostatic. The coefficient of fluid pressure in layers enveloped by aqui-

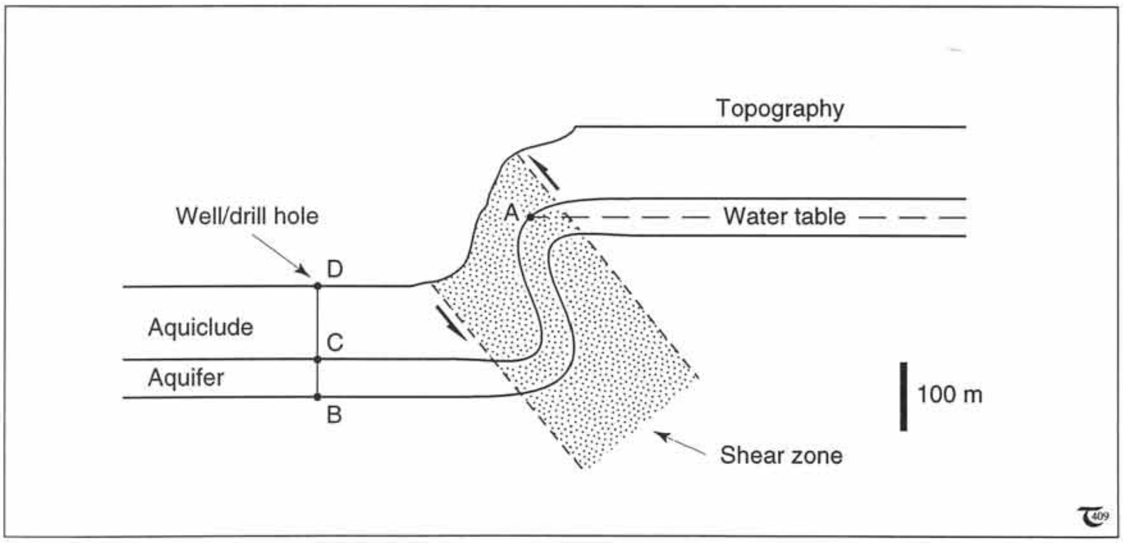

***Figure 3-11:** Cross-section of aquifer, displaced by a shear zone. See exercise 3-9.*

cludes is likely to be higher than the hydrostatic pressure; such rocks are described as *overpressured*. Overpressured rocks cause potential danger in drilling for hydrocarbons, because the fluid pressure is difficult to account for in calculations compensating for bore mud density. Overpressure may be related to the dehydration of montmorillonite to illite during non-equilibrium compaction, i.e., expulsion of lattice-locked water and the resulting rise of fluid pressure. Other overpressuring mechanisms may be rapid burial, thermal expansion of pore fluid, and hydrocarbon gas generation during subsidence.

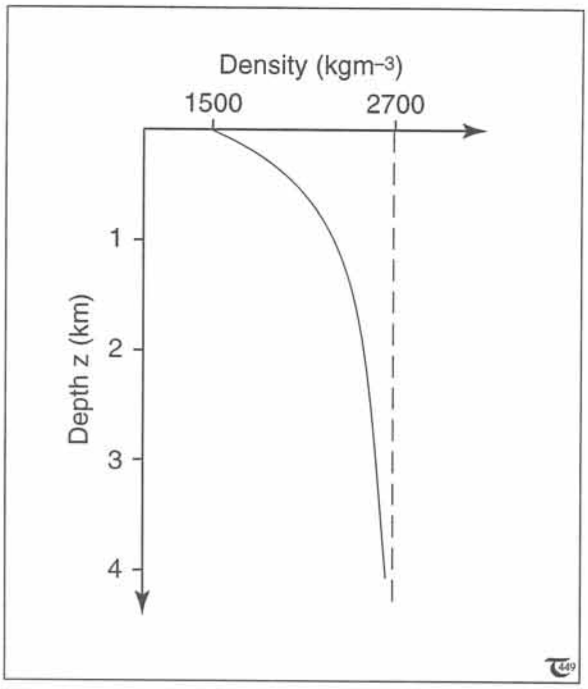

Figure 3-12: *Generalized compaction curve of clay series, showing density versus depth.*

□Exercise 3-10: Consider the bore mud density required for well control in petroleum geology. Two vertical wells A and B have been drilled through a horizontally stratified basin (Fig. 3-13). The density of the rocks is continuously monitored from chips collected at the wellhead from the circulating drilling mud. Jumps in density are as graphed. The skillfull drilling master has adjusted the density of the drilling mud several times to prevent collapse of the walls of both wells. a) Well A was drilled first. Calculate the average density of the mud column at the time drill hole A was abandoned. b) Well B penetrates the same layers as well A but hits a gas reservoir at 4 km depth. The gas-pressure of the reservoir is 120 MPa, and this overpressure closed the safety-valve of the drilling rig when first penetrated. The bore mud was replaced by a much denser one through the safety valve to compensate for the overpressure. What was the correct mud density when the safety valve could be opened again?

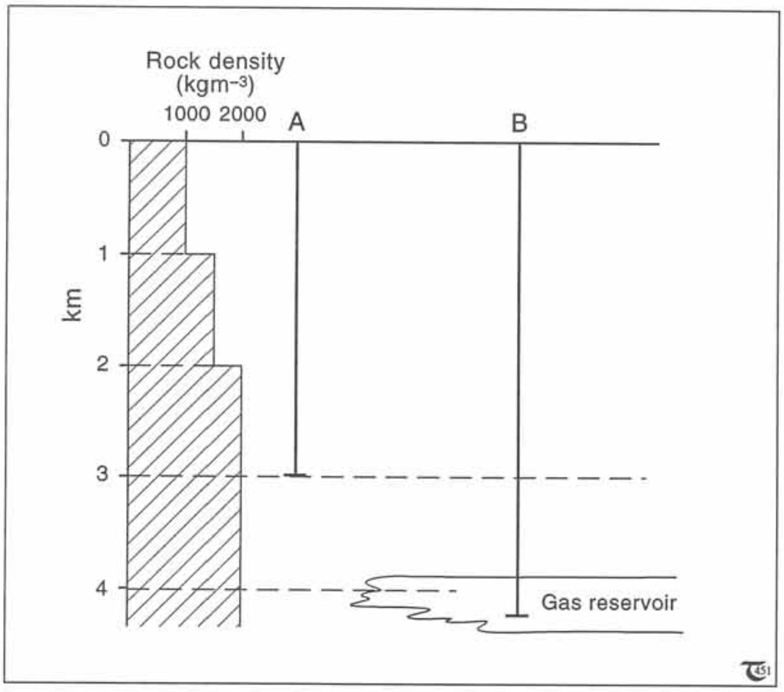

Figure 3-13: *Horizontally stratified basin with formation density as graphed, penetrated by two wells, A and B. See exercise 3-10.*

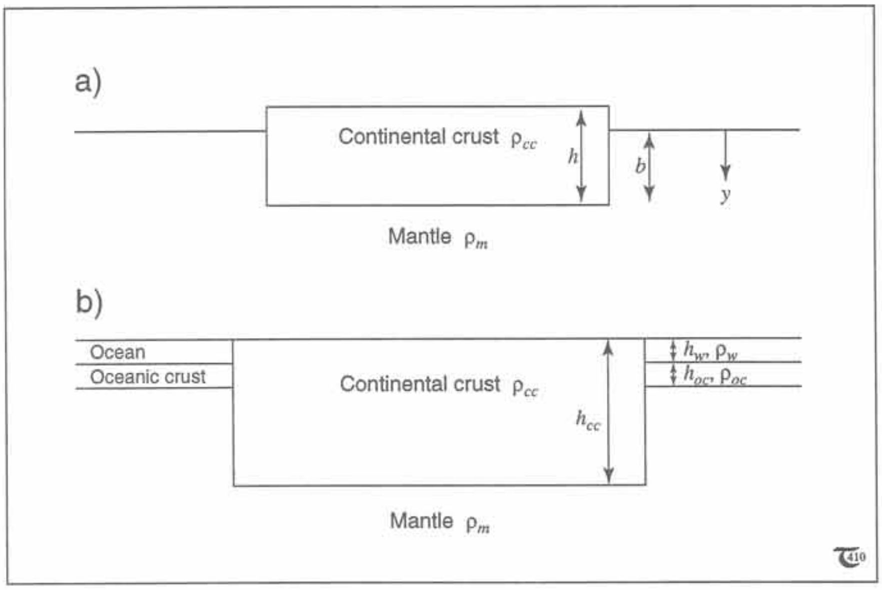

Figure 3-14: a) & b) Isostatic models of continental crust, floating in mantle rocks. See exercises 3-11 and 3-12.

□Exercise 3-11: Consider the principle of isostasy. The continents can be most simply thought of as layers, floating on mantle rocks (Fig. 3-14a). Archimedes' principle implies that continents should be isostatically balanced. The pressure at the base of a continent is equal to that at similar levels in the adjoining oceanic area. Use this principle to calculate the elevation of the continental area above the ocean floor. The crustal thickness h=50 km, ρ_{cc}=2,700 kg m^{-3} and ρ_m= 3,300 kg m^{-3}.

□Exercise 3-12: Consider further the principle of isostasy. A more realistic section of oceanic and crustal crust, including both the oceanic water body and the difference of density between the oceanic crustal rocks and the underlying mantle (from which they were geochemically fractionated), is sketched in Figure 3-14b. a) Apply the principle of isostasy or hydrostatic equilibrium to show that the depth of the ocean basin relative to the continent is given by:

$$h_w = [(\rho_m - \rho_{cc})/(\rho_m - \rho_w)]h_{cc} - [(\rho_m - \rho_{oc})/(\rho_m - \rho_w)]h_{oc} \quad (3\text{-}10)$$

b) Calculate the isostatic depth, h_w, using h_{cc}=35 km, h_{oc}=6 km, ρ_m=3,300 kg m^{-3}, ρ_{cc}=2,800 kg m^{-3}, ρ_{oc}=2,900 kg m^{-3}, ρ_w=1,000 kg m^{-3}. c) Sea level has displayed maximum fluctuations of about 200 m during the Cenozoic. The sea level near the end of the Cretaceous was about 200 meters higher than today. The rise in sea level must have been isostatically compensated for by an increase in the mean depth of the ocean basins. Calculate the isostatic depth increase, using the parameters given in (b).

References

Articles

The following work by John Byerlee has contributed enormously to the generalization of friction coefficients for slip on faults in rocks (see, also, chapter six):

Byerlee, J.D. (1968, *Journal of Geophysical Research*, volume 73, pages 4,741 to 4,750). Brittle-ductile transition in rocks.

Byerlee, J.D. (1975, *International Journal of Rock Mechanics and Mining Science*, volume 12, pages 1 to 4). The fracture strength and frictional strength of Weber sandstone.

Byerlee, J.D. (1978, *Pure and Applied Geophysics*, volume 116, pages 615 to 626). Friction of rocks.

Interesting work, relating to fluid pressure in rock formations, is found in the following papers:

Berg, R.R. and Habeck, M.F. (1982, *Transactions-Gulf Coast Association of Geological Societies*, volume 32, pages 247 to 253). Abnormal pressures in the lower Vicksburg, McAllen Ranch Field, South Texas. *Comment:* Well data from the McAllen Ranch Field, Texas, suggest that overpressure in this reservoir is related to the dehydration of montmorillonite to illite during non-equilibrium compaction.

Ham, H.H. (1966, *Oil and Gas Journal*, volume 64, pages 58 to 63). New charts help estimate formation pressures. *Comment*: Estimates of pore pressures in formations, due to lithostatic loads, are displayed in simple terms in this early work.