Chapter 5: Elasticity

AT LOW temperatures and shallow crustal levels, rock behaves brittly and, consequently, will fracture at high stress and is elastic in response to low stress. At the higher temperatures of deeper crustal levels, rocks initially, also, deform by instantaneous elastic distortion if subjected to a deviatoric stress. However, the elastic distortion is negligible compared to the large distortions caused by the slow flow or crystalline creep, occurring when the stress is maintained for geologically significant periods. In order to understand why rock can both fracture in a brittle fashion and flow in a ductile fashion, it is necessary to understand some basic principles of rock rheology. This chapter provides, together with chapters six to eight, a state-of-the-art outline on rock rheology, and highlights which parameters have a principal influence on the mechanical response of rocks.

Contents: Section 5-1 outlines mechanical model analogs for simulating elastic behavior. Section 5-2 introduces basic measures of strain and stress-strain and strain-time graphs. The elastic moduli and their relationship are discussed in sections 5-3 to 5-7. Some practical applications of crustal stress calculations, associated with elastic loading, are outlined in section 5-8. Anelasticity and the standard linear solid are introduced in section 5-9.

Practical hint: **Elasticity theory becomes much more digestible and interesting by investigating the rheology of a variety of strips of natural and synthetic rubbers, both compressible and incompressible. Attempt to measure or estimate some of the elastic moduli.**

5-1 Elasticity models

The initial response of any rock subjected to a deviatoric stress is instantaneous elastic distortion. The amount of elastic strain is extremely small, rarely more than a few percent, but elasticity provides a mechanism for propagating various types of elastic waves through the Earth's interior. Elastic distortion is *recoverable*, because the rock will resume its initial shape if the stress is removed. This behavior is commonly portrayed by a *mechanical spring model* or *Hookean element* (Fig. 5-1a). Shallow crustal rocks principally deform elastically, possibly followed by *permanent* or *irreversible distortion*, due to rock failure if the distortion exceeds the elastic limit. The frictional plastic movement over any fault plane after failure can be represented by a *frictional plastic unit* (Fig. 5-1b). The failure itself, or, in case of a pre-existing fault, the static friction, is, perhaps, best resembled by a *plug-pull model*, first introduced here (Fig. 5-1c). The complete process of elastic distortion, failure, and subsequent frictional plasticity over fault planes can be summarily represented by the mechanical array of Figure 5-1d, showing a spring, plug, and frictional unit in series. The model will be slightly modified later to account for anelastic behavior observed in rocks (section 5-9).

Elasticity of rocks arises from the interatomic forces, maintaining each atom in its lattice position. A crystal lattice will aim at a low energy state by balancing *repulsive and attractive interatomic forces*. If placed under tension, the interatomic attraction resists the extension, and, similarly, shortening is resisted by interatomic repulsion. Volume change is limited by a general resistance to any changes in lattice spacings. Individual crystals will contribute to the overall elastic behavior of a rock. The elastic nature of crystal lattices may be represented by emplacing a mechanical spring model inside the crystal lattice and representing each interatomic bond by a spring of a particular stiffness (Fig. 5-2). Because it is impractical to sum up all the individual elastic stiffnesses of the bonds between atoms, rock mechanics provides so-called *elastic moduli*, valid for a particular type of macroscopic rock deformation and the specific stress applied.

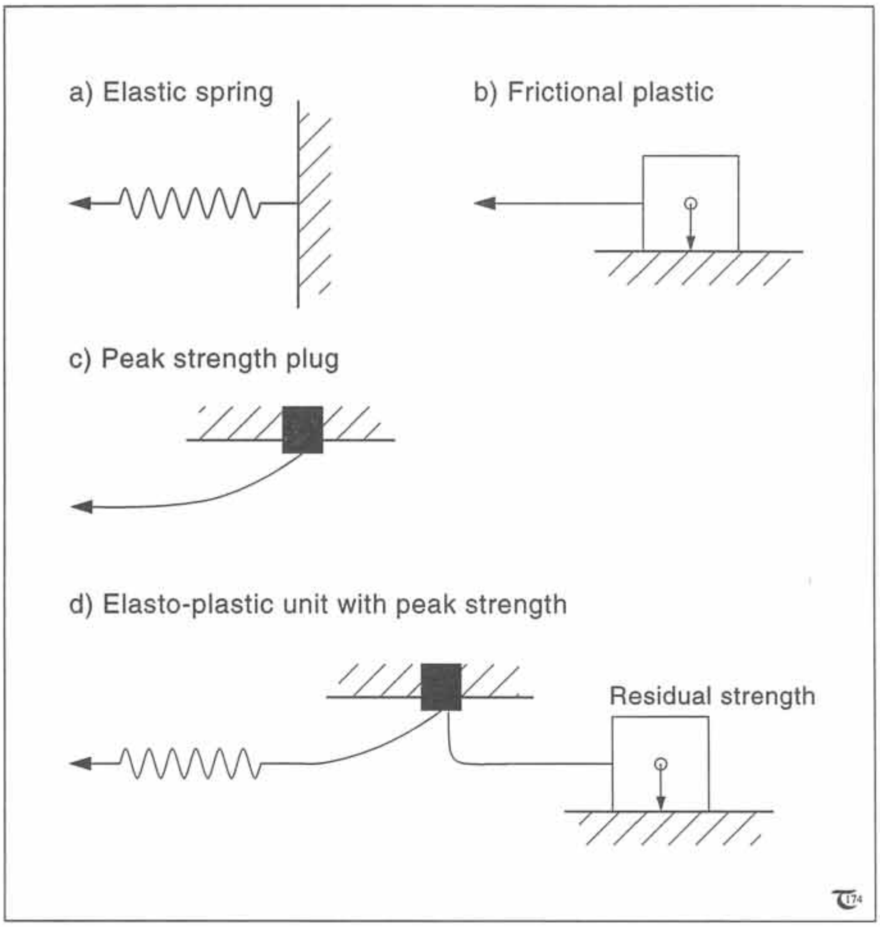


Figure 5-1: *a) to d) Mechanical models used as analogs to visualize the rheological behaviour of rocks in the elasto-plastic regime.*

☐**Exercise 5-1: Produce a qualitative graph, showing the force required to maintain motion at all stages for the case of an elasto-plastic unit with peak strength (Fig. 5-1d).**

5-2 Elastic strains, creep tests, and constant strain-rate tests

The elastic moduli for rocks, defined in sections 5-3 to 5-7, are based on laboratory measurements of two types of strain or distortion. These are the elongation, e, and the angular shear strain, γ. Either of these strain measures are tensor quantities, discussed in detail in chapters eleven and twelve. However, if the direction of elongation is known, a simple definition can be used (Fig. 5-3a & b):

$$e=(L-L_0)/L_0 \quad (5\text{-}1)$$

with the intial line length, L_0, and the deformed line length, L. Any elastic strains in rocks are extremely small, unlike that suggested by Figures 5-3a & b. Elastic strains in rocks are treated as infinitesimal quantities.

In three dimensions, three principal elongations may exist, mutually perpendicular and denoted by subscripts, e_1, e_2, and e_3. Extension is counted positive, and shortening by compression is negative. If the deformation involves no volume change, $e_1+e_2+e_3=0$. Any fractional change of the volume, V, can be estimated from:

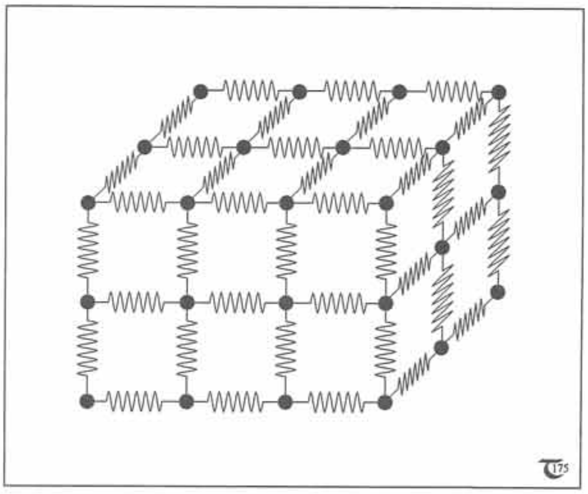


Figure 5-2: Spring model of crystal lattice. This representation may help to understand the significance of elastic moduli used in rock mechanics.

$$\delta V/V=e_1+e_2+e_3 \quad (5\text{-}2)$$

Expression (5-2) is valid only for infinitesimal strains ($e_1,e_2,e_3<<1$). The other measure of strain, the *angular shear strain*, γ, was intro-

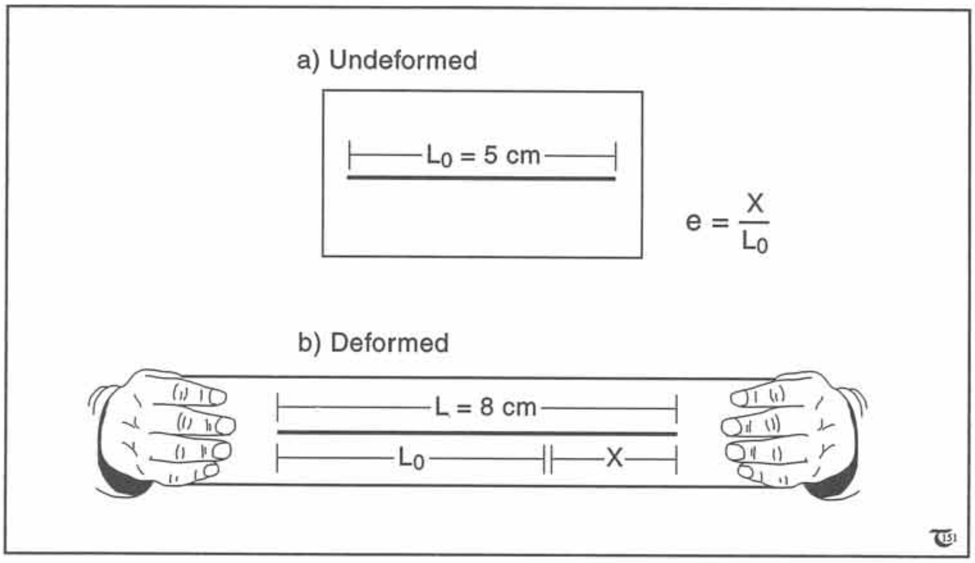


***Figure 5-3:** a) & b) Pure elongation. (a) Undeformed line of initial length, L_o. (b) Deformed line of length $L=L_o+x$. The elongation equals the length increase, x, normalized by L_o.*

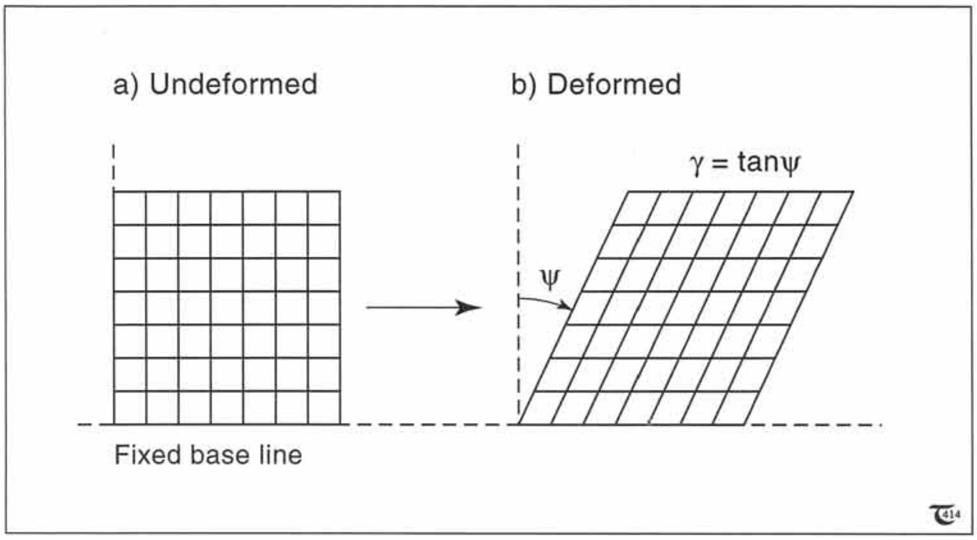


***Figure 5-4:** a) & b) Definition sketch of angular shear strain, $\gamma = \tan \psi$.*

The rheological behavior of visco-elastic materials is determined in laboratory tests, using either a *constant load (creep test)* or a *constant strain-rate* test. In the constant load test, the strain accumulation is monitored versus time and an instantaneous strain results if the rheology is truly elastic (Fig. 5-5a). A test with a constant strain-rate means that the monitored stress increases linearly with strain over time (Fig. 5-5b).

duced to measure the *distortional rotation*, rather than the change in length of material lines. The angular shear strain is established by measuring the angle, ψ, by which the angle between two initially perpendicular lines, changes (one of these two reference lines is assumed to remain stationary during the deformation) (Fig. 5-4a & b):

$$\gamma = \tan \psi \qquad (5\text{-}3)$$

5-3 Poisson ratio

Elastic volume changes may occur in stressed rocks. The measure defined to quantify the volumetric distortion of a block with free lateral boundaries is the *Poisson ratio*, ν:

$$\nu = -e_3/e_1 \qquad (5\text{-}4)$$

The magnitude of ν is determined by uniaxial shortening of a specimen in the σ_1-direction with *free lateral boundaries* and monitoring all three principal elongations, $e_{1,2,3}$ (Fig. 5-7). It should be noted that ν is a non-dimensional material constant. No elastic volume change will occur if ν equals 0.5,

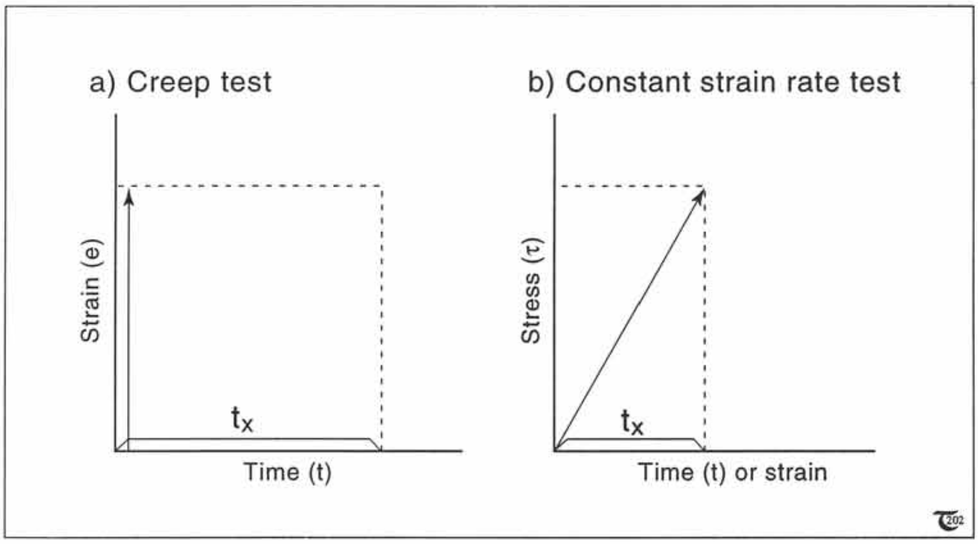


***Figure 5-5:** a) & b) Typical graphs used to portray the elastic response of rock samples deformed under (a) constant load or creep test, and (b) constant strain-rate test.*

because then all shortening in the e_1-direction will be compensated for by proportional extensions of $e_3=-0.5e_1$ and $e_2=-0.5e_1$, so that the volume remains constant, i.e.:

$$e_1+e_2+e_3=0 \qquad (5\text{-}5)$$

It is worth noting that e_1 is negative for shortening, as follows from equation (5-1). The Poisson ratio for natural rubber is 0.49, implying that the resistance to compression is much greater than the resistance to shearing (i.e., $\kappa >> G$, see later). Poisson's ratio of rocks typically is 0.25

□Exercise 5-2: a) Consider a constant load or creep test, and indicate in Figure 5-5a what happens if the load is removed after time lapse, t. b) Consider a constant strain-rate test, and indicate in Figure 5-5b what will happen with the stress if the strain rate or deformation rate suddenly ceases after some time, $t=t_x$ (or finite strain, $e=e_x$). c) A third type of rheological test, a so-called *stress-relaxation test* applies a particular instantaneous strain and monitors the resulting stress versus time. Plot the stress-time graph for a stress relaxation test in an ideal elastic material.

□Exercise 5-3: Consider the elastic spring model combined with a plug model (Fig. 5-6). a) Subject the plug-elastic unit to a creep test, where the load used exceeds the plug strength. Sketch the strain-time graph, recording the event. b) Subject the plug-elastic unit to a *constant strain-rate test*, and graph the stress-time (or stress-strain) curve.

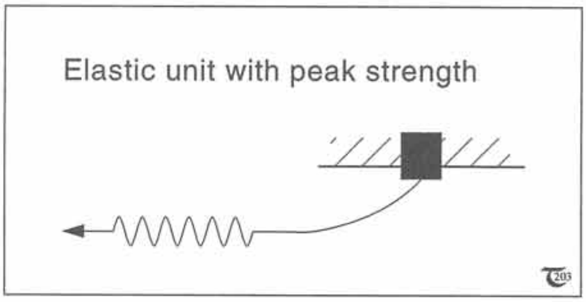


Figure 5-6: Elastic spring model, combined with a peak strength plug model. See exercise 5-3.

□Exercise 5-4: Consider the frictional plastic unit of Figure 5-1b. A creep test will reveal two cases: the load applied is either too small or large enough to cause movement. If the load is large enough, the shear stress on the fault plane is constant (equal to the frictional strength). Plot a graph of stress versus time.

□Exercise 5-5: Consider the elasto-plastic unit with plug peak strength, as portrayed in Figure 5-1d. The peak strength of the plug is taken here as twice the frictional strength of the plastic unit. Produce the stress-time plot for a constant strain-rate test.

Table 5-1: *Elastic parameters for common rocks.*

Rock	Young modulus (GPa)	Shear modulus (GPa)	Poisson ratio
Ice		9.2	0.03
Halite		30	0.15
Shale	10-30	14	-
Sandstone	10-60	4-30	0.20-0.30
Limestone	60-80	20-30	0.25-0.30
Dolomite	50-90	30-50	-
Marble	30-90	20-35	0.10-0.40
Gneiss	4-70	10-35	0.04-0.15
Amphibole	-	50-100	0.40
Basalt	60-80	30	0.25
Granite	40-70	20-30	0.10-0.25
Diabase	80-110	30-45	0.25
Gabbro	60-100	20-35	0.15-0.20
Diorite	60-80	30-35	-
Dunite	140-160	60-70	-

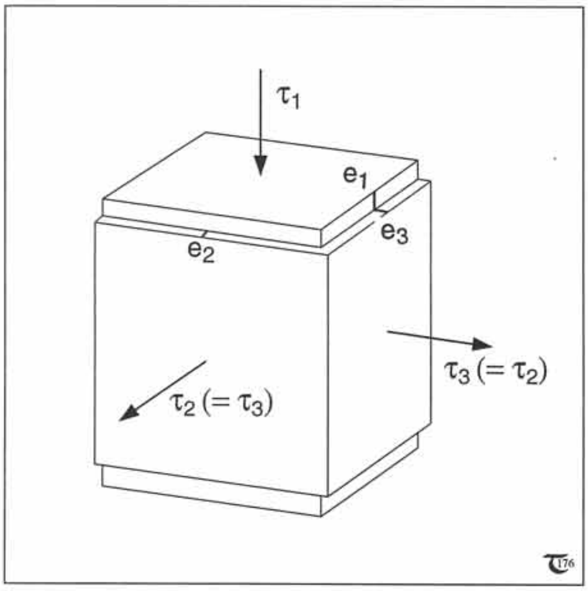


Figure 5-7: *Orientation of principal elongations in response to uniaxial stress test for establishing the Poisson ratio.*

(Table 5-1), implying that $e_2 = e_3 = -0.25e_1$, only half the extension expected if volume were to remain constant. However, although this suggests large compressibility, any elastic volume changes remain minuscule, simply because the total amount of elastic strain in rocks always remains very, very small indeed.

The Poisson ratio is commonly determined under atmospheric pressure (because of the free lateral boundaries), but due to linear elasticity any state of stress could be superimposed. Thus, the Poisson effect may be significant to the deformation of mantle rocks under extremely high pressures. Also, ν increases if the temperature comes near the solidus. The Poisson ratio, further, is of principal interest to engineering applications. However, the Poisson effect is insignificant for the deformation of crustal rocks. Occasionally, the Poisson number, m, defined as the reciprocal of the Poisson ratio, i.e., $m = 1/\nu$, is used in the literature.

Table 5-2: *Stress-strain data for every fifth increment of loading in uniaxial compression test of Berea sandstone at constant strain-rate. See graph of Figure 5-8. Courtesy Zaki Al-Harari.*

Load (MPa)	Axial strain	Lateral strain	Volumetric strain
-0.67	-6.00×10^{-6}	-5.02×10^{-6}	-1.60×10^{-5}
-2.27	-1.56×10^{-4}	2.61×10^{-5}	-1.04×10^{-4}
-4.10	-3.13×10^{-4}	6.53×10^{-5}	-1.82×10^{-4}
-6.50	-4.48×10^{-4}	1.09×10^{-4}	-2.29×10^{-4}
-8.83	-6.10×10^{-4}	1.72×10^{-4}	-2.66×10^{-4}
-11.39	-7.44×10^{-4}	2.46×10^{-4}	-2.52×10^{-4}
-14.90	-8.78×10^{-4}	3.27×10^{-4}	-2.23×10^{-4}
-18.01	-1.04×10^{-3}	4.34×10^{-4}	-1.69×10^{-4}
-20.80	-1.19×10^{-3}	5.68×10^{-4}	-5.39×10^{-5}
-24.16	-1.34×10^{-3}	7.24×10^{-4}	1.14×10^{-4}
-27.23	-1.49×10^{-3}	9.01×10^{-4}	3.13×10^{-4}
-30.65	-1.63×10^{-3}	1.13×10^{-3}	6.27×10^{-4}
-33.12	-1.80×10^{-3}	1.42×10^{-3}	1.04×10^{-3}
-36.04	-1.95×10^{-3}	1.83×10^{-3}	1.70×10^{-3}
-38.09	-2.13×10^{-3}	2.43×10^{-3}	2.73×10^{-3}
-39.09	-2.29×10^{-3}	3.45×10^{-3}	4.62×10^{-3}

Because of the extremely small elastic strains of 10^{-3} in rocks subjected to large stresses and pressures on the order of 100 MPa, experiments in rock mechanics require high precision for determining the elastic moduli. The uniaxial compression test of exercise 5-7 allows estimation of the Poisson ratio and other elastic moduli (see, also, exercise 5-10). However, greater accuracy in measurement of the elastic moduli is most practically achieved measuring wave velocities, using either ultrasonic waves or by measuring resonance frequencies of vibrating samples.

□Exercise 5-6: Show that the Poisson ratio for homogeneous strains must be 0.5 if a rock is to be incompressible.

□Exercise 5-7: Figure 5-8 illustrates a constant strain-rate test by uniaxial loading of a sample of Berea sandstone at a rate of 0.689 MPa per second (or 100 psi/sec). Table 5-2 is a sample list of the recorded data. Volume change occurs. Positive volumetric strains indicate volume increase and negative values denote volume decrease. Compute the Poisson ratio for Berea sandstone for the early load increments. The Poisson ratio is defined only for small or infintesimal strains, for which the assumption of linear elasticity is valid.

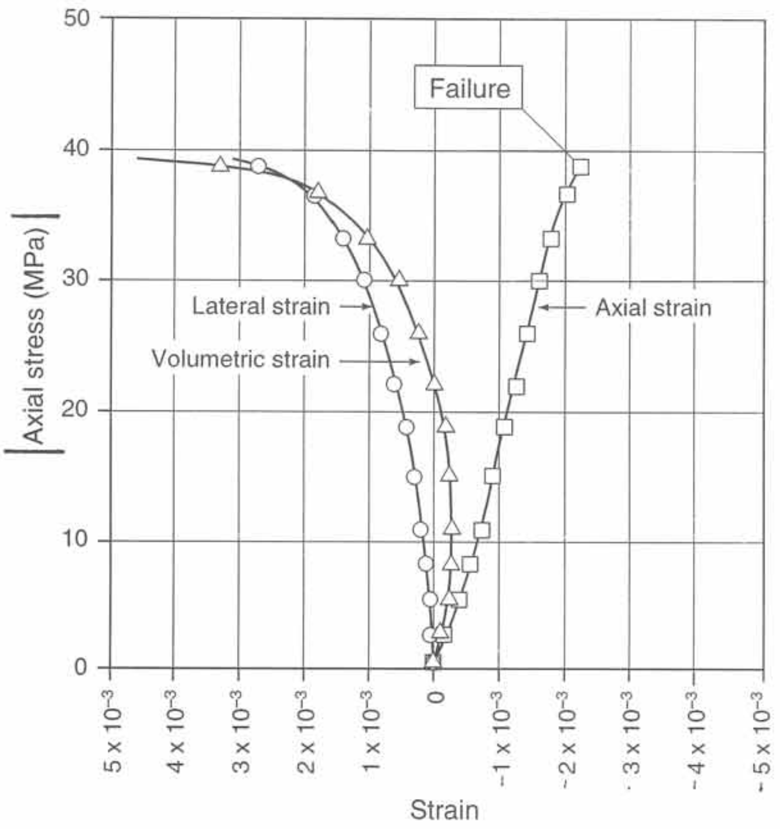


Figure 5-8: Stress-strain graph of the axial and lateral elongations in uniaxial creep test on Berea sandstone at room conditions. See, also, Figure 6-19.

5-4 Compressibility and bulk modulus

Another measure for elastic volume change, but with confined lateral boundaries and therefore due to a pressure and not a deviatoric stress, is the *compressibility*, β:

$$\beta = -\delta V/(V\delta P) \qquad (5\text{-}6a)$$

with $\delta V/V(=e_1+e_2+e_3)$ measuring the fractional volume change in response to a change in pressure, δP. No volume change or dilation occurs if $\delta V/V=0$. An alternative measure is the *bulk modulus* or *incompressibility*:

$$\kappa = 1/\beta \qquad (5\text{-}6b)$$

All matter exhibits a bulk modulus. Gases are easily compressible and elastic, but these have no elastic moduli, except for the bulk modulus, which typically is 0.1031 MPa for ideal gases at atmospheric pressure. Natural rubber has a bulk modulus of 19 MPa. The bulk moduli for liquids are of the order of 1 GPa, with the exception of mercury with $\kappa=25.9$ GPa. Diamond, possibly the least compressible substance in the Universe, exhibits a bulk modulus of 550 GPa.

The bulk modulus for common crustal rocks typically has the order of magnitude of 100 GPa. This means that upon burial to 10 km depth, where the lithostatic pressure is 250 MPa or 0.25 GPa, the elastic volume decrease is only $\delta V/V = \delta P/K = 0.25$ GPa/100 GPa$=0.0025$ times. For spatially uniform dilation this means each elongation, $e_i=0.0025/3=0.0008$ or 0.08% (or 0.8^3 mm^3 per 1 m^3); and, therefore, it remains entirely negligible in crustal deformations where the depth scale limits δP. However, in the lower mantle, elastic compression amounts up to 30 to 40 percent.

In crustal rocks, any volumetric compression by pressure increase is largely counteracted by geothermal expansion upon burial. This becomes clear, considering the *thermal expansivity*, α:

$$\alpha = \delta V/(V\delta T) \qquad (5\text{-}7)$$

with temperature change δT. The thermal expansivity of rocks typically is of the order 10^{-5} K^{-1}. The implied fractional expansion occurring in crustal rocks buried to 10 km depth, using a moderate geothermal gradient of 30 K km^{-1}, is $\delta V/V=\alpha(\delta T)=10^{-5}[K^{-1}]\text{x}300\,[K]=0.003$ or 0.3%.

□Exercise 5-8: Calculate the volume change of air in a bicycle pump if the air has a bulk modulus of $\kappa=0.1$ MPa and the pressure reaches 10 atmosphere or about 1 MPa.

5-5 Young modulus

The elastic modulus, which relates the elastic elongation, e_1, to the normal stress, here σ_1, in Poisson's shortening experiment (Fig. 5-7), is given by *Hooke's law*:

$$\sigma_1 = Ee_1 \qquad (5\text{-}8)$$

using E for the *Young modulus*.

Robert Hooke, who published the *True Theory of Elasticity* in 1678, considered only *linear elastic* materials, where doubling of the tension (or deviatoric stress τ_1) leads to doubling of the extension (e_1). Many materials comply with linear elasticity. The Young modulus for natural rubber is 0.86 MPa, compared to 100 Pa for light, open-pore foam rubbers. Young's modulus in crustal rocks is approximately linear and generally varies between 10 and 100 GPa (Table 5-1).

In any case, deviatoric stresses of tectonic origin in the crust reach magnitudes of 10 to 100 MPa and the implied elastic elongation is $e_1 = \tau_1/E = 100$ MPa/10 GPa$=0.01$ or 1%, truly negligibly small.

□Exercise 5-9: a) Estimate the elastic elongation, occurring if you stretch foam rubber (E=100 Pa) with a tensional stress, τ_1=1 kPa. b) Alternatively, estimate the stress, τ_1, that would arise if you extended the same foam by 100%, so that e_1=1.

□Exercise 5-10: Determine the Young modulus for Berea sandstone, at each load increment, using the data referred to in exercise 5-7.

□Exercise 5-11: Suppose a cylinder of diabase, 10 cm in diameter and 25 cm long, is placed under an axial compression of -10 MPa. How much will it shorten axially and how much will it expand transversely, given a Young modulus of 10^{11} Pa and a Poisson ratio of 0.25?

5-6 Shear modulus

The *shear modulus* or *rigidity*, G, relates any elastic angular shear strain, γ, to the applied shear stress, τ_S:

$$\tau_S = G\gamma \qquad (5\text{-}9)$$

The shear modulus for natural rubber is 0.29 MPa, and for rocks it typically is of the order of 10 GPa (Table 5-1). Crustal shear stresses are up to 100 MPa, so that the maximum elastic shear strain in the crust is $\gamma=\tau_S/G=0.01$, implying a possible angular distortion of only half a degree, again negligibly small. However, accumulation of stress, by localized, elastic shear before frictional slip, features centrally in the so-called stick-slip model for episodic seismicity, and, therefore, it is important in geoscience (Figs. 5-9 & 5-10). Initially, the elastic shear increases with time, due to a uniform velocity occurring at a distance from the fault. Fault slip occurs when the shear stress of equation (5-9), resulting from the accumulated shear strain, matches the frictional strength of the fault surface. The elastic shear strain is then released coeval with a drop in the stress, but both may build up again provided the differential velocity of plate segments is maintained.

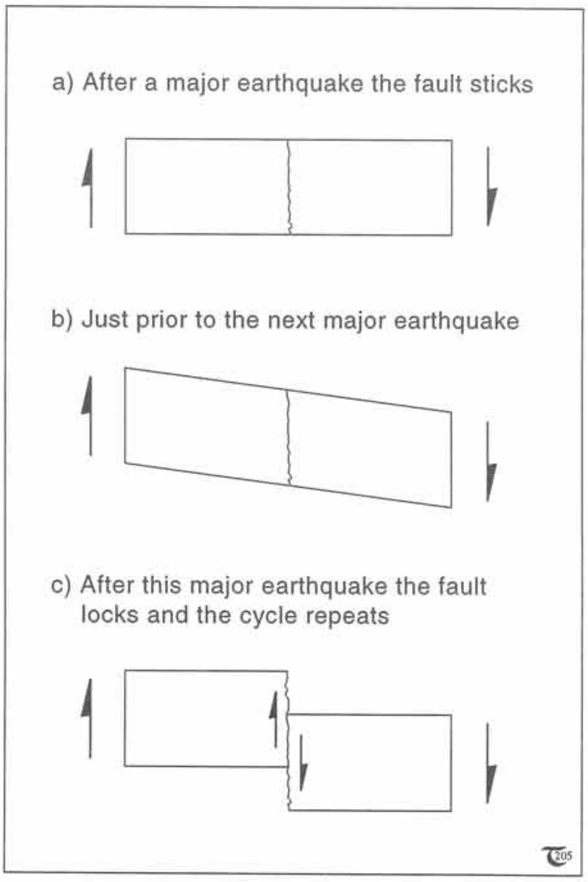


Figure 5-9: a) to c) Three basic stages in the evolution of seismic fault motion, according to the stick-slip model. The relative motion of plate segments is indicated by the half-arrows.

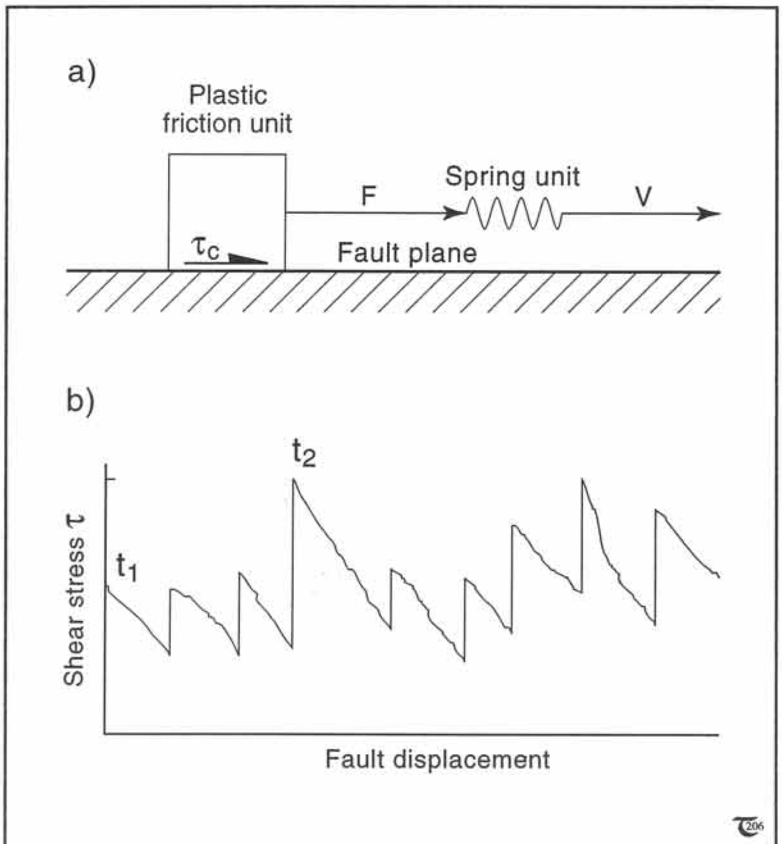


Figure 5-10: *a) & b) Mechanical analog for the stick-slip mode of fault motion. (a) Spring, pulled with constant velocity, v, exerts a pull force, F, on the plastic unit. Slip occurs when F results in stress exceeding the critical shear stress,* τ_c*. (b) The stress-slip graph for stick-slip fault motion. Critical shear stress for times,* t_1 *and* t_2*, differ because of differences in sticking strength, due to variations in sealing bonds by mineralization across the fault surface.*

□Exercise 5-12: The angular shear strain on the San Andreas Fault system, associated with the 1906 San Francisco earthquake, was 2.5x10^{-4}. The local shear modulus of the rocks involved is 30 GPa. How much was the system's stress reduction by the earthquake if all the shear strain is due to elastic relaxation only (Figs. 5-9 & 5-10)?

5-7 Relationship of elastic moduli

The *elastic moduli and constants* (ν,κ,E,G) for isotropic rocks *are interdependent*, and, if any two are known, the others follow:

$$G=E/(2+2\nu) \quad (5\text{-}10)$$
$$\kappa=E/(3\text{-}6\nu) \quad (5\text{-}11)$$
$$\nu=(3\kappa\text{-}2G)/(6\kappa+2G) \quad (5\text{-}12)$$

quations (5-10) and (5-11) allow the plotting of the ratios, E/G, E/κ, and G/κ, as a function of the Poisson ratio (Fig. 5-11). The ratio of the Young modulus to the shear modulus varies only between 2 and 3, implying that, in any elastic material, the Young modulus is only between twice to thrice the value of the shear modulus. The ratios, containing the bulk modulus, also plotted in Figure 5-11, appear to vary greatly with the Poisson ratio. Materials with smaller Poisson ratios tend to have larger bulk moduli as compared to those of materials with larger Poisson ratios. Actually, many materials with ν close to 0.5 have bulk moduli, κ, smaller than those for materials with $\nu<<0.5$, that is, they are *more compressible* than materials with smaller Poisson ratios. The behavior of liquids is particularly interesting. Their shear modulus G is zero, and κ is small, so that ν is close to 0.5 [cf. eq. (5-12)]. But it is incorrect to conclude that liquids are *incompressible*. Rather, their resistance to compression is much greater than their resistance to shearing, i.e., $\kappa>>G$. Generally, materials of Poisson ratios close to 0.5, possess Young and shear moduli up to two orders of magnitude smaller than the bulk modulus.

In addition to the elastic moduli outlined above, we introduce the Lamé constant, λ:

$$\lambda=(\nu E)/[(1+\nu)(1-2\nu)] \qquad (5\text{-}13)$$

Combining equations (5-13) and (5-11) shows that the Lamé constant approaches to the value of κ for Poisson ratios close to 0.5. The elastic moduli of mantle rocks are estimated by a range of geophysical techniques. Figure 5-12 illustrates a classical graph by Sir Edward Bullen with estimates of the moduli for the Earth mantle.

□Exercise 5-13: Examine Figure 5-11 and explain why materials of Poisson ratio 0.5 have infinitely small ratios G/κ.

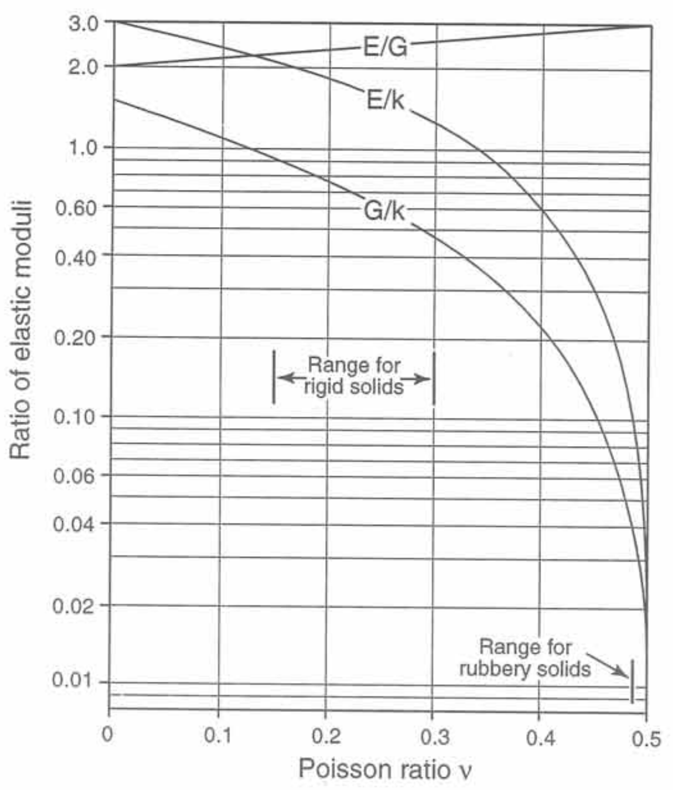


Figure 5-11: Relationship between ratios of the various elastic moduli and the Poisson ratio.

5-8 Elasticity and lithostatic pressure

Although the elastic distortion of crustal rocks remains small (Fig. 5-13), elastic deformations become of tectonic significance if occurring over extremely large length scales. Examples are the elastic bending or flexing of the upper layers of crustal plates, due to sedimentary loading, glacial loading, tidal forces, or slab subduction. However, this elastic deformation is recoverable and causes negligible distortion on the outcrop scale if the elastic stress remains below the yield stress. Two important aspects of elasticity are: (1) failure occurs when the yield stress exceeds the elastic tolerance, and (2) deviatoric stresses, arising from the elastic energy, may explain jointing of surface rocks if deloaded.

The mean pressure in incompressible rocks due to the lithostatic load is simply given by P= $|\sigma_{mean}|=|(1/3)(\sigma_1+\sigma_2+\sigma_3)|=\rho gz$. (For this sign convention $\tau_i=\sigma_i+P$ if σ is negative.) However, the *pressure* will be affected if the Poisson ratio indicates that rocks are compressible, which can be physically interpreted as follows: The loading of jointed rock and associated elastic vertical compression and prevention of lateral elongation cause horizontal stresses upon burial. A relationship between the resulting total stress and the Poisson ratio, is:

$$\sigma_{2,3}=[\nu/(1-\nu)]\sigma_1 \qquad (5\text{-}14)$$

with $\sigma_1=-\rho gz$ (Fig. 5-14). The pressure at any depth is given by:

$$P=|(1/3)(\sigma_1+\sigma_2+\sigma_3)|=[(1+\nu)/(3-3\nu)]\rho gz \qquad (5\text{-}15)$$

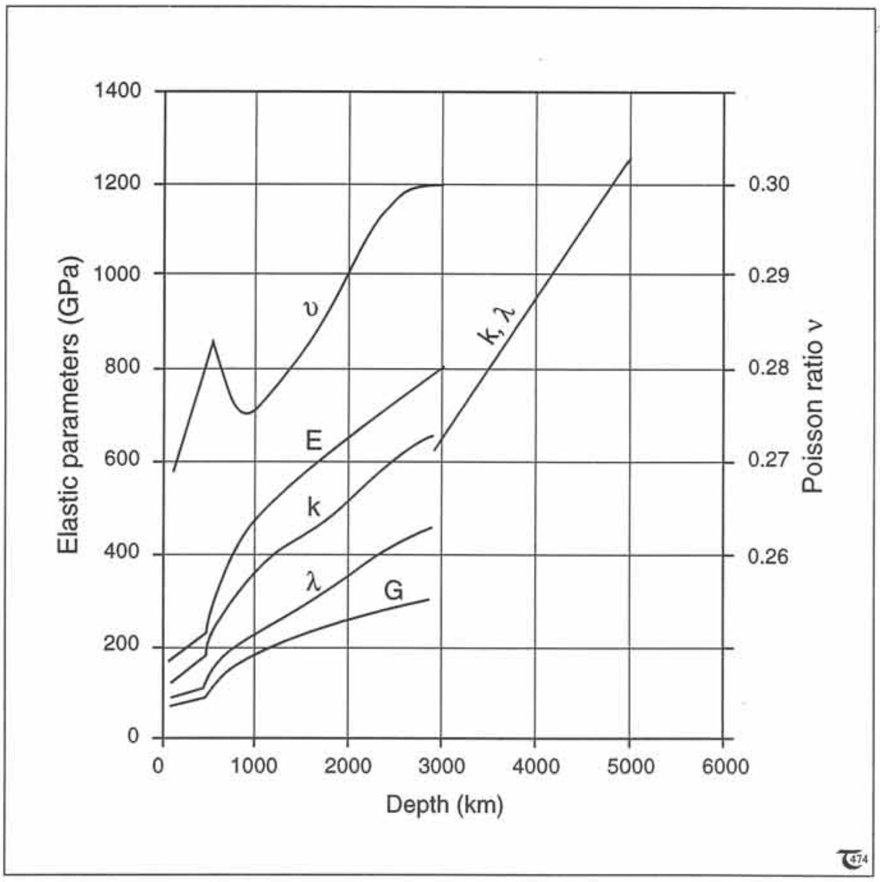


Figure 5-12: Estimates of the elastic moduli (E, G, and κ) and elastic constants (ν and λ) for the mantle of the Earth. The Poisson ratio is scaled at the right-hand ordinate; other elastic parameters are scaled at the left ordinate.

Expression (5-15) indicates that any pressure P for $\nu=0.25$ will be only 5/9 times the lithostatic stress. This essentially implies the existence of a deviatoric stress, comprising vertical compression and horizontal tension:

$$\tau_1=\sigma_1+P=-[(2-4\nu)/(3-3\nu)]\rho gz \qquad (5\text{-}16a)$$
$$\tau_{2,3}=\sigma_{2,3}+P=[(1-2\nu)/(3-3\nu)]\rho gz \qquad (5\text{-}16b)$$

It must be concluded that lithostatically loaded rocks experience tiny volumetric distortions, leading to elastic deviatoric stresses. Their magnitude for z=5 km, $\rho=3{,}000$ kg m^{-3}, and $\nu=0.25$ gives $\tau_1=-66$ MPa and $\tau_{2,3}=33$ MPa. These stresses then reverse upon deloading by erosion and may contribute to jointing of shallow crustal rocks.

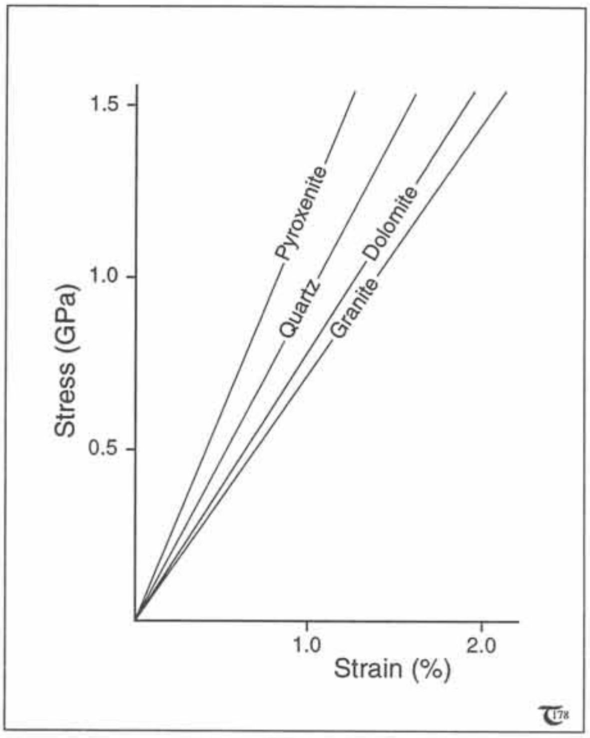
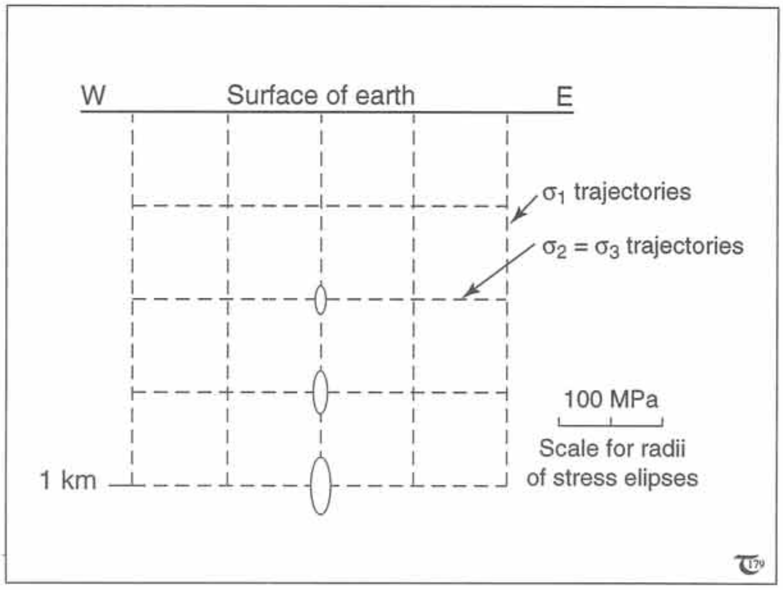


Figure 5-13: Stress-strain graph for constant strain-rate tests on common rock types.

Figure 5-14: Stress trajectories and stress magnitude in vertical crustal section, due to lithostatic load only.



□Exercise 5-14: Determine the maximum possible stress that will relax after erosion of a 10 km thickness of granite. Assume that the initial state of stress is lithostatic and that ρ=2,700 kg m^{-3} and ν=0.25.

□Exercise 5-15: Consider a lithostatically loaded rock column of ν=0.25. The tectonic stress is a uniaxial horizontal compression, $\tau_{TECTONIC}$=-100 MPa, parallel to τ_3. a) Plot the magnitude of the three principal total deviatoric stresses, τ_1, τ_2, and τ_3, versus depth. b) Next, assume the rock is part of an area subjected to a regional deviatoric stress of tectonic origin. Figure 5-15 shows a vertical section of the region's stress trajectories and stress ellipse shapes, in qualitative fashion, perpendicular to the τ_3-trajectories (compare with Fig. 5-14). For the data considered here, at which depth occurs the level of isotropic points (where $\tau_1=\tau_2$)?

□Exercise 5-16: An initially stress-free, horizontal surface is covered and buried by the deposition of 5 km of sediment with density 2,500 kg m^{-3}. If the surface is laterally constrained and has a Poisson ratio of 0.25, what are the three principal stresses at the original surface?

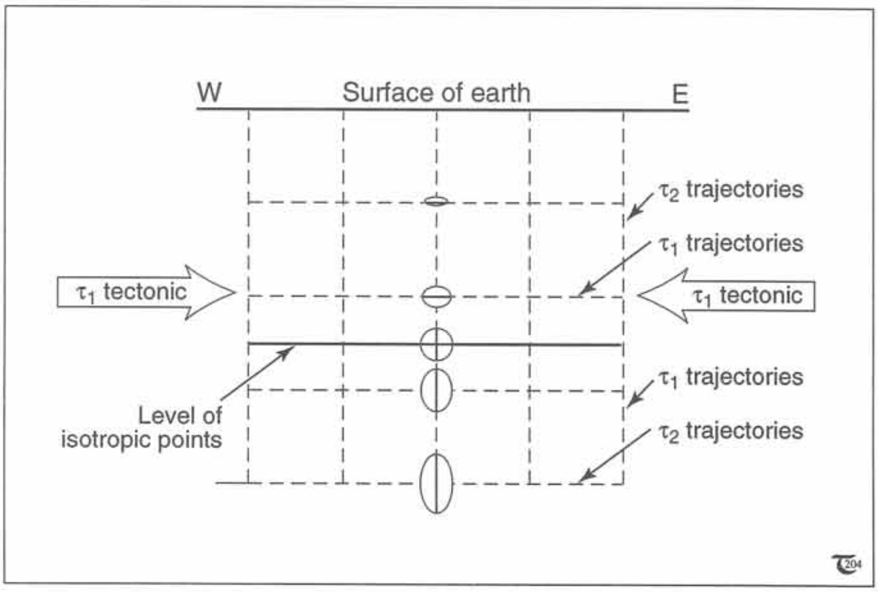


Figure 5-15: Stress trajectories and stress magnitude in crustal section of Figure 5-14 after superposition of a uniform horizontal compression.

5-9 Anelasticity

It appears that, during laboratory tests on rock samples, recoverable deformation, which is not instantanous, occurs. This time-dependent elasticity, or so-called *anelasticity*, is apparent from the generalized curve of the elastic strain, accumulating with time in an axially loaded sample (Fig. 5-16a). Anelasticity can be best represented by the mechanical model of a *Standard Linear Solid* (SLS), comprising a spring in series with a *Kelvin-Voigt unit*, itself comprising a spring and parallel viscous element, in which the deviatoric stress is proportional to the strain rate (Fig. 5-16b). The anelasticity in rocks is, also, termed recoverable *transient creep*, but, notably, it does not involve any permanent crystalloplastic deformation. The time, after which the anelastic strain reaches 1/e of its final value, is called the *strain retardation time*. The anelastic behavior in rocks is of great importance for the attenuation of seismic waves, passing through the upper mantle, as described by the *quality factor*, commonly designated as Q. Anelastic behavior of rocks, also, needs to be taken into account in mechanical engineering of tunnels, quarries, and dam constructions.

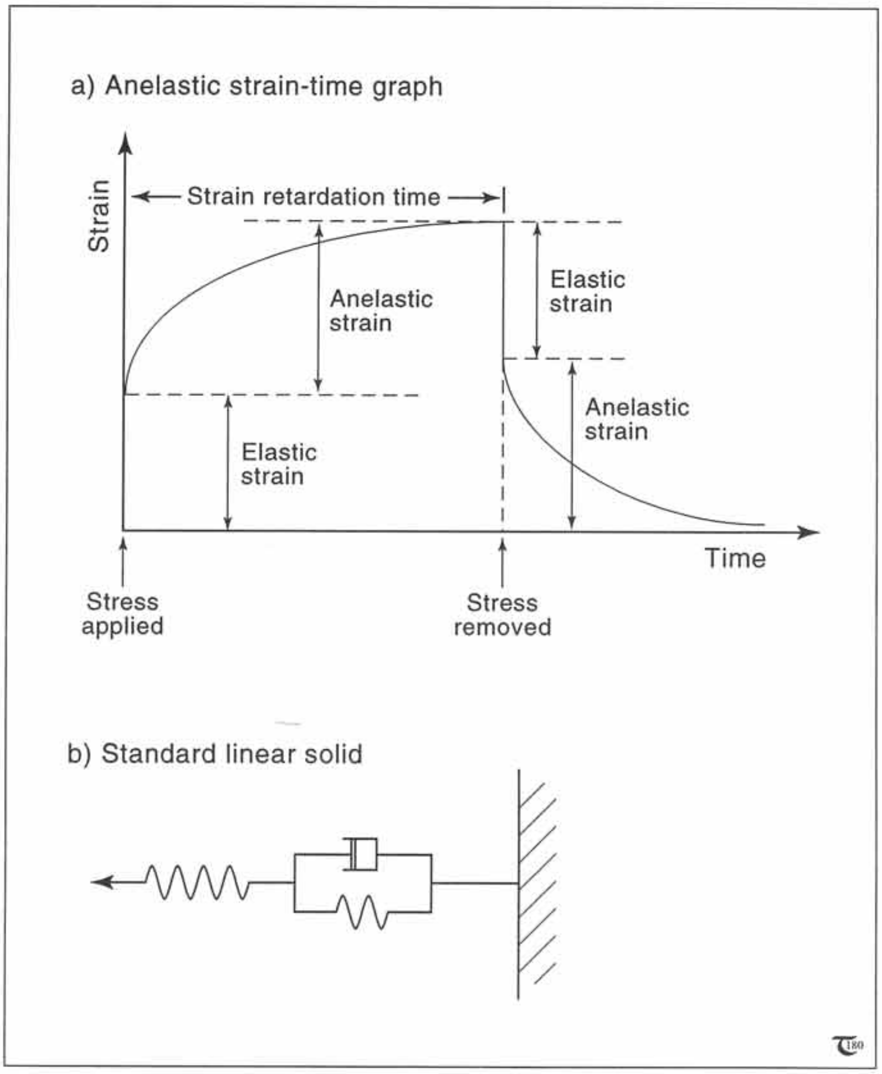


Figure 5-16: *a) Anelastic strain-time graph, and (b) mechanical model for standard linear solid.*

□Exercise 5-17: Plot the strain-time graph for a stress-relaxation test, similar to that shown in Figure 5-16a, but now for a Kelvin-Voigt unit only.

References

Detailed introductions to elasticity theory can be found in the following textbooks:

Theory of Elasticity (1970, McGraw-Hill, 567 pages), by S.P. Timoshenko and J.N. Goodier.

Elasticity, Fracture, and Flow (1971, Methuen, 3rd edition), by J.C. Jaeger.

Fundamentals of Rock Mechanics (1979, Chapman and Hall, 593 pages), by J.C. Jaeger and N.G.W. Cook.

Geodynamics (1981, Wiley, 450 pages), by G. Schubert and D. Turcotte.