FIRST PAGE MISSING

$e^{-\frac{i\mathcal{H}t}{\hbar}}$ is unitary $\rightarrow \left(e^{-\frac{i\mathcal{H}t}{\hbar}}\right)^\dagger \left(e^{-\frac{i\mathcal{H}t}{\hbar}}\right) = I$

$$e^A = I + A + \frac{A^2}{2!} + \frac{A^3}{3!} + \cdots$$

$$e^{-i\mathcal{H}} = I + (-i\mathcal{H}) + \frac{(-i\mathcal{H})^2}{2!} + \frac{(-i\mathcal{H})^3}{3!} + \cdots$$

$$(e^{-i\mathcal{H}})^\dagger = I + (-i\mathcal{H})^\dagger + \frac{(-i\mathcal{H})^{2\dagger}}{2} + \frac{(-i\mathcal{H})^{3\dagger}}{3!} + \cdots$$

$$-i^\dagger = i$$

$$\mathcal{H}^\dagger = \mathcal{H}$$

$$(AB)^\dagger = B^\dagger A^\dagger$$

$$\mathcal{H}^{2\dagger} = \mathcal{H}^\dagger \mathcal{H}^\dagger = \mathcal{H}^2$$

$$\cdots$$

$$= I + i\mathcal{H} + \frac{(i\mathcal{H})^2}{2!} + \frac{(i\mathcal{H})^3}{3!} + \cdots$$

$$= e^{+i\mathcal{H}}$$

$$(\quad)^\dagger(\quad) = e^{+i\mathcal{H}} e^{-i\mathcal{H}} = e^0 = I$$

Pauli matrices

$$\sigma_x = \begin{bmatrix} & 1 \\ 1 & \end{bmatrix} \qquad \sigma_y = \begin{bmatrix} & -i \\ i & \end{bmatrix} \qquad \sigma_z = \begin{bmatrix} 1 & \\ & -1 \end{bmatrix}$$

$$\sigma_x^2 = I \quad ; \quad \sigma_y^2 = I \quad ; \quad \sigma_z^2 = I$$

$$\sigma_k^\dagger = \sigma_k$$

$$[\sigma_x, \sigma_y] = 2i\,\sigma_z \qquad \ldots$$

$$I_x = \frac{\sigma_x}{2} \qquad I_y = \frac{\sigma_y}{2} \qquad I_z = \frac{\sigma_z}{2}$$

$$\mathcal{H} = n_0 \sigma_I + n_x \sigma_x + n_y \sigma_y + n_z \sigma_z$$

1 - qubit rotations

$$e^{-i\frac{\theta}{2}\sigma_x} = \cos\frac{\theta}{2}\, I - i\sin\frac{\theta}{2}\,\sigma_x = \begin{bmatrix} \cos\theta/2 & -i\sin\theta/2 \\ -i\sin\theta/2 & \cos\theta/2 \end{bmatrix}$$

↳ rotation about $\hat{x}$, over angle θ

$$e^{-i\frac{\theta}{2}\sigma_y} = \cos\frac{\theta}{2}\, I - i\sin\frac{\theta}{2}\,\sigma_y : \begin{bmatrix} \cos\theta/2 & -\sin\theta/2 \\ \sin\theta/2 & \cos\theta/2 \end{bmatrix}$$

$$e^{-i\frac{\theta}{2}\sigma_z} = \qquad = \begin{bmatrix} \cos\frac{\theta}{2} - i\sin\frac{\theta}{2} & 0 \\ 0 & \cos\frac{\theta}{2} + i\sin\frac{\theta}{2} \end{bmatrix}$$

$$\exp\left[-i\frac{\theta}{2}\left(n_x\sigma_x + \dots\right)\right]$$

$\hat{y}$

$\theta = 0 \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

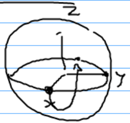
$\theta = \pi \quad \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ i \end{bmatrix} = \begin{bmatrix} -i \\ 1 \end{bmatrix}$

$|0\rangle \to |1\rangle$

$|1\rangle \to -|0\rangle$

$|0\rangle + |1\rangle \to -|0\rangle + |1\rangle$

$\theta = \frac{\pi}{2} \quad \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$

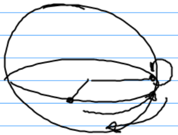


rotation axis ↔ eigenstates

Universal 1-q gates

$\forall U, \ \exists \alpha, \beta, \gamma, \delta : \ U = e^{i\alpha} R_z(\beta) R_y(\gamma) R_z(\delta)$

Thm 4.1 N & C



$R_z(-90) R_y(\theta) R_z(90)$

Any 2 axis suffice

Two - qubits

$$\frac{\sigma_z^1}{2} \otimes \frac{\sigma_z^2}{2}$$

Ising : $\mathcal{H}_I = 2\pi\hbar J \, I_z^1 I_z^2$

$$\exp\left(-i \, \frac{2\pi\hbar J \, I_z^1 I_z^2}{\hbar} \, \frac{1}{2J}\right) = \sqrt{-i}\begin{bmatrix}1&&&\\&i&&\\&&i&\\&&&1\end{bmatrix}$$

$$\text{CPHASE} = \begin{bmatrix}1&&&\\&i&&\\&&i&\\&&&1\end{bmatrix}\begin{bmatrix}1&&&\\&-i&&\\&&1&\\&&&-i\end{bmatrix}\begin{bmatrix}1&&&\\&1&&\\&&-i&\\&&&-i\end{bmatrix} = \begin{bmatrix}1&&&\\&1&&\\&&1&\\&&&-1\end{bmatrix}$$

$$\text{CNOT} = \sqrt{i}\; R_z^1(90)\; R_z^2(-90)\; \underbrace{R_x^2(90)\left(\frac{1}{2J}\right) R_y^2(90)} = \begin{bmatrix}1&&&\\&1&&\\&&&1\\&&1&\end{bmatrix}$$

Heisenberg $\mathcal{H}_H = 2\pi\hbar J \left(I_x^1 I_x^2 + I_y^1 I_y^2 + I_z^1 I_z^2\right)$

$$\exp\left(-i \, \frac{\mathcal{H}_H}{\hbar} \, \frac{1}{2J}\right) = \sqrt{-i}\begin{bmatrix}1&0&0&0\\0&0&1&0\\0&1&0&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}0\\1\\0\\0\end{bmatrix}$$

$$\left.\begin{array}{l}|00\rangle \rightarrow |00\rangle\\ |01\rangle \rightarrow |10\rangle\\ |10\rangle \rightarrow |01\rangle\\ |11\rangle \rightarrow |11\rangle\end{array}\right\} \text{SWAP}$$

CNOT + 1 qubit rots

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N&C 4.2, 4.3