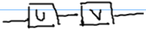
Quantum circuits

N&C 4.2, 4.3

⟶ time



⟶ time

↳ $VU|\psi_{in}\rangle$

3 qubit { 

control 1

target 2

↓

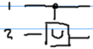
qubits

$CNOT_{12}$

$$\begin{bmatrix} 1 & & & \\ & 1 & & \\ & & & 1 \\ & & 1 & \end{bmatrix}$$

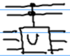
$CNOT_{21}$

$$\begin{bmatrix} 1 & & & \\ & & & 1 \\ & & 1 & \\ & 1 & & \end{bmatrix}$$

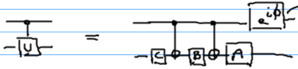


controlled-U   if qubit 1 = $|1\rangle$, then apply U to q.2

doubly controlled NOT → flips 3 iff. 1 and 2 are $|1\rangle$



Elements of universality

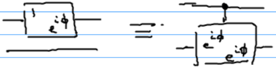


$\begin{bmatrix} 1 & \\ & e^{i\phi} \end{bmatrix}$

$\begin{bmatrix} & 1 \\ 1 & \end{bmatrix}$

$\forall U, \exists \phi, A, B, C : \begin{cases} U = e^{i\phi} A X B X C \\ ABC = I \end{cases}$

↳ N&C corollary 4.2.

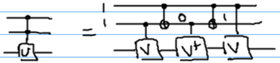
① 

$|00\rangle \rightarrow |00\rangle$
$|01\rangle \rightarrow |01\rangle$
$|10\rangle \rightarrow e^{i\phi}|10\rangle$
$|11\rangle \rightarrow e^{i\phi}|11\rangle$

② if qubit 1 is $|0\rangle \rightarrow ABC = I$ applied to q-2
$|1\rangle \rightarrow e^{i\phi} A X B X C = U$

what if q. 1 starts from $|0\rangle + |1\rangle$?
$(|0\rangle + |1\rangle)|0\rangle \rightarrow (|0\rangle + e^{i\phi}|1\rangle)|0\rangle = |00\rangle + e^{i\phi}|10\rangle$

One more example



choose $V^2 = U$

$00 \rightarrow I$

$01 \rightarrow V^{\dagger} V = I$

$10 \rightarrow V V^{\dagger} = I$

$11 \rightarrow V V = U$

$$= \begin{pmatrix} \begin{bmatrix} I & 0 \\ 0 & V \end{bmatrix} & \\ & \begin{bmatrix} I & 0 \\ 0 & V \end{bmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & & & & & & \\ & 1 & & & & & \\ & & 1 & & & & \\ & & & 1 & & & \\ & & & & 1 & & \\ & & & & & 1 & \\ & & & & & & \boxed{V} \end{pmatrix}$$

Remarks on universality

- it doesn't say anything efficiency
- some gates are easier to implement given H's available

Teleportation

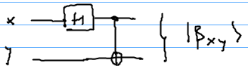
N&C 1.3.6 - 1.3.7

Transmit $|\psi\rangle$ using classical communication only

why difficult

can't measure
can't copies

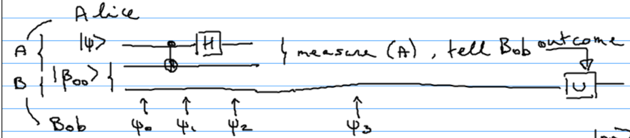
Bell states



$$
\begin{array}{llllll}
|00\rangle & \xrightarrow{H} & (|0\rangle + |1\rangle)\,|0\rangle & \xrightarrow{CNOT} & |00\rangle + |11\rangle & = \beta_{00} \\
|01\rangle & \rightarrow & (\qquad)\,|1\rangle & & |01\rangle + |10\rangle & = \beta_{01} \\
|10\rangle & \rightarrow & (|0\rangle - |1\rangle)\,|0\rangle & \rightarrow & |00\rangle - |11\rangle & = \beta_{10} \\
|11\rangle & \rightarrow & & \rightarrow & |01\rangle - |10\rangle & = \beta_{11}
\end{array}
$$

↑ computational basis

↑ Bell basis



$|00\rangle + e^{i\varphi}|11\rangle$

$|00\rangle + |11\rangle$

$\sqrt{\frac{1}{3}}|00\rangle + \sqrt{\frac{2}{3}}|11\rangle$

$$|\psi_0\rangle = (\alpha|0\rangle + \beta|1\rangle)\,(|00\rangle + |11\rangle)$$
$$= \alpha|0\rangle\,(|00\rangle + |11\rangle) + \beta|1\rangle\,(|00\rangle + |11\rangle)$$
$$|\psi_1\rangle = \qquad\qquad + \beta|1\rangle\,(|10\rangle + |01\rangle)$$
$$|\psi_2\rangle = \alpha(|0\rangle + |1\rangle)\,(|00\rangle + |11\rangle) + \beta\,(|0\rangle - |1\rangle)\,(|10\rangle + |01\rangle)$$
$$= |00\rangle(\alpha|0\rangle + \beta|1\rangle) + |01\rangle(\alpha|1\rangle + \beta|0\rangle)$$
$$+ |10\rangle(\alpha|0\rangle - \beta|1\rangle) + |11\rangle(\alpha|1\rangle - \beta|0\rangle)$$

Measure : $1/4 \to 00 \;\to\; \alpha|0\rangle + \beta|1\rangle \;\to\; I$

$1/4 \to 01 \;\to\; \alpha|1\rangle + \beta|0\rangle \;\to\; X$

$1/4 \to 10 \;\to\; \alpha|0\rangle - \beta|1\rangle \;\to\; Z$

$1/4 \to 11 \;\to\; \alpha|1\rangle - \beta|0\rangle \;\to\; ZX$

$= U$ Bob should apply

NOTE : · not faster than light ; NO copy