The DiVincenzo criteria

Five criteria that any candidate quantum computer implementation must satisfy.

Two additional criteria for quantum communication
Implementation of quantum computers
D. DiVincenzo

1. Well-defined qubits

2. Initialization to a pure state

3. Universal set of quantum gates

4. Qubit-specific measurement

5. Long coherence times
Well-defined qubits

- two-level quantum systems
  - $^1\text{H}, ^{13}\text{C}, ^{19}\text{F}, \ldots$
  - electron spin

- two-dimensional subspaces of larger systems

+ : auxiliary levels
- : leakage
$n$ 2-level systems vs. one $2^n$-level system

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<tr>
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<th>1</th>
<th>0</th>
<th>$111$</th>
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<th>110</th>
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<th>011</th>
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energy or precision $\sim n$  
scalable

energy or precision $\sim 2^n$  
NOT scalable
Ensemble quantum computer

Many identical copies of a quantum computer

Works fine -- only read-out must be modified

Image: Krenner et al, cond-mat/0505731
Many copies of each qubit, but not a quantum computer
Encoded qubits

|0⟩_L = |01⟩ - |10⟩
|1⟩_L = |10⟩ + |01⟩

Decoherence free subspace

|0⟩_L = (|01⟩+|10⟩) |0⟩
|1⟩_L = \sqrt{2/3} |001⟩ + \sqrt{1/3}(|01⟩+|10⟩)|0⟩

Trade qubits for Hamiltonian terms (e.g. exchange only QC)

Beware: leakage
Initialization to a pure state

To $|000\rangle$  
Equilibration at low temperature ($h\nu >> kT$)

Other physical mechanisms:

- Ferromagnet
- Laser cooling
- Optical pumping
- . . .

To $|\psi\rangle$  
Perform a hard, non-destructive measurement

Other physical processes

If you want qubits in $|000\rangle$, simply rotate the qubits from $|\psi\rangle$ to $|000\rangle$
Initialization timescale

e.g. equilibration can be slow (> $5\ T_1$)

Why initialization anyways?

Computation = garbage in $\Rightarrow$ garbage out

Ancilla qubits ("help" qubits)

Error correction = removing entropy from the qubits

Either initialize fast or build qubit "conveyor belt"

one-time

need continuous fresh supply
Are mixed states acceptable?

Equilibrium at moderate or high temperature (hν ≫ kT)

Mixed state

↑↑  ↑↓  ↓↑  ↓↓

Effective pure state

↑↑  ↑↓  ↓↑  ↓↓

Signal same as for pure state but amplitude ~ 1/2^n

Gershenfeld & Chuang, Science 97, Cory, Havel & Fahmi, PNAS 97
Effective pure state preparation

(1) Add up $2^N-1$ experiments (Knill, Chuang, Laflamme, PRA 1998)

\[
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+ & \\
\uparrow\uparrow & \quad \downarrow\downarrow & \quad \downarrow\downarrow & \quad \downarrow\downarrow \\
+ & \\
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\end{align*}
\]

Later $\approx (2^N - 1) / N$ experiments (Vandersypen et al., PRL 2000)

(2) Work in subspace (Gershenfeld & Chuang, Science 1997)

(3) Average over space (Cory et al., Phys. D 1998)
“Scalable” QC with hot qubits

**Goal:** obtain $k$ cold qubits from $n$ hot qubits

**Idea:** reduce the entropy of $k$ qubits, and increase the entropy of the remaining qubits (total entropy remains constant)

$$ n H(p) = k H(0) + (n-k) H(1/2) $$

$$ k = n (1 - H(p)) \approx n \varepsilon^2 $$

(Pr[0] = $p = \frac{1+\varepsilon}{2}$)

**Overhead:**

- # qubits $n \sim k$
- # operations $\sim k \log k$
- "Efficient bootstrapping" (as $k$ and $n \rightarrow \infty$)

Building block Schulman-Vazirani cooling

Step 1: \( \text{CNOT}_{12} \)

<table>
<thead>
<tr>
<th>Prob.</th>
<th>bc</th>
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<th>bc</th>
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<tbody>
<tr>
<td>( p^2 )</td>
<td>00</td>
<td>( p^2 )</td>
<td>00</td>
</tr>
<tr>
<td>( p(1-p) )</td>
<td>01</td>
<td>( p(1-p) )</td>
<td>01</td>
</tr>
<tr>
<td>( (1-p)p )</td>
<td>10</td>
<td>( (1-p)p )</td>
<td>11</td>
</tr>
<tr>
<td>( (1-p)^2 )</td>
<td>11</td>
<td>( (1-p)^2 )</td>
<td>10</td>
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If qubit \( c \) is 1, qubit \( b \) is at \( \infty \) T. If qubit \( c \) is 0, bias qubit \( b \) is \( 2\varepsilon \).

Step 2: \( \text{Fredkin}_{c,ab} \) (swap qubits \( a \) and \( b \) iff qubit \( c \) is 0)

If qubit \( b \) is 1, qubit \( a \) has bias \( \varepsilon \).
If qubit \( b \) is 0, qubit \( a \) has bias \( 2\varepsilon \).

On average, qubit \( a \) has bias \( 3\varepsilon/2 \), so it has been “cooled”.

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Universal set of quantum gates

Selective single-qubit rotations

\[ R_{\vec{n}}(\theta) = \exp(i\alpha) \exp(-i\theta \vec{n} \cdot \sigma/2) \]

Sufficient: rotations about two different axis

Almost any two-qubit gate is universal
E.g. quantum-XOR or Controlled-NOT
Coupling networks

Switchable and direct:

\[ J_{12}(t) \]
\[ J_{13}(t) \]
\[ J_{23}(t) \]

Indirect (e.g. nearest neighbour):

\[ J_{12}(t) \]
\[ J_{23}(t) \]

Fixed:

\[ J_{12} \]
\[ J_{13} \]
\[ J_{23} \]

Fixed and indirect:

\[ J_{12} \]
\[ J_{23} \]

Bus:

\[ J_1(t) \]
\[ J_2(t) \]
\[ J_3(t) \]
Indirect coupling: Shuffle qubits around

SWAP_{12} = CNOT_{12} CNOT_{21} CNOT_{12}

“only” a linear overhead ...
Removing effect of fixed couplings: refocusing

- There exist efficient extensions for arbitrary coupling networks
  Leung et al, PRA 00, Jones&Knill, JMR 99
- Refocusing can also be used to remove unwanted single-qubit terms
Even individual addressing is not strictly needed

\[ \psi_1 X A_1 B_1 C_1 A_2 B_2 C_2 A_3 B_3 C_3 \ldots A_n B_n C_n \]

\[ \psi_1 0 1 \psi_2 0 0 \psi_3 0 0 \ldots \psi_n 0 0 \]

\[ X_A \text{ flips all } A\text{'s} \]
\[ \text{CNOT}_{CA} \text{ selectively flips } A_2 \]
\[ \text{Fredkin}_{C\_AB} \text{ swaps } A_1 \text{ and } B_1 \]
\[ \text{etc.} \]

Distinct qubit at the end is needed for setting up a unique “1”

S. Lloyd, Science 261, 1569, 1993
Other requirements

Gates must be precise (systematic errors)

Calibration
Cross-talk
Left-over terms in Hamiltonian
Non-commuting terms in Hamiltonian
Hardware limitations (pulse timing, phase noise etc)

Gates must be fast

> 10000 faster than coherence time
Parallelization
Qubit measurement

Ideally: reliable hard measurement of all qubits

Acceptable in principle:

- ensemble averaged measurement of each qubit (next)
- unreliable measurement (next)
- hard measurement of a single qubit
  (swap consecutive qubits into read-out site, while maintaining coherence)
Dealing with a limited measurement fidelity

1) Repeat calculation

OK for “decision problems” (1-bit answer)
Not convenient

2) “Quantum FAN-OUT”

\[ a|0\rangle + b|1\rangle \]

\[ a|000\rangle + b|111\rangle \] majority vote

Measurement of one qubit must not disturb state of others, apart from collapse.
Ensemble averaged measurements

Say at the end of Shor’s algorithm, we have state

\[ |0110\rangle + |0011\rangle \]

Measurement on a single system gives

0100 or 0011

From either outcome, can find prima factor (e.g. “5”) classically

Measurement on ensemble would give

0 \(\frac{1}{2}\) 1 \(\frac{1}{2}\)

Instead, perform classical postprocessing on quantum computer

now get always “5”, and averaging no longer hurts
Decoherence

The “coherence time” summarizes many aspects of state degradation

- Decoherence ($T_2$) → maximum time for computation
- Relaxation ($T_1$) → maximum time for measurement
- Leakage → detect, and replace by random qubit

Uncorrelated errors
Correlated errors → Over time, or between qubits

Random errors → detect and correct
Systematic errors → unwind
$T_1$ and $T_2$ (and terminology)

$T_1$
Longitudinal relaxation
Spin-lattice relaxation
Relaxation
Energy relaxation
...

$T_2$
Transverse relaxation
Spin-spin relaxation
Decoherence
Phase randomization
Dephasing
...

By definition: $T_2 < 2T_1$
In practice, often $T_2 << T_1$
Two additional criteria

6. Interconvert stationary and flying qubits
   - Repeater stations
   - Distributed quantum computing

7. Transmit flying qubits between distant locations
   - Quantum communication
   - Error correction (communication between different parts of the computer)
Key challenge

combine access to qubits (initialization, control, readout) with high degree of isolation (coherence) in a scalable system

Message about DiVincenzo requirements

(almost) everything goes

Can trade off one requirement for another