The DiVincenzo criteria

Five criteria that any candidate quantum computer implementation must satisfy.

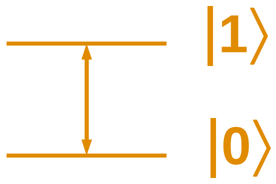
Two additional criteria for quantum communication

IBM Research

Implementation of quantum computers

D. DiVincenzo

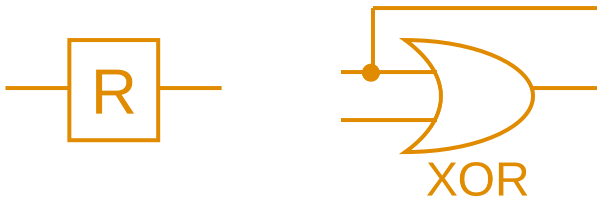
1. Well-defined qubits



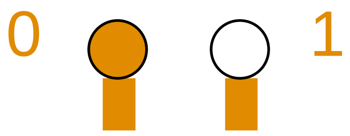
2. Initialization to a pure state

$|00 \ldots 0\rangle$

3. Universal set of quantum gates



4. Qubit-specific measurement



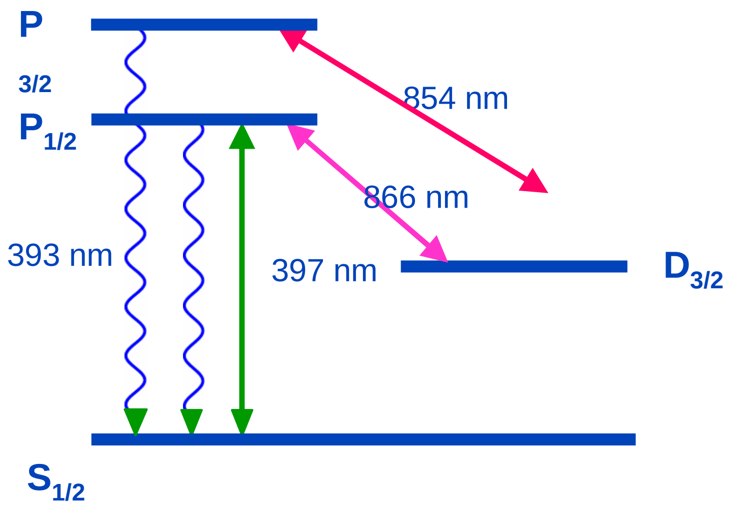
5. Long coherence times

$|0\ldots 0\rangle + |1\ldots 1\rangle$

Well-defined qubits

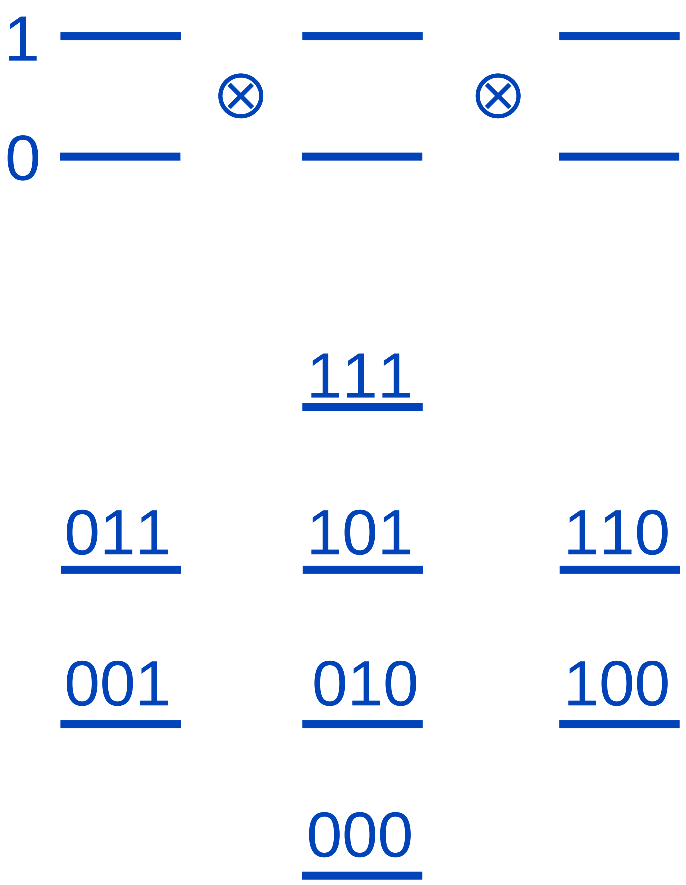
- two-level quantum systems
    - ^{1}H, ^{13}C, ^{19}F, ...
    - electron spin

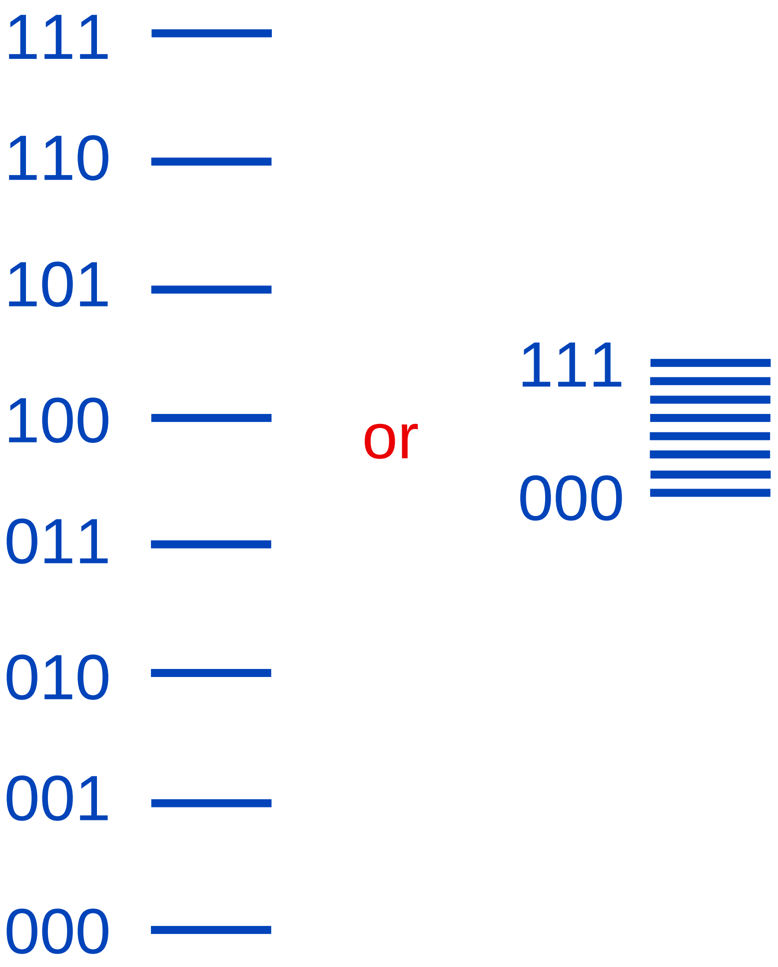
- two-dimensional subspaces of larger systems



+: auxiliary levels
- : leakage

n 2-level systems vs. one 2^n-level system

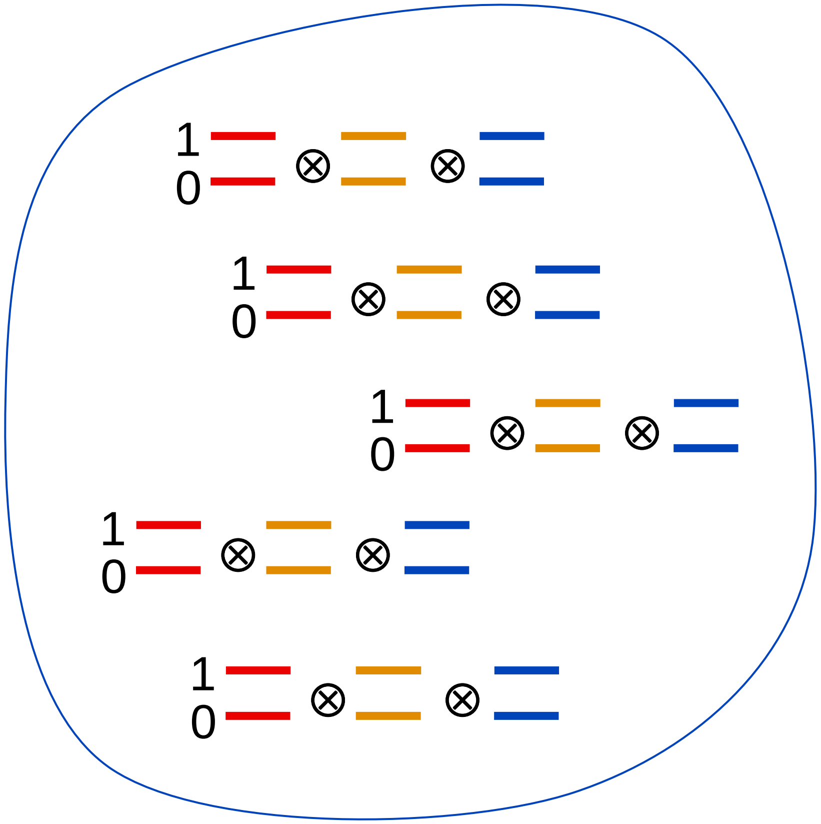




energy or precision ~ *n*
→ ***scalable***

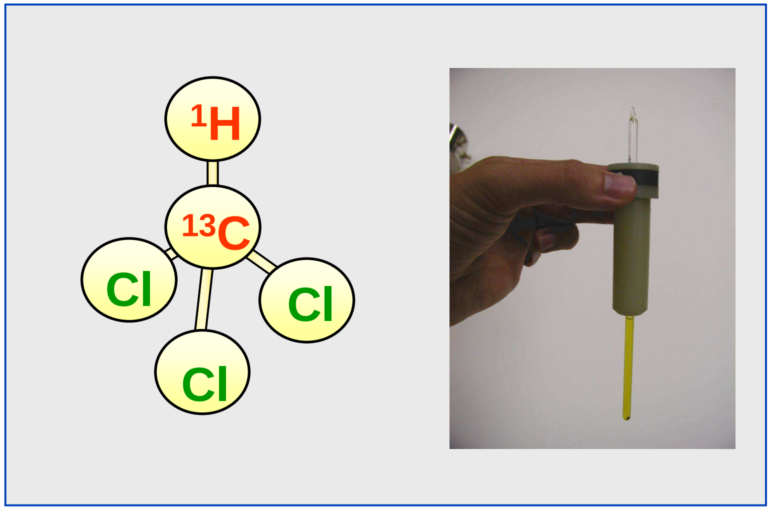
energy or precision ~ 2^n
→ ***NOT scalable***

Ensemble quantum computer



Many identical copies of a quantum computer

Works fine --
only read-out must be modified



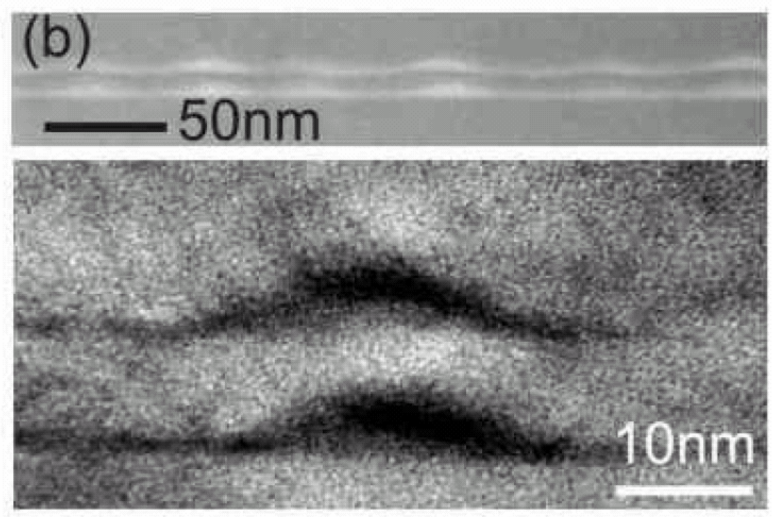
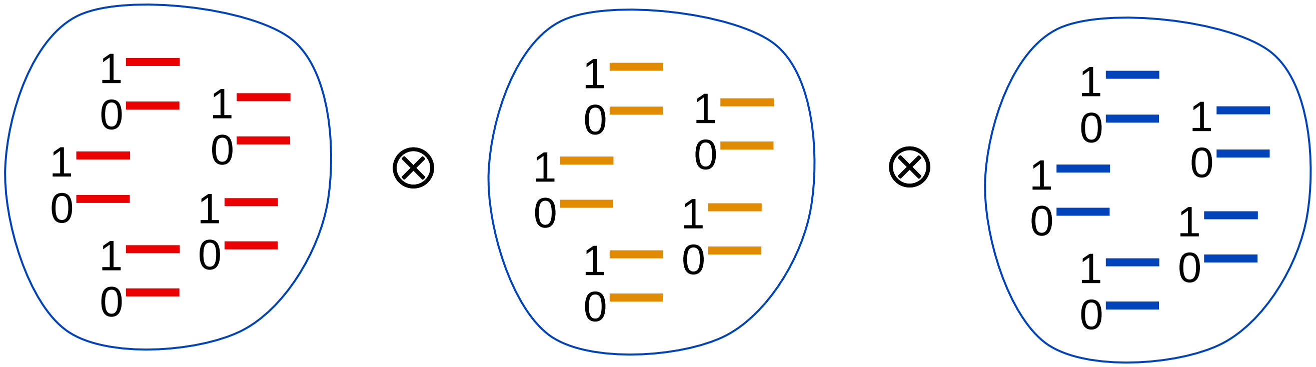


Image: Krenner et al, cond-mat/0505731

Useless device



Many copies of each qubit,
but not a quantum computer

Encoded qubits

$|0\rangle_L = |01\rangle - |10\rangle$

$|1\rangle_L = |10\rangle + |01\rangle$

→ Decoherence free subspace

$|0\rangle_L = (|01\rangle + |10\rangle)\,|0\rangle$

$|1\rangle_L = \sqrt{2/3}\,|001\rangle + \sqrt{1/3}(|01\rangle + |10\rangle)|0\rangle$

→ Trade qubits for
Hamiltonian terms
(e.g. exchange only QC)

Beware: leakage

Initialization to a pure state

To $|000\rangle$ Equilibration at low temperature ($h\nu >> kT$)

Other physical mechanisms:

- Ferromagnet
- Laser cooling
- Optical pumping
- . . .

To $|\psi\rangle$ Perform a hard, non-destructive measurement

Other physical processes

If you want qubits in $|000\rangle$, simply rotate
the qubits from $|\psi\rangle$ to $|000\rangle$

Initialization timescale

e.g. equilibration can be slow (> 5 T_1)

Why initialization anyways?

Computation = garbage in ⇒ garbage out } one-time

Ancilla qubits (“help” qubits)

Error correction = removing entropy from the qubits

} need continuous fresh supply

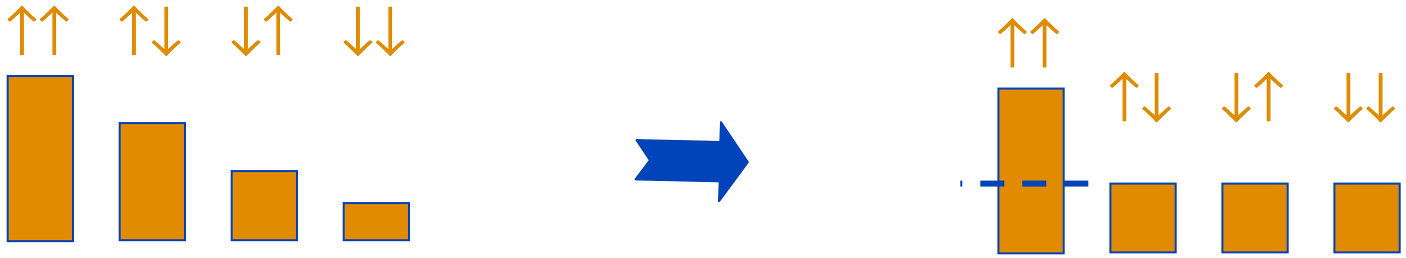
Either initialize fast
or build qubit “conveyer belt”

Are mixed states acceptable?

Equilibrium at moderate or high temperature ($h\nu \not\gg kT$)

Mixed state

Effective pure state



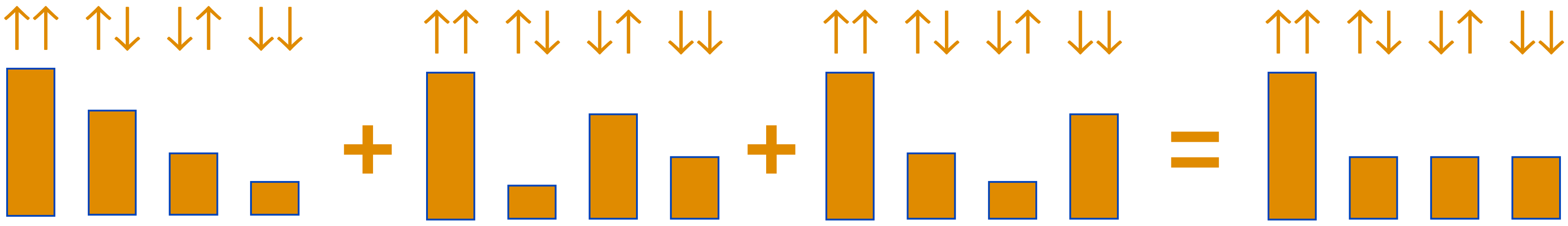
polarization *p*

Signal same as for pure state
but amplitude ~ $1/2^n$

Gershenfeld&Chuang, Science 97,
Cory, Havel & Fahmi, PNAS 97

Effective pure state preparation

(1) Add up 2^N-1 experiments (Knill,Chuang,Laflamme, PRA 1998)



Later ≈ (2^N - 1) / N experiments (Vandersypen *et al.*, PRL 2000)

(2) Work in subspace (Gershenfeld&Chuang, Science 1997)

(3) Average over space (Cory *et al.*, Phys. D 1998)

“Scalable” QC with hot qubits

Goal: **obtain *k* cold qubits from *n* hot qubits**

Idea: reduce the entropy of *k* qubits, and increase the entropy of the remaining qubits (total entropy remains constant)

$$n\,H(p) = k\,H(0) + (n-k)\,H(1/2)$$

$$k = n\,(1 - H(\mathrm{p})) \cong n\,\varepsilon^2$$

$$\left(\Pr[0] = p = \frac{1+\varepsilon}{2} \right)$$

Overhead: # qubits $n \sim k$

operations $\sim k \log k$

(as k and $n \rightarrow \infty$)

“Efficient bootstrapping”

Schulman & Vazirani, Proc. 31st ACM Symp Theory of Computing, p.322 (1999)

Building block Schulman-Vazirani cooling

Step 1: CNOT$_{12}$

Prob.	*bc*
p^2	00
$p(1-p)$	01
$(1-p)p$	10
$(1-p)^2$	11

Prob.	*bc*
p^2	00
$p(1-p)$	0**1**
$(1-p)p$	1**1**
$(1-p)^2$	10

If qubit *c* is 1,
qubit *b* is at ∞ T.

If qubit *c* is 0,
bias qubit *b* is 2ε.

Step 2: Fredkin$_{c,ab}$ (swap qubits *a* and *b* iff qubit *c* is 0)

If qubit *b* is 1, qubit *a* has bias ε.

If qubit *b* is 0, qubit *a* has bias 2ε.

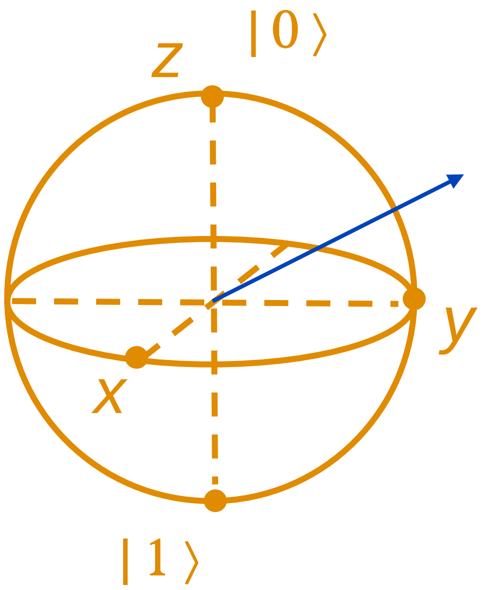
On average, qubit *a* has bias 3ε/2 , so it has been “cooled”.

Universal set of quantum gates

Selective single-qubit rotations

$$R_{\vec{n}}(\theta) = \exp(i\alpha)\exp(-i\theta\vec{n}\cdot\sigma/2)$$

Sufficient: rotations about two different axis



Almost any two-qubit gate is universal
E.g. quantum-XOR or Controlled-NOT

In	Out
00	00
01	01
10	11
11	10

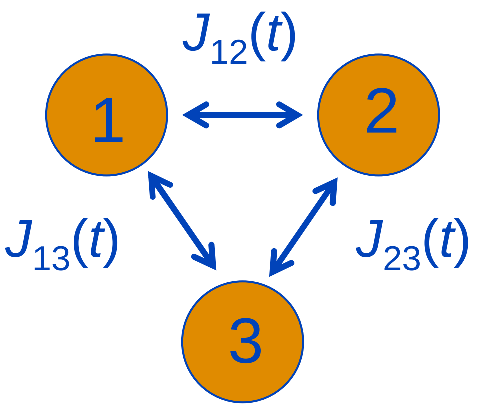
$$\begin{array}{cccc|c} 00 & 01 & 10 & 11 & \\ \hline 1 & 0 & 0 & 0 & 00 \\ 0 & 1 & 0 & 0 & 01 \\ 0 & 0 & 0 & 1 & 10 \\ 0 & 0 & 1 & 0 & 11 \end{array}$$

$$\frac{(|0\rangle + |1\rangle)}{\sqrt{2}}|0\rangle = \frac{|00\rangle + |10\rangle}{\sqrt{2}}$$

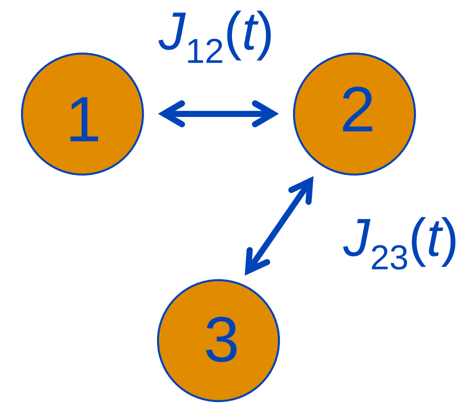
$$\Downarrow$$

$$\frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

Coupling networks

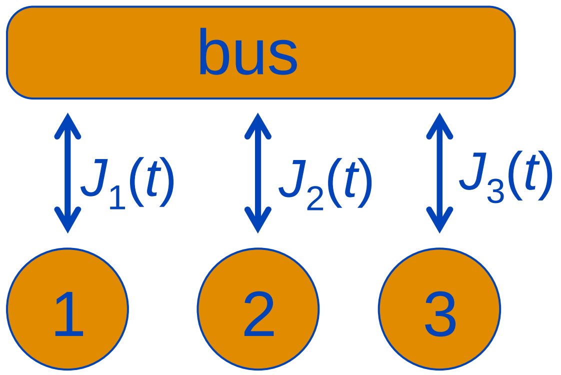


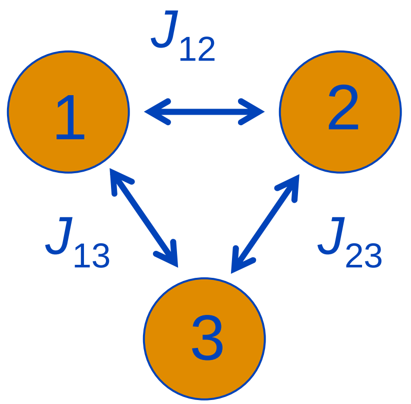
switchable and direct



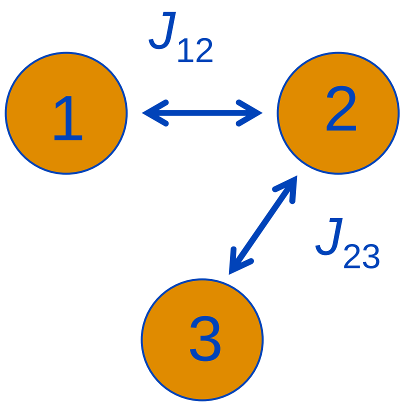
indirect

e.g. nearest neighbour

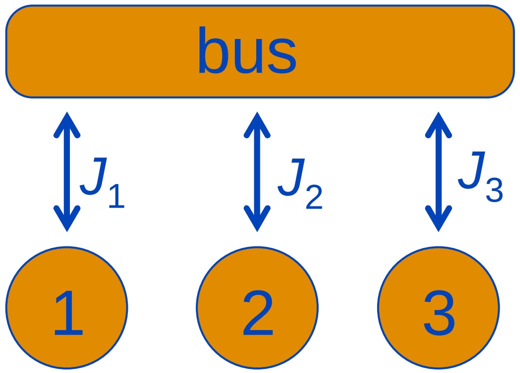




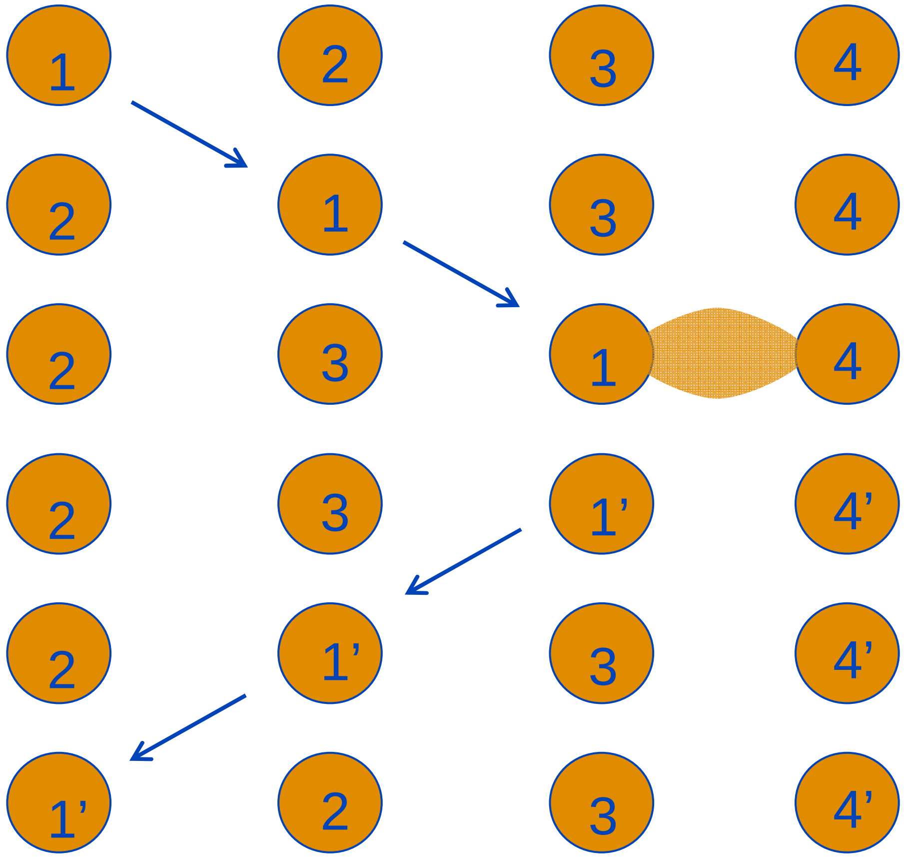
fixed



fixed and indirect



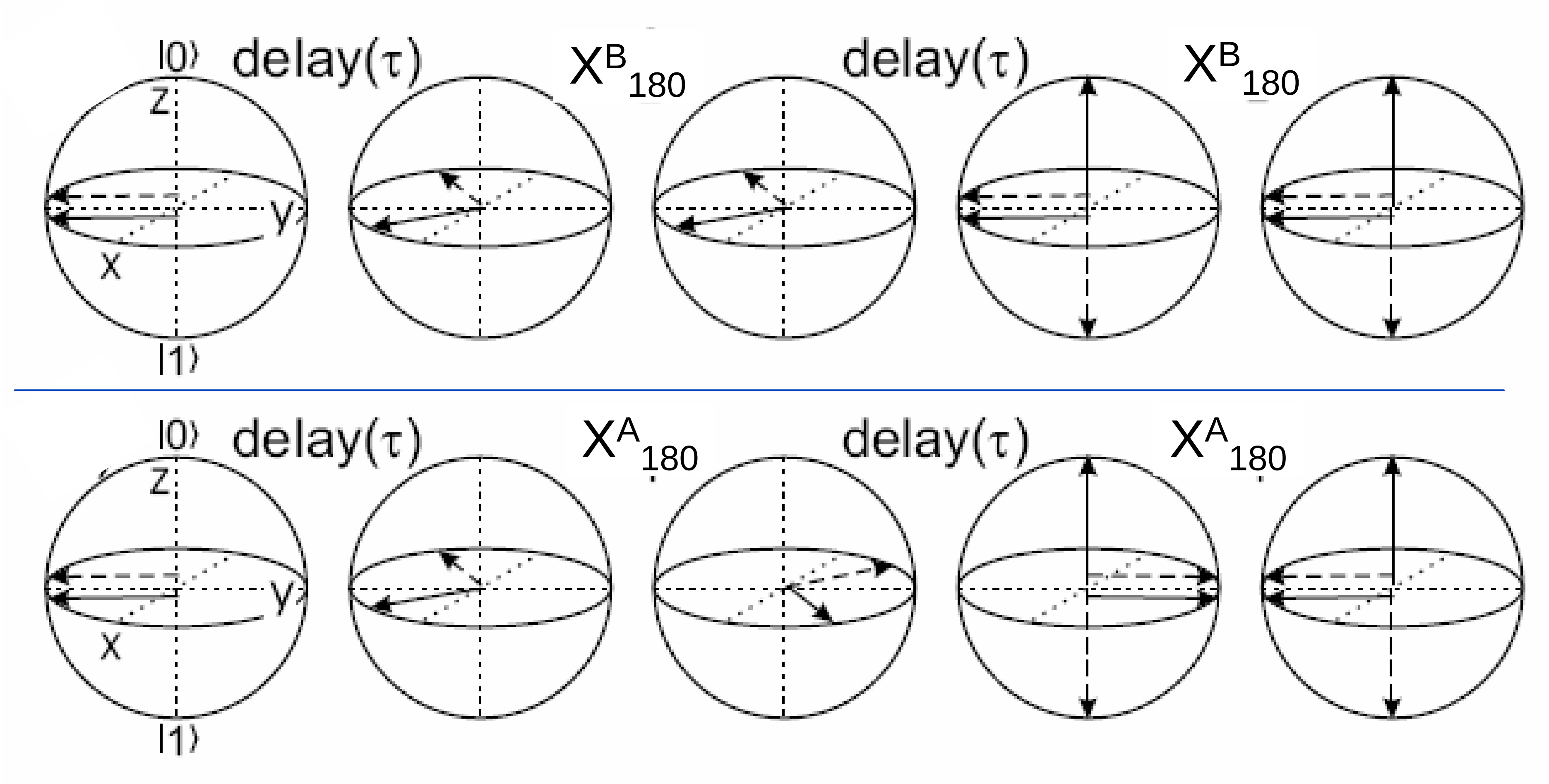
Indirect coupling: Shuffle qubits around



$$\mathrm{SWAP}_{12} = \mathrm{CNOT}_{12}\ \mathrm{CNOT}_{21}\ \mathrm{CNOT}_{12}$$

“only” a linear overhead ...

Removing effect of fixed couplings: refocusing

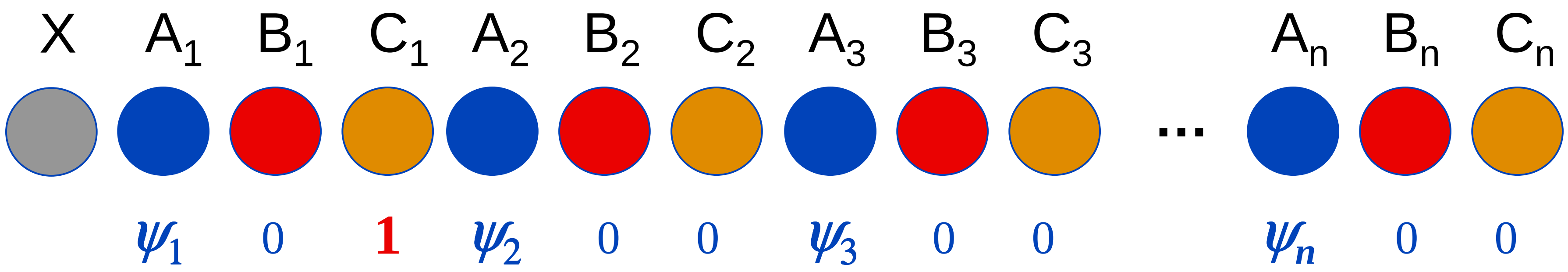


- There exist efficient extensions for arbitrary coupling networks

Leung et al, PRA 00, Jones&Knill, JMR 99

- Refocusing can also be used to remove unwanted single-qubit terms

Even individual addressing is not strictly needed



X_A flips *all* A's
CNOT_{CA} *selectively* flips A_2
Fredkin_{C_AB} swaps A_1 and B_1
etc.

Distinct qubit at the end is needed for setting up a unique "1"

S. Lloyd, Science 261, 1569, 1993

Other requirements

Gates must be precise (systematic errors)

- Calibration
- Cross-talk
- Left-over terms in Hamiltonian
- Non-commuting terms in Hamiltonian
- Hardware limitations (pulse timing, phase noise etc)

Gates must be fast

- > 10000 faster than coherence time
- Parallelization

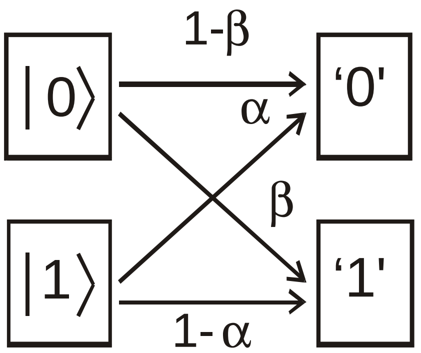
Qubit measurement

Ideally: reliable hard measurement of all qubits

Acceptable in principle:

- ensemble averaged measurement of each qubit (next)
- unreliable measurement (next)
- hard measurement of a single qubit
  (swap consecutive qubits into read-out site,
  while maintaining coherence)

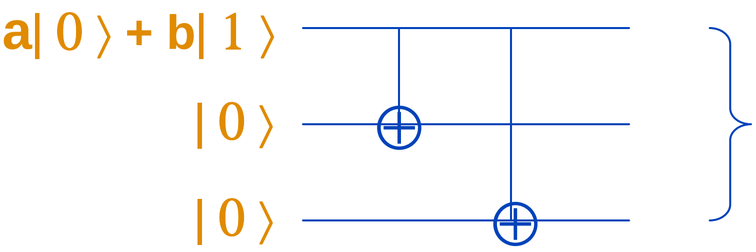
Dealing with a limited measurement fidelity



1) Repeat calculation

OK for “decision problems” (1-bit answer)
Not convenient

2) “Quantum FAN-OUT”


$\mathbf{a}|0\rangle + \mathbf{b}|1\rangle$
$|0\rangle$
$|0\rangle$

$\mathbf{a}|000\rangle + \mathbf{b}|111\rangle$ majority vote

Measurement of one qubit must not disturb state of others, apart from collapse.

Ensemble averaged measurements

Say at the end of Shor’s algorithm, we have state

$$|0110\rangle + |0011\rangle$$

Measurement on a single system gives

0100 or 0011

From either outcome, can find prima factor (e.g. “5”) classically

Measurement on ensemble would give

0 ½ 1 ½

Instead, perform classical postprocessing on quantum computer

now get always “5”, and averaging no longer hurts

Decoherence

The “coherence time” summarizes
many aspects of state degradation

Decoherence (T_2) → maximum time for computation
Relaxation (T_1) → maximum time for measurement
Leakage → detect, and replace by random qubit

$\left.\begin{array}{l}\text{Uncorrelated errors}\\ \text{Correlated errors}\end{array}\right\}$ Over time, or between qubits

Random errors → detect and correct
Systematic errors → unwind

T_1 and T_2 (and terminology)

T_1

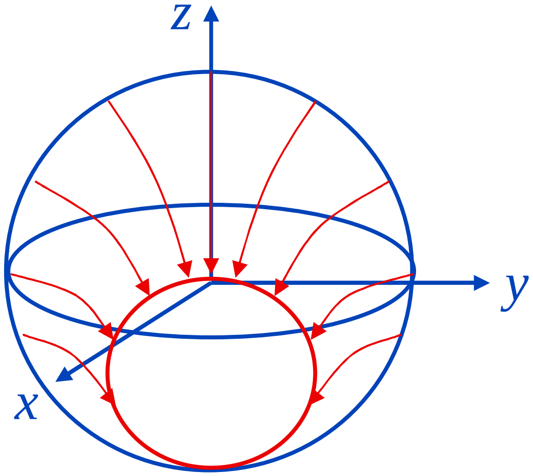
Longitudinal relaxation

Spin-lattice relaxation

Relaxation

Energy relaxation

…



T_2

Transverse relaxation

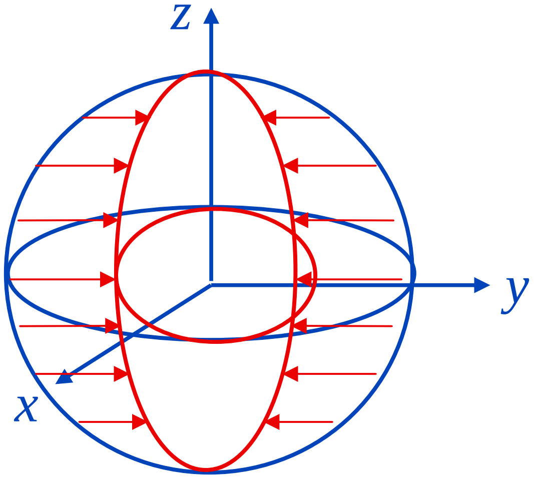
Spin-spin relaxation

Decoherence

Phase randomization

Dephasing

…



By definition: $T_2 < 2T_1$ In practice, often $T_2 \ll T_1$

Two additional criteria

6. Interconvert stationary and flying qubits

   Repeater stations
   Distributed quantum computing

7. Transmit flying qubits between distant locations

   Quantum communication
   Error correction (communication between different parts of the computer)

Key challenge

combine access to qubits (initialization, control, readout)
with high degree of isolation (coherence)
in a scalable system

Message about DiVincenzo requirements

(almost) everything goes

Can trade off one requirement for another