

Quiz #4. Cooperative Games in Characteristic Function Form
 EPA 2142: Policy and Decision Models
 6 January 2010

Consider the following three player game in characteristic function form.

$$\begin{aligned}
 v\{\phi\} &= 0 \\
 v\{A\} &= 21 & v\{B\} &= 18 & v\{C\} &= 8.5 \\
 v\{AB\} &= 60 & v\{AC\} &= 40 & v\{BC\} &= 37 \\
 v\{ABC\} &= 100
 \end{aligned}$$

HINT: You might find it easier to convert this game to its strategic equivalent, and solve this game instead. If you do so, please convert your answer back to the original units.

Solve the game using four solution concepts from the text. You may select any four of your choice, but the concepts must be suitable for analyzing games in characteristic functions.

You will receive four points for the first correct answer, three points for the second correct answer, two points for the third correct answer, and one for the fourth answer. Your responses will be sorted so that the highest quality answer is graded first. Should you attempt more than four solutions, the least correct solution will be dropped from further grading.

Use additional paper as needed, including triangle diagrams. Please write your name and student number on each sheet.

A correct solution may involve graphical analysis, logical analysis, and algebra. Be both clear and complete. A brief justification in a sentence of how you might use the technique is required. See the table below for additional guidance. A comparison of the answers is appreciated, but not required.

	<u>Triangle Diagram</u>	<u>Algebraic Work</u>	<u>Logical or Rhetorical Defense</u>
Core	Map the associated inequalities. If there is a feasible region, identify the region. If the core is empty, say so.	Derive the associated inequalities	Justify the use of the core in a single sentence.
Stable Set	Display the associated stable sets.	Not needed.	Defend your selection as being both internally and externally stable. Justify the use of the stable set in a single sentence.
Shapley Value	Not needed.	Provide the imputation. The complete solution including the permutations, marginal value calculations or the swing voting combinatorics.	Justify the use of the Shapley value in a single sentence.
Bargaining Set	Show the set of possible bargaining solutions if there are multiple possible solutions.	Provide a list of the possible partitions. Provide the equalities resulting from the bargaining setting, and their solution. For each partition state whether the imputation is a single value or a set. If it is a single value, then provide this value. If it is a set, then describe the set in a series of inequalities.	Justify the use of the bargaining set in a single sentence. Logically defend the presence or absence of set-valued solutions.
Nucleolus	Identify the core on the diagram. Geometrically identify the nucleolus point.	Provide the imputation. If you have not already identified the associated inequalities for the core, do so now. Calculate the excess for all players at this point.	Justify the use of the nucleolus in a single sentence. Defend the idea that the identified point minimizes the maximum excess.
Gately Point	Not needed.	A calculation of the marginal values associated with each of the players. A calculation of the appropriate ratios of division to minimize disruption. Calculate the associated disruption at this point.	Justify the use of the Gately point in a single sentence. Defend the idea that the identified point minimizes the maximum disruption.

Table 1: Providing a Clear and Complete Answer