Chapter 4
DETECTION LIMIT IN DIFFERENTIAL MEASUREMENTS

- ***Why differential measurements?***
- ***How is the quality of a differential measurement specified?***
- ***What is the detection limiting signal in a differential measurement?***
- ***How is the instrumentation amplifier used?***

4.1 Introduction

The sub-systems discussed so far are composed of components or circuits with a single-ended input and/or output. Information is transferred over two wires, but with one at ground potential. One of the terminals of each signal source (voltage or current) is at ground potential. Moreover, one of the input terminals of a read-out circuit is at ground potential. As a consequence, the information content is determined by the absolute value of the signal at the other wire (the non-grounded wire that is interconnecting the source and read-out). Many practical signal sources, however, use the two wires actively and supply a **differential signal** that is superimposed on a **common signal**. Such a signal source has a differential output. Each of the two output terminals carries a signal and the information content is determined by the difference between these signals.

A suitable circuit for the read-out of a differential source should be equipped with a differential input (i.e. two active terminals, none directly forced to ground potential). This read-out circuit should be designed to be sensitive only to the difference between the signals at the input terminals (= the differential signal), and should be immune to the average value relative to ground potential (= the common-mode signal). A practical amplifier satisfies this requirement up to a certain extent, which results in an additive error and thus in a detection limit. This limited suppression of the common-mode signal is specified in the **Common-Mode Rejection Ratio (CMRR)**.

Common-mode rejection in a measurement system is particularly important in medical instrumentation. Recording an electro-cardiogram (ECG) involves the positioning of two signal electrodes plus one reference electrode on the skin of a patient, as shown schematically and highly simplified in Fig. 4.1a.

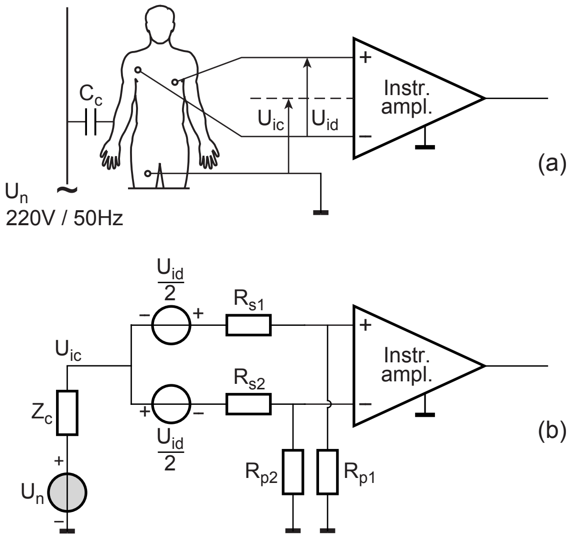

Figure 4.1, Application of the instrumentation amplifier for measuring the ECG: (a) set-up and (b) equivalent circuit of the input.

The series resistance between the electrode and the skin is considerable and poorly-defined due to the skin conductivity and its dependence on the patient's condition. This contact resistance can be reduced using conductive and moisturising cream before placement of the electrode. The read-out has a parallel resistance to ground for biasing purposes. As is demonstrated in Section 4.3.2, a very suitable differential amplifier is the **instrumentation amplifier**.

The equivalent electrical circuit of the measurement set-up is shown in Fig. 4.1b. The signals in an ECG are differential bio-potentials of typically U_{id}= 10 μV which are superimposed on a common-mode signal (typically U_{ic}> 1 mV, due to capacitive coupling of the mains voltage). The challenge in differential read-out design is to obtain a sufficient CMRR for reproducible detection of the differential signal, despite the fact that the magnitude is two orders of magnitude lower than that of the common-mode signal.

4.2 Detection limit due to finite common-mode rejection

4.2.1 Differential-mode and common-mode sensitivity

The Wheatstone bridge shown in Fig. 4.2 is an example of a signal source that supplies a relatively small differential signal, U_{oWd}, superimposed on a relatively large common-mode signal, U_{oWc}. A fully balanced bridge results in a zero value for the differential signal, U_{oWd}= 0 (ΔR= 0; see also Section 2.5.5). This differential signal is superimposed on a non-zero common signal U_{oWc}= $(U_1+U_2)/2$= $U_{exc}/2$. A bridge imbalance yields:

$$U_1 = \frac{R+\Delta R}{2R} U_{exc}$$
$$U_2 = \frac{R-\Delta R}{2R} U_{exc} \qquad (4\text{-}1)$$

Thus: U_{oWd}= $(\Delta R/R)U_{exc}$ and U_{oWc}= $U_{exc}/2$.

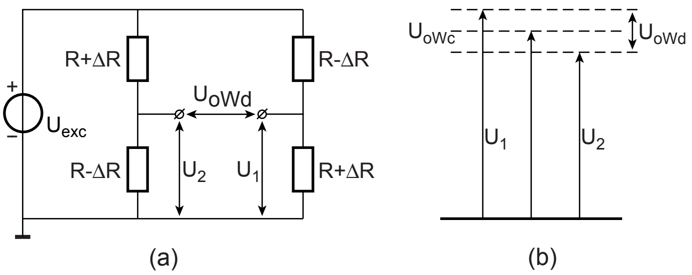

Figure 4.2, Bridge circuit with differential-mode and common-mode signal components at the output.

The circuit for the read-out of the differential signal is shown schematically in Fig. 4.3a. The relevant specifications are the differential gain, G_d= U_o/U_{id}, and the common-mode rejection ratio, *CMRR*= *H*.

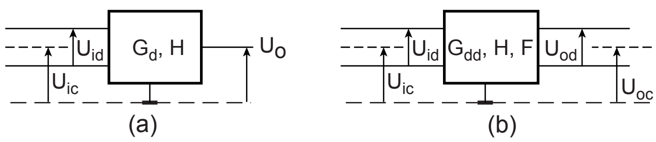

Figure 4.3, Definition of common-mode rejection in the case of: (a) a single-ended output and (b) a differential output.

The read-out should only be sensitive to U_{id}= U_{oWd} (U_o= G_dU_{id}). However, a practical circuit also exhibits a non-zero sensitivity to U_{ic}. Therefore, the output voltage, U_o, is more adequately described by:

$$U_o = G_d U_{id} + G_c U_{ic} \tag{4-2}$$

The unwanted second term is due to the limited CMRR.

The CMRR is defined as the ratio between the sensitivity to the differential input signal, U_{id}, as expressed by the differential gain, G_d, and the sensitivity to the common input signal, U_{ic}, as expressed by the common-mode gain, G_c.

When using Fig. 4.3 this definition yields:

$$H = \frac{G_d}{G_c} = \frac{\dfrac{U_o}{U_{id}}}{\dfrac{U_o}{U_{ic}}} = \left(\frac{U_{ic}}{U_{id}}\right)_{U_o = const.} \tag{4-3}$$

This expression indicates that CMRR can also be interpreted as the differential input signal, U_{id}, which is required to have the same effect on the output (i.e. U_o= constant) as any given common-mode signal, U_{ic}. Note that the CMRR should be maximised to yield a minimum additive error, ε:

$$U_o = G_d U_{id} + G_c U_{ic} = G_d \left(U_{id} + \frac{G_c}{G_d} U_{ic}\right) = G_d \left(U_{id} + \frac{U_{ic}}{H}\right) = G_d \left(U_{id} + \varepsilon\right), \tag{4-4}$$

with $H = U_{ic}/\varepsilon$.

4.2.2 Maximising the CMRR

The common-mode behaviour of an electronic measurement system is largely determined by the input circuit. A simplified schematic of the input stage of a differential read-out circuit is shown in Fig. 4.4. Obviously, the CMRR is maximised if U_{ic} is unable to cause a differential voltage, $U_+ - U_-$. A qualitative inspection of the circuit reveals two opportunities to provide such an advantage:

- No common-mode voltage would be generated across the differential input in case the Voltage across R_{d1} due to U_{ic} is equal to that generated across R_{d2}, so that the effects are cancelled out ($R_{d1} = R_{d2}$).
- No common-mode current would flow in the case of an extremely high impedance between U_{ic} and ground ($R_c \rightarrow \infty$).

These qualitative considerations translate into two important design targets:

- **symmetry** by using a fully balanced input circuit and
- **isolation** using a floating input with respect to the common ground potential.

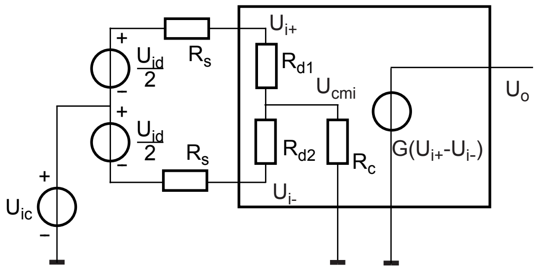

Figure 4.4, Input circuit of a differential amplifier with finite isolation from ground potential and unequal differential input resistors.

These conclusions are confirmed by calculating the common-mode signal at the input stage, U_{cmi}, which results in:

$$U_{cmi} = \frac{R_c}{R_c + \dfrac{(R_s + R_{d1})(R_s + R_{d2})}{2R_s + R_{d1} + R_{d2}}} U_{ic} + \left(\frac{\dfrac{R_c(R_s + R_{d2})}{R_c + R_s + R_{d2}}}{\dfrac{R_c(R_s + R_{d2})}{R_c + R_s + R_{d2}} + R_s + R_{d1}} - \frac{\dfrac{R_c(R_s + R_{d1})}{R_c + R_s + R_{d1}}}{\dfrac{R_c(R_s + R_{d1})}{R_c + R_s + R_{d1}} + R_s + R_{d2}} \right) \frac{U_{id}}{2} \qquad (4\text{-}5)$$

The differential signal between the non-inverting input (U_+) and inverting input (U_-) can be derived as:

$$U_+ - U_- = \frac{U_{ic} - U_{cmi}}{R_s + R_{d1}} R_{d1} - \frac{U_{ic} - U_{cmi}}{R_s + R_{d2}} R_{d2} + \frac{R_{d1} + R_{d2}}{R_{d1} + R_{d2} + 2R_s} U_{id} \qquad (4\text{-}6)$$

Assuming: $R_{d1} = R_d + \Delta R_d/2$, $R_{d2} = R_d - \Delta R_d/2$ and $R_c \gg R_d$ yields:

$$U_{cmi} = \frac{R_c}{R_c + \dfrac{\left(R_s + R_d + \dfrac{\Delta R_d}{2}\right)\left(R_s + R_d - \dfrac{\Delta R_d}{2}\right)}{2(R_s + R_d)}} U_{ic} + \frac{\Delta R_d}{R_s + R_d + R_s(1 + R_s/R_c + R_d/R_c)} U_{id} = \frac{2R_c U_{ic}}{2R_c + R_s + R_d} + \frac{\Delta R_d U_{id}}{2R_s + R_d} \approx \frac{2R_c U_{ic}}{2R_c + R_s + R_d} \qquad (4\text{-}7)$$

Note that at the detection limit: $U_{ic} \gg [\Delta R_d/(2R_s + R_d)]U_{id}$.

Hence:

$$\frac{U_+ - U_-}{U_{ic}} \approx \frac{\Delta R_d}{R_s + R_d}\left(1 - \frac{R_c}{R_c + \left(R_s + R_d\right)/2}\right)$$
$$\frac{U_+ - U_-}{U_{id}} \approx \frac{R_d}{R_s + R_d} \tag{4-8}$$

The common-mode rejection follows via Eqn. 4.1 as:

$$CMRR = H = \frac{\dfrac{U_+ - U_-}{U_{id}}}{\dfrac{U_+ - U_-}{U_{ic}}} = \frac{R_d}{\Delta R_d} \times \frac{2R_c + R_s + R_d}{R_s + R_d} \tag{4-9}$$

Equation 4-9 confirms that two approaches are available for maximising common-mode rejection. The first is by maximising the impedance between U_{cmi} and the common ground: $R_c \gg R_s + R_d$ (i.e. high isolation). The second contribution involves good matching between the input components: $\Delta R_d = 0$ (i.e. symmetry). A practical operational amplifier can be designed for $(R_c/R_d)_{max}$ to exceed 10^3. Moreover, matching could be better than 1%. Hence, the maximum achievable common-mode rejection can be in the 10^5-10^6 range (100-120 dB).

The read-out circuit shown in Fig. 4.3a provides a single-ended output directly at the output of the first gain stage. A fully differential system offers a superior suppression of noise and interference. A gain stage with differential input and differential output should, therefore, be used, as is shown in Fig. 4.3b. However, such a gain stage is only fully specified if both the differential and the common level of the output signal are specified in terms of the differential and common input signals.

The definition of the CMRR specification of a differential input/output gain stage does not fundamentally differ from the one provided for the gain stage with differential input and single-ended output. The obvious change is that in a fully differential system the CMRR relates the **differential output signal** to the signals supplied at the input terminals. Hence, G_{cd} (Gain common-input-to-differential-output) is used instead of G_c.

The common output level is generally considered to be determined by the common input signal only ($U_{oc}/U_{id} = 0$). The common-mode signal transfer function is specified by the **discrimination factor, *F***.

The discrimination factor, *F*, is specified in terms of the gain for the common-mode input signal to common-mode output signal relative to the differential gain: *F*= G_{dd}/G_{cc}.

In equation:

$$H = \frac{G_{dd}}{G_{cd}} = \frac{\frac{U_{od}}{U_{id}}}{\frac{U_{od}}{U_{ic}}} = \left(\frac{U_{ic}}{U_{id}}\right)_{U_{od}=const.} \qquad F = \frac{G_{dd}}{G_{cc}} = \frac{\frac{U_{od}}{U_{id}}}{\frac{U_{oc}}{U_{ic}}} \tag{4-10}$$

The terms *CMRR* and *F* are used for a fully-differential system throughout this book. Similar to the *CMRR*, a high value for *F* is also preferred.

The fundamental difference between the CMRR (*H*) and *F* is that a finite value for the CMRR results in mixing of the differential and common input signals at the output and thus directly imposes a detection limit on the system. A finite value for *F* does not result in an additional differential output signal in the case of a non-zero common-mode input, but rather fails to block the transfer of this common-mode input level to the differential output.

Most importantly, in the case of a finite *F*, the differential and common-mode signals remain distinguishable and the detectivity is not yet impaired. A relatively high value for *F* is nevertheless very important, as the (partial) removal of any common-mode signal reduces the requirements to be imposed on the CMRR of the next stage, which contributes to a high value of the overall CMRR of the system.

It should be noted that the English literature generally uses only the common-mode gain, G_{cc}, and has not adopted the more convenient dimensionless notation in terms of the discrimination factor, F.

4.2.3 Practical aspects of the CMRR in bridge readout

The limited CMRR in a differential amplifier has consequences for the Wheatstone bridge; Fig. 4.5a shows the circuit. The differential and common-mode signals are:

$$U_{id} = \frac{\Delta R}{R_o} U_{exc} \ , \ U_{ic} = \frac{U_{exc}}{2} \tag{4-11}$$

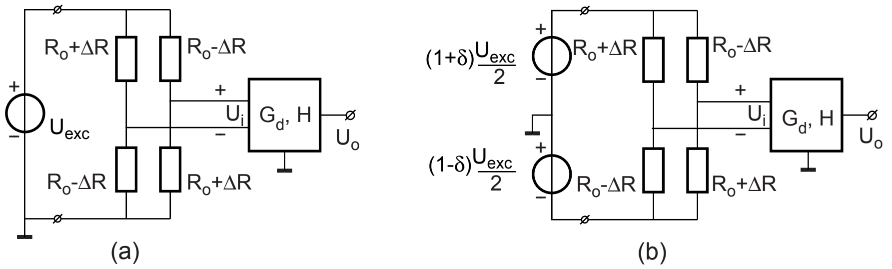

Figure 4.5, Read-out of the full Wheatstone bridge using a differential amplifier with a finite *CMRR*: (a) single-ended excitation and (b) differential excitation.

An expression results from the definition of CMRR for the minimum CMRR value required to detect of $(\Delta R/R_o)_{det}$. The inaccuracy specification, ε, is included by requiring the output voltage due to U_{id}(min.) to be equal to 1/ε times that output voltage due to U_{ic}:

$$H_{min} = \left(\frac{U_{ic}}{U_{id}}\right)_{U_o=const.} = \frac{1}{\varepsilon} \times \frac{\frac{U_{exc}}{2}}{\left(\frac{\Delta R}{R_o}\right)_{det} U_{exc}} = \frac{1}{2\varepsilon\left(\frac{\Delta R}{R_o}\right)_{det}} \qquad (4\text{-}12)$$

For $(\Delta R/R_o)_{det}$.= 2×10^{-6} (which is the result of 1 μstrain when measured using a metal film strain gauge with k_ε= 2) and ε= 1%, the result is H_{min}= 25×10^6 (= 148 dB), which is an extremely demanding specification. The reason for this high CMRR requirement is the presence of a large common-mode input signal, U_{ic}. Reducing this value would reduce the required read-out specifications. This objective is achieved in a differential driving scheme, as shown in Fig. 4.5b.

The differential input signal remains unchanged, U_{id}= $\Delta R/R_o$, whereas the common-mode signal is at ground potential, U_{ic}= 0, in the case of perfectly balanced excitation voltages (δ= 0). Hence, a common-mode rejection H=1 would in principle be sufficient. The reason for this somewhat unrealistic outcome is the assumed perfect differential drive (δ= 0). In practice the two excitation voltage sources are not perfectly matched and any mismatch results in a common-mode input signal. A mismatch δ results in:

$$U_{i+} = -(1-\delta)\frac{U_{exc}}{2} + \frac{R_o + \Delta R}{2R_o}U_{exc} = \left(\delta + \frac{\Delta R}{R_o}\right)\frac{U_{exc}}{2}$$

$$U_{i-} = -(1-\delta)\frac{U_{exc}}{2} + \frac{R_o - \Delta R}{2R_o}U_{exc} = \left(\delta - \frac{\Delta R}{R_o}\right)\frac{U_{exc}}{2} \qquad (4\text{-}13)$$

$$U_{id} = U_{i+} - U_{i-} = \frac{\Delta R}{R_o}U_{exc},\ U_{ic} = \frac{U_{i+} + U_{i-}}{2} = \delta\frac{U_{exc}}{2}$$

The required CMRR of the read-out results in:

$$H_{\min} = \left(\frac{U_{ic}}{U_{id}}\right)_{U_o=C} = \frac{1}{\varepsilon} \times \frac{\delta\frac{U_{exc}}{2}}{\left(\frac{\Delta R}{R_o}\right)_{\det} U_{exc}} = \frac{\delta}{2\varepsilon\left(\frac{\Delta R}{R_o}\right)_{\det}} \qquad (4\text{-}14)$$

and is proportional to the degree of mismatch between the two excitation voltage sources. Limiting the mismatch to δ= 1% is well feasible and results in $H_{\min}$= 25×10^4 (108 dB), which is an acceptable specification. As a general rule it can be stated that an increased symmetry (in this case balancing of excitation sources) helps in reducing CMRR requirements.

Example 4.1
A Wheatstone bridge is imbalanced due to the resistance values shown in Fig. 4.6. Calculate the differential bridge output signal, U_{id}, the common-mode signal, U_{ic}, and the required CMRR of the differential amplifier used, to ensure that U_{ic} has the same influence on the amplifier output signal as U_{id} for versions (a) and (b).

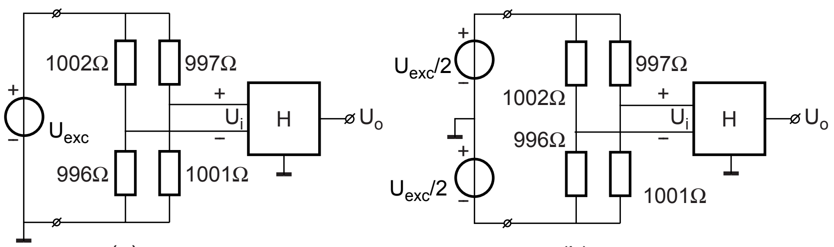

Figure 4.6, Bridge circuit with differential-mode and common-mode signal levels at the output (U_{exc}= 10V).

Solution version (a):
U_{i+} = (1001/1998)×10V and U_{i-} = (996/1998)×10 V.
U_{id} = U_{i+}-U_{i-} = [(1001-996)/1998]×10 = 50/1998 = 25 mV.

$U_{ic} = (U_{i+}+U_{i-})/2 = (1001+996)/3996 = 4.9975$ V.
$U_o = G_d(U_{id}+U_{ic}/H) \rightarrow H_{min} = U_{ic}/U_{id} = 4.9975/0.025 = 199.9$.

Solution version (b):
$U_{i+} = (1001/1998 - 997/1998)\times 5$V and $U_{i-} = (996/1998 - 1002/1998)\times 5$ V.
$U_{id} = U_{i+} - U_{i-} = [(4+6)/1998]5 = 50/1998 = 25$ mV.
$U_{ic} = (U_{i+}+U_{i-})/2 = (4-6)/3996 = 2.5$ mV.
$U_o = G_d(U_{id}+U_{ic}/H) \rightarrow H_{min} = U_{ic}/U_{id} < 1$.

4.2.4 Using the CMRR for noise reduction

A differential sensor structure combined with a differential read-out system offers huge benefits in terms of the detection limit, as it enables the conversion of an interfering signal into a common-mode signal, which is subsequently suppressed using the *CMRR* of the differential read-out.

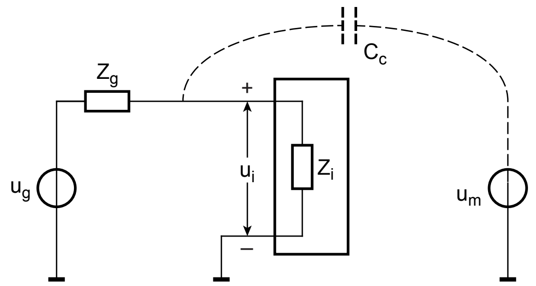

Figure 4.7, Capacitive injection of the mains voltage in a single-ended system.

Figure 4.7 shows a single-ended circuit for the read-out of u_g with capacitive coupling. The transfer function from the source to input, $u_{i,g}/u_g$, results in an expression for the signal at the input terminals:

$$u_{i,g} = \frac{R_i}{R_g + R_i} u_g \simeq \left(1 - \frac{R_g}{R_i}\right) u_g \qquad (4\text{-}15)$$

Similarly, for $R_g \ll R_i$, the noise signal at the input terminals due to the mains voltage, $u_{i,m}$, is expressed as:

$$u_{i,m} = \frac{Z_g // Z_i}{Z_g // Z_i + \frac{1}{j\omega C_c}} u_m = \frac{j\omega \frac{R_i R_g}{R_i + R_g} C_c}{1 + j\omega \frac{R_i R_g}{R_i + R_g} C_c} u_m \simeq j\omega R_g C_c u_m \qquad (4\text{-}16)$$

The Electro-Magnetic Interference (EMI-see Section 5.8) is directly coupled to the input and imposes an unacceptable detection limit.

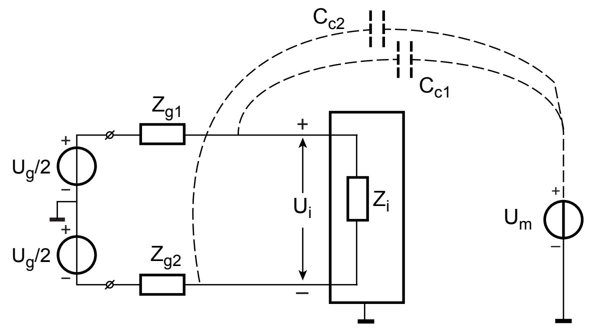

Figure 4.8, Capacitive injection of the mains voltage on a differential input.

Figure 4.8 shows the differential read-out of a differential voltage source, U_g, with a relatively large source impedance, Z_g. Since also Z_i is large enough to enable voltage read-out without source loading, the mains voltage, U_m, is coupled to the signal wires. However, for fully balanced source impedances, differential read-out and equal coupling capacitances ($Z_{g1}= Z_{g2}$ and $C_{c1}= C_{c2}$), the injected noise voltages are equal and thus only contribute to the common-mode signal. In the case of a sufficiently high CMRR, these are eliminated in the differential signal.

However, any differential signal due to unequal source impedances or coupling capacitances can in principle not be distinguished from the source signal (unless filtering or other signal conditioning is possible- see Chapter 6) and can affect the detection limit.

Example 4.2

The single-ended circuit in Fig. 4.7 uses the following component values: $Z_i= R_i = 1$ MΩ and $C_c = 1$pF. Calculate the signal level, $U_{i,g}$ and the level of the injected voltage, $U_{i,m}$, at the input of the readout where $Z_g = R_g = 50$ kΩ and the mains voltage is the external source of error (U_m= 230 V and frequency 50 Hz).

Solution:

Equation (4-15) yields: $U_{i,g} = 0.95\times U_g$.

Similarly, for $R_g \ll R_i$, Equation (4-16) yields $U_{i,m}= 100\pi\times50\times10^3\times10^{-12}\times 230$= 3.6 mV. Assuming an inaccuracy specification at ε= 5% results in a detection limit at $U_{i,g}$(min.)= $U_{i,m}/\varepsilon$= 72 mV. Referring this voltage level to the input U_g yields: U_g(min.)= $72\times10^{-3}/0.95$= 75.8 mV. Obviously, this is too high a detection limit.

The differential system, as shown in Fig. 4.8 with *CMRR*= 80 dB (the effect of coupling to the output is reduced by a factor 10^4) yields U_g(min.)= $(72\times10^{-3}\times10^{-4})/0.95$= 7.58 μV, which is much more acceptable.

4.3 OPAMP circuits for differential readout

4.3.1 The 1-OPAMP differential amplifier

An operational amplifier (OPAMP) is equipped with a differential input, U_+-U_-, which should make the OPAMP suitable for the direct read-out of a floating signal source.

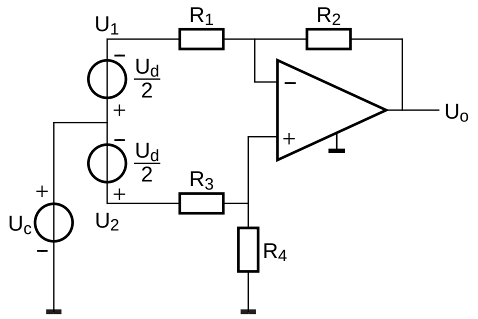

Figure 4.9, Elementary opamp differential amplifier.

Figure 4.9 shows the basic differential amplifier. It is instructive to note that this circuit is a combination of an inverting amplifier, U_o/U_1, and a non-inverting amplifier, U_o/U_2. Recognising these topologies simplifies the calculation of the transfer function using superposition:

$$\left.\begin{array}{l} U_2 = 0;\ U_{o1} = -\dfrac{R_2}{R_1}U_1 = -\dfrac{R_2}{R_1}\left(U_c - \dfrac{U_d}{2}\right) \\ U_1 = 0;\ U_{o2} = \dfrac{R_4}{R_3+R_4}\cdot\dfrac{R_1+R_2}{R_1}U_2 = \dfrac{R_4\left(R_1+R_2\right)}{\left(R_3+R_4\right)R_1}\left(U_c + \dfrac{U_d}{2}\right) \end{array}\right\}$$

$$U_o = U_{o1} + U_{o2} = -\frac{R_2}{R_1}U_1 + \frac{R_4\left(R_1+R_2\right)}{\left(R_3+R_4\right)R_1}U_2 \tag{4-17}$$

$$For:\ R_1R_4 = R_2R_3 \rightarrow U_o = \frac{R_2}{R_1}\left[U_2 - U_1\right] = \frac{R_2}{R_1}U_d$$

In the case of ideal OPAMP characteristics and a perfect resistor matching in pairs (R_1R_4= R_2R_3), a perfect differential amplifier results in a gain of: G_d= U_o/ $(U_2$-$U_1)$= U_o/U_d= R_2/R_1= R_4/R_3. However, this circuit shows three unfavourable properties:

- Practical resistors are matched within a certain tolerance.
- The limitations due to the finite and frequency-dependent open-loop gain of the OPAMP, $A(\omega)$, do apply.
- The two terminals of the floating signal source are unevenly loaded by the inputs of the differential amplifier. The non-inverting input (connected to U_2) has a large input impedance, $Z_{i1} = R_3 + R_4$. The inverting input (connected to U_1) a small input impedance, $Z_{i2} = R_1$. Problems can be expected in the read-out of a differential source with relatively large source impedances.

The most significant problem is the resistor mismatch. Assume the effect resistor mismatch is expressed as: $(R_1+R_2)/R_2 = (1+\delta)(R_3+R_4)/R_4$. In this case the transfer function of the differential amplifier is expressed as:

$$\begin{aligned}
U_o &= -\frac{R_2}{R_1}U_1 + \frac{R_4\left(R_1+R_2\right)}{\left(R_3+R_4\right)R_1}U_2 = -\frac{R_2}{R_1}U_1 + \frac{R_2}{R_1}\left(1+\delta\right)U_2 = \\
&\frac{R_2}{R_1}\left(1+\frac{\delta}{2}\right)\left(U_2-U_1\right) + \frac{\delta R_2}{R_1}\frac{U_1+U_2}{2} = A_{dd}\left(U_2-U_1\right) + A_{cd}\frac{(U_1+U_2)}{2} \\
&\rightarrow H = \frac{A_{dd}}{A_{cd}} = \frac{\frac{R_2}{R_1}\left(1+\frac{\delta}{2}\right)}{\frac{R_2}{R_1}\delta} = \frac{1+\frac{\delta}{2}}{\delta} \approx \frac{1}{\delta}
\end{aligned} \tag{4-18}$$

Equation (4-18) confirms that the CMRR becomes infinitely large for perfectly matched resistors. However, it also demonstrates that the *CMRR* deteriorates with increasing component tolerance. Assume δ= 5% limits the *CMRR* to: H_{max}= 1/0.05= 20. This is insufficient to detect a differential input signal more than two orders of magnitude smaller than the common-mode level, which is the case in many applications. Therefore, a more versatile circuit, the instrumentation amplifier, is used.

4.3.2 The 3-OPAMP instrumentation amplifier

Combining a special 2-OPAMP differential-to-differential voltage pre-amplifier with the 1-OPAMP differential amplifier shown in Fig. 4.9, yields the instrumentation amplifier shown in Fig. 4.10. This circuit features a superior *CMRR* performance.

Both inputs, U_{i1} and U_{i2}, are connected to the non-inverting input of an opamp with the output fed back to the inverting input. The input impedance is extremely high due to the local feedback, which attempts to reproduce the input voltage at the inverting input node. **Therefore, the input signal voltage sources are not affected, which is desirable in voltage read-out**.

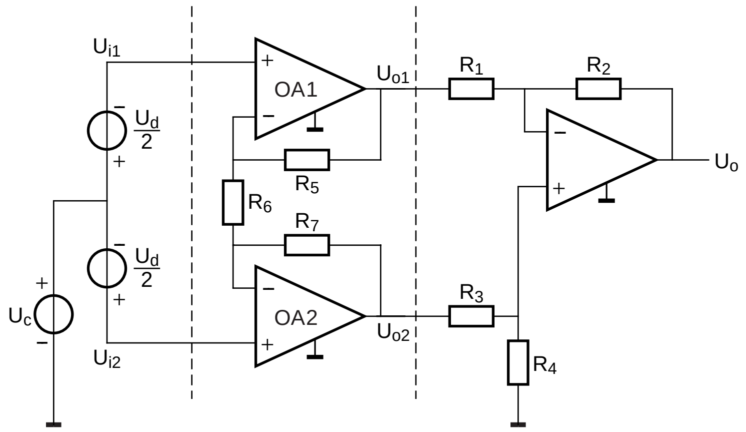

Figure 4.10, Instrumentation amplifier.

The expressions for G_{dd}, *CMRR* and *F* of the differential-to-differential voltage pre-amplifier can be derived via the transfer functions U_{o1}/U_{i1}, U_{o2}/U_{i1}, U_{o1}/U_{i2} and U_{o2}/U_{i2}.

It is, again, instructive to identify the various inverting amplifiers and non-inverting amplifiers within this circuit. The transfer function U_{o1}/U_{i1} can be derived using superposition (U_{i2}=0). The transfer function can basically be derived directly from Fig. 4.10. Since U_{i2}= 0, node R_6, R_7 is at (virtual) ground potential and thus OA1 and the associated local feedback circuit is a non-inverting amplifier with gain: $U_{o1}/U_{i1}= (R_5+ R_6)/R_6$.

A similar argument applies to the transfer function U_{o2}/U_{i1}. OA1 ensures that U_{i1} is (virtually) reproduced at node R_5, R_6. With respect to this node OA2 and the associated local feedback circuit are an inverting amplifier with a gain of: $U_{o2}/U_{i1}= -R_7/R_6$.

The transfer function U_{o2}/U_{i2} is derived similar to the approach used for U_{o1}/U_{i1} and U_{o1}/U_{i2} is derived similar to U_{o2}/U_{i1}, by exchanging the roles of OA1 and OA2: $U_{o2}/U_{i2}= (R_7+ R_6)/R_6$ and $U_{o1}/U_{i2}= -R_5/R_6$.

The differential-mode gain is calculated using $G_{dd}= (U_{o1}-U_{o2})/(U_{i1}-U_{i2})$, the differential-to-common gain using $G_{cd}= (U_{o1}-U_{o2})/[(U_{i1}+U_{i2})/2]$ and the common-mode gain using $G_{cc}= (U_{o1}+U_{o2})/(U_{i1}+U_{i2})$. Finally, the common-mode rejection and discrimination factor of this pre-amplifier result from: $H_1= G_{dd}/G_{cd}$ and $F_1= G_{dd}/G_{cc}$, respectively.

Combining the four transfer functions results in U_{o1} and U_{o2} expressed as:

$$U_{o1} = \frac{R_5 + R_6}{R_6} U_{i1} - \frac{R_5}{R_6} U_{i2}$$
$$U_{o2} = -\frac{R_7}{R_6} U_{i1} + \frac{R_6 + R_7}{R_6} U_{i2} \tag{4-19}$$

The differential and common-mode components of the output signal result in:

$$U_{o1} - U_{o2} = \frac{R_5 + R_6 + R_7}{R_6} U_{i1} - \frac{R_5 + R_6 + R_7}{R_6} U_{i2} = \frac{R_5 + R_6 + R_7}{R_6} U_d$$
$$= G_{dd} U_d + G_{cd} U_c \rightarrow G_{dd} = \frac{R_5 + R_6 + R_7}{R_6}, \; G_{cd} = 0$$
$$\frac{U_{o1} + U_{o2}}{2} = \frac{R_5 + R_6 - R_7}{2R_6} U_{i1} + \frac{R_6 + R_7 - R_5}{2R_6} U_{i2} = U_c + \frac{R_5 - R_7}{R_6} \frac{U_d}{2}$$
$$= G_{cc} U_c + G_{dc} U_d \simeq G_{cc} U_c \rightarrow G_{cc} = 1 \tag{4-20}$$

Since the finite CMRR is an issue related to the differential output voltage when $U_d \ll U_c$, the effect of G_{dc} can be disregarded. Usually $R_5 = R_7$, while R_6 is used to vary the differential gain, G_{dd}. The CMRR and discrimination factor of the pre-amplifier can be derived as:

$$H_1 = \frac{G_{dd}}{G_{cd}} = \frac{R_5 + R_6 + R_7}{0 \times R_6} \rightarrow \infty$$
$$F_1 = \frac{G_{dd}}{G_{cc}} = \frac{R_5 + R_6 + R_7}{R_6} \tag{4-21}$$

Equation (4-21) indicates that the pre-amplifier has a favourable effect on the CMRR. For deriving an expression for the overall gain and CMRR, the cascaded system composed of the differential voltage pre-amplifier and the 1-OPAMP differential amplifier has to be considered.

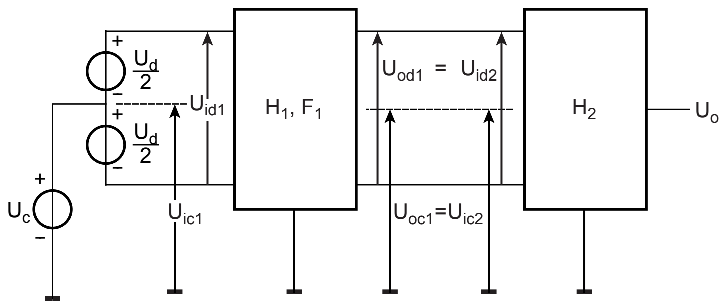

Figure 4.11, Common-mode rejection in a cascaded system.

An expression for the overall *CMRR* of such a cascaded system as a function of H_1 and F_1 of the pre-amplifier and the common-mode rejection H_2 of the 1-OPAMP differential amplifier is derived using Fig. 4.10:

$$
\begin{aligned}
U_o &= G_{d2}U_{id2} + G_{c2}U_{ic2} = G_{d2}\left(U_{id2} + \frac{A_{c2}}{A_{d2}}.U_{ic2}\right) = G_{d2}\left(U_{id2} + \frac{U_{ic2}}{H_2}\right) \\
U_{id2} &= G_{dd1}U_{id1} + G_{cd1}U_{ic1} = G_{dd1}\left(U_{id1} + \frac{G_{cd1}}{G_{dd1}}U_{ic1}\right) = G_{dd1}\left(U_{id1} + \frac{U_{ic1}}{H_1}\right) \\
U_{ic2} &= G_{cc1}U_{ic1} = G_{dd1}\frac{G_{cc1}}{G_{dd1}}U_{ic1} = G_{dd1}\times\frac{U_{ic1}}{F_1}
\end{aligned}
\qquad (4\text{-}22)
$$

Inserting the expressions for U_{id2} and U_{ic2} into that of U_o yields:

$$
\begin{aligned}
&U_0 = G_{dd1}\times G_{d2}\left(U_{id1} + \frac{U_{ic1}}{H_1} + \frac{U_{ic1}}{F_1H_2}\right) = G_d\left(U_{id1} + \frac{U_{ic1}}{H_{tot}}\right) \\
&\text{with :}\begin{cases} G_d = G_{dd1}\times A_{d2} \\ \dfrac{1}{H_{tot}} = \dfrac{1}{H_1} + \dfrac{1}{F_1H_2} \end{cases}
\end{aligned}
\qquad (4\text{-}23)
$$

In the instrumentation amplifier $H_1 \rightarrow \infty$, which yields: $H_{tot} = F_1H_2 \gg H_2$. Therefore, the poor common-mode performance of the 1-OPAMP differential amplifier is improved by the differential-mode signal gain in the pre-amplifier. The CMRR of the instrumentation amplifier in terms of the passive components follows from equations (4-21) and (4-23) as:

$$
\frac{1}{H_{tot}} = \frac{1}{H_1} + \frac{1}{F_1H_2} \simeq \frac{1}{F_1H_2} \rightarrow H_{tot} \simeq F_1H_2 = \frac{R_5 + R_6 + R_7}{R_6}\times\frac{1}{\delta} \qquad (4\text{-}24)
$$

In summary: The merit of the differential-to-differential voltage pre-amplifier is that it combines a large CMRR with a large discrimination factor. The combined effect is an overall common-mode rejection of the instrumentation amplifier that is equal to the differential-mode gain in the preamplifier and the inverse of the component mismatch in the 1-OPAMP differential amplifier ($H_{tot} = F_1 \times H_2$).

It should be noted that the operation of the instrumentation amplifier strongly relies on the floating input circuit. Recognising that the inverting and non-inverting inputs of an OPAMP circuit in feedback are at the same potential ($U_+ - U_- = 0$), leads to the conclusion that the differential input voltage, U_{id}, is across R_6 and no current loop is available to U_{ic}, as shown in Fig. 4.12a. This isolation criterion is discussed in Section 4.2.2. This benefit is lost in the modified version of the differential-to-differential input stage of the instrumentation amplifier, as shown in Fig. 4.12b, in which resistor R_6 has been split into two equal parts.

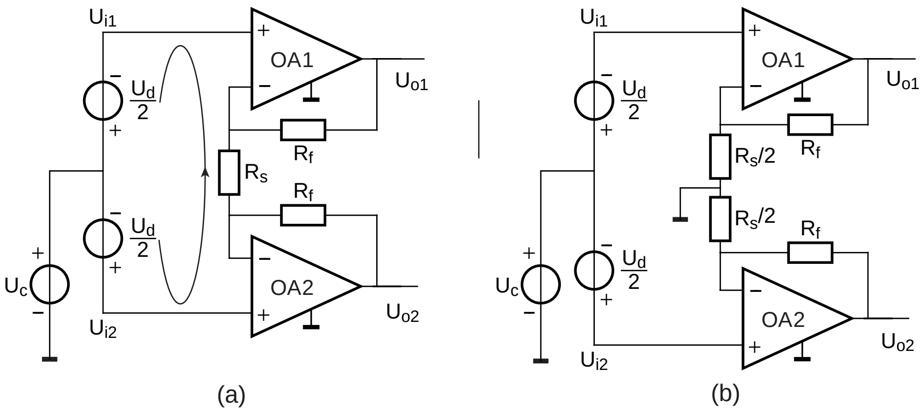

Figure 4.12, Two different versions of the differential-to-differential voltage pre-amplifier.

The modified input stage of the instrumentation amplifier in Fig. 4.12b is in fact composed of two separate non-inverting amplifiers with the gain $U_{o1}/U_{i1}= U_{o2}/U_{i2}= (R_f+R_s/2)/(R_s/2)= (2R_f+R_s)/R_s$. Therefore, the differential gain is equal to that of the basic instrumentation amplifier, as expressed in equation (4-19) for $R_f= R_5= R_7$. However, the transfer function is without the cross-coupling terms U_{o2}/U_{i1} and U_{o1}/U_{i2}.

The mismatch between the upper $R_s/2$ and its lower counterpart should be considered in the *CMRR* performance analysis. When assuming $R_{s,upper}= R_s(1+\varepsilon)/2$ and $R_{s,lower}= R_s(1-\varepsilon)/2$, the result is a transfer function described by:

$$\left.\begin{aligned} U_{o1} &= \frac{2R_f + R_s(1+\varepsilon)}{R_s(1+\varepsilon)} U_{i1} \\ U_{o2} &= \frac{2R_f + R_s(1-\varepsilon)}{R_s(1-\varepsilon)} U_{i2} \end{aligned}\right\} \rightarrow$$

$$U_{o1} - U_{o2} = \frac{2R_f(1-\varepsilon) + R_s(1-\varepsilon^2)}{R_s(1-\varepsilon^2)} U_{i1} - \frac{2R_f(1+\varepsilon) + R_s(1-\varepsilon^2)}{R_s(1-\varepsilon^2)} U_{i2} \approx \tag{4-25}$$

$$\left(\frac{2R_f + R_s}{R_s}\right)(U_{i1} - U_{i2}) - \frac{4\varepsilon R_f}{R_s}\frac{(U_{i1} + U_{i2})}{2} = G_{dd}U_d + G_{cd}U_c$$

Hence $G_{dd}= (2R_f+R_s)/R_s$.and $G_{cd}= 4\varepsilon R_f/R_s$. The common-mode gain can be derived as:

$$\frac{U_{o1}+U_{o2}}{2}=\frac{2R_f(1-\varepsilon)+R_s(1-\varepsilon^2)}{2R_s(1-\varepsilon^2)}U_{i1}+\frac{2R_f(1+\varepsilon)+R_s(1-\varepsilon^2)}{2R_s(1-\varepsilon^2)}U_{i2}\approx \left(\frac{2R_f+R_s}{R_s}\right)\frac{(U_{i1}+U_{i2})}{2}=G_{cc}U_c \tag{4-26}$$

which results in G_{cc}= $(2R_f+R_s)/R_s$. Consequently, *H* and *F* can be expressed as:

$$H=\frac{G_{dd}}{G_{cd}}=\frac{(2R_f+R_s)/R_s}{(4\varepsilon R_f)/R_s}=\frac{R_f+R_s/2}{2\varepsilon R_f} \quad \text{and} \quad F=\frac{G_{dd}}{G_{cc}}=1 \tag{4-27}$$

Although the modified version provides the same differential-to-differential gain, the *CMRR* performance is significantly reduced, which is primarily due to the loss of isolation. The only remaining parameter for a high CMRR is symmetry. However, mismatch is introduced by using two components.

Forcing a (virtual) potential across the terminals of a sensor could be important in some applications, such as the read-out of an electro-chemical cell. However, modifying the basic instrumentation amplifier to the circuit shown in Fig. 4.12b to achieve this is not a good approach.

Example 4.3
Figure 4.13 shows a 2-OPAMP differential amplifier.

1. Derive the expression for the differential gain G_{dd} of the 2-OPAMP differential amplifier and Dimension the components in the circuit for a differential gain $G= U_o/(U_{i1}-U_{i2})= 20$ and $R_2= R_3= 1$ kΩ.
2. Derive the expression for the *CMRR* when considering practical tolerances in the resistors (assume $R_1= R_4(1+\delta)$, $R_2= R_3(1+\varepsilon)$, G= 20 and $\varepsilon= -\delta=$ 5%). The OPAMP's can in a first approximation be considered ideal (no offset or bias and infinitely large CMRR).

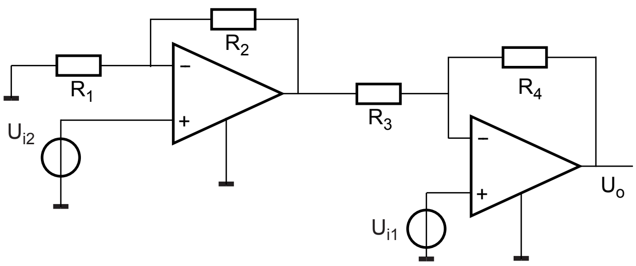

Figure 4.13, 2-OPAMP differential amplifier.

Solution:
The transfer function can be derived using superposition:

$$U_o = \frac{R_3+R_4}{R_3}U_{i1} + \frac{R_1+R_2}{R_1}\times(-)\frac{R_4}{R_3}U_{i2} = \frac{R_3+R_4}{R_3}\left(U_{i1} - \frac{R_1+R_2}{R_3+R_4}\times\frac{R_4}{R_1}U_{i2}\right) \quad (4\text{-}28)$$

For $R_2R_4= R_1R_3$ the result is: $U_o= G(U_{i1}-U_{i2})$ with $G= (R_3+R_4)/R_3= (R_1+R_2)/R_2= 20$. Hence, $R_1= R_4= 19$ kΩ.

2. The CMRR results from G_d and G_c:

$$U_o = G\left(U_{i1} - \frac{R_4(1+\delta)+R_3(1+\varepsilon)}{R_3+R_4}\times\frac{R_4}{R_4(1+\delta)}U_{i2}\right) =$$
$$G\left(U_{i1}-U_{i2}+\left[\frac{(\delta-\varepsilon)R_3}{(R_3+R_4)(1+\delta)}\right]U_{i2}\right) \simeq G\left(U_{i1}-U_{i2}+\left[\frac{(\delta-\varepsilon)}{G(1+\delta)}\right]U_{i2}\right) = \quad (4\text{-}29)$$
$$G\left[1-\frac{(\delta-\varepsilon)}{2G(1+\delta)}\right](U_{i1}-U_{i2})+G\left[\frac{(\delta-\varepsilon)}{G(1+\delta)}\right]\frac{U_{i1}+U_{i2}}{2} = G_dU_{id}+G_cU_{ic}$$

Via $H= G_d/G_c$ the result is: $H= [2G(1+\delta)-\delta+\varepsilon]/[2(\delta-\varepsilon)]= (40+1.95-0.05)/(0.2)= 209.5$ (= 46.4 dB). Since there is no floating input, this circuit does not provide the benefits of the instrumentation amplifier.

4.3.3 Using the instrumentation amplifier in medical applications

In the ECG measurement shown in Fig. 4.1b and presented in the introduction of this chapter, the common-mode signal is capacitively coupled to the input circuit, and resistors R_{p1} and R_{p2} are required for biasing purposes. These resistors basically ‘spoil’ the floating input of the instrumentation amplifier, the consequences of which are discussed in this section.

The transfer function of the differential ECG signal and the common-mode signal to the non-inverted and inverted inputs is basically similar to that of a differential-to-differential input stage of a read-out circuit and is expressed as (Fig. 4.1b):

$$U_+ = \frac{R_{p1}}{R_{p1}+R_{s1}}\left(U_{ic}+\frac{U_{id}}{2}\right)$$
$$U_- = \frac{R_{p2}}{R_{p2}+R_{s2}}\left(U_{ic}-\frac{U_{id}}{2}\right) \quad (4\text{-}30)$$

Which yields:

$$U_+ - U_- = \left(\frac{R_{p1}}{R_{p1}+R_{s1}} - \frac{R_{p2}}{R_{p2}+R_{s2}}\right)U_{ic} + \left(\frac{R_{p1}}{R_{p1}+R_{s1}} + \frac{R_{p2}}{R_{p2}+R_{s2}}\right)\frac{U_{id}}{2}$$
$$H = \frac{(U_+ - U_-)/U_{id}}{(U_+ - U_-)/U_{ic}} = \frac{1}{2}\times\frac{2R_{p1}R_{p2}+R_{p1}R_{s2}+R_{p2}R_{s1}}{R_{p1}R_{s2}-R_{p2}R_{s1}} \qquad (4\text{-}31)$$

Introducing $R_{p1}= R_p+\Delta R_p$, $R_{p2}= R_p-\Delta R_p$, $R_{s1}= R_s+\Delta R_s$ and $R_{s2}= R_s-\Delta R_s$, while disregarding second-order terms (ΔR^2), yields:

$$H = \frac{1}{2}.\frac{2R_p^2+\left(R_p+\frac{\Delta R_p}{2}\right)\left(R_s-\frac{\Delta R_s}{2}\right)+\left(R_p-\frac{\Delta R_p}{2}\right)\left(R_s+\frac{\Delta R_s}{2}\right)}{\left(R_p+\frac{\Delta R_p}{2}\right)\left(R_s-\frac{\Delta R_s}{2}\right)-\left(R_p-\frac{\Delta R_p}{2}\right)\left(R_s+\frac{\Delta R_s}{2}\right)} \rightarrow$$
$$H = \frac{R_p\left(R_p+R_s\right)}{R_s\Delta R_p - R_p\Delta R_s} = \frac{\frac{R_p+R_s}{R_s}}{\frac{\Delta R_p}{R_p}-\frac{\Delta R_s}{R_s}} \qquad (4\text{-}32)$$

The expression for the discrimination factor, F, can be derived via $(U_+ + U_-)/2$ as a function of U_{ic} and U_{id} using a similar approach:

$$U_+ + U_- = \left(\frac{R_{p1}}{R_{p1}+R_{s1}} + \frac{R_{p2}}{R_{p2}+R_{s2}}\right)U_{ic} + \left(\frac{R_{p1}}{R_{p1}+R_{s1}} - \frac{R_{p2}}{R_{p2}+R_{s2}}\right)\frac{U_{id}}{2}$$

$$F = \frac{\frac{U_+ - U_-}{U_{id}}}{\frac{(U_+ + U_-)/2}{U_{ic}}} = \frac{\frac{R_{p1}}{R_{p1}+R_{s1}} + \frac{R_{p2}}{R_{p2}+R_{s2}}}{\frac{R_{p1}}{R_{p1}+R_{s1}} + \frac{R_{p2}}{R_{p2}+R_{s2}}} = 1 \qquad (4\text{-}33)$$

The combination of this differential-to-differential input attenuator and instrumentation amplifier can be considered a cascaded system, and the overall differential gain and CMRR can be calculated using equation (4-23).

Since the common-mode rejection of a practical instrumentation amplifier exceeds 10^4 at low frequencies, it can be concluded that the common-mode rejection of the input resistive divider is a limiting factor to the overall common-mode rejection and not to the instrumentation amplifier (equation (4-23): $1/H_t$= $1/H_1+1/(F_1H_2)$= $1/H_1+1/H_2 \sim 1/H_1$). Also in this case the primary cause is the limited isolation combined with component mismatch.

The *CMRR* of the input circuit, therefore, fully determines CMRR performance. As shown in equation (4-32), the CMRR is determined by the factor R_p/R_s and component tolerances, $\Delta R_p/R_p$ and $\Delta R_s/R_s$. The uncertainties in contact resistivity to the human body are much larger than those in bias resistors. Therefore, $\Delta R_s/R_s$ is usually the most significant.

The discrimination factor of the input circuit does not contribute to an increased overall common-mode rejection (F_1= 1). However, in this particular case the value of F_1 is insignificant due to $H_2 \gg H_1$. In an instrumentation amplifier typically CMRR(DC) > 80 dB.

Assuming R_p=10 MΩ ± 1% and R_s= 10 kΩ ± 20% yields:
$H_t = [1/H_1+1/H_2]^{-1} = [(0.01+0.2)\times 10^{-3} + 10^{-4}]^{-1} = 3226$ (about 70 dB), which confirms that the overall *CMRR* critically depends on the contact resistance between electrode and skin. Actually, this value for the common-mode rejection is not good enough. The common-mode signal is the capacitively coupled mains voltage and depends strongly on the coupling capacitance, C_c. Assuming C_c= 0.1 pF yields:

$$\frac{U_{ic}}{U_m} \simeq \frac{j\omega C_c\left(R_s+R_p\right)/2}{1+j\omega C_c\left(R_s+R_p\right)/2} \rightarrow \left|\frac{U_{ic}}{U_m}\right| = \frac{\omega C_c\left(R_s+R_p\right)/2}{\sqrt{1+\left(\omega C_c\left(R_s+R_p\right)\right)^2/4}} = 157\times 10^{-6} \qquad (4\text{-}34)$$

A mains voltage U_m= 230 V_{rms} at 50 Hz yields U_{ic}= 36 mV. Assume a differential gain G_{dd}= 100 and a differential input signal U_{id}= 10 µV superimposed on this common-mode level. The resulting output signal is described by:

$$U_o = G_d\left(U_{id} + \frac{U_{ic}}{H_t}\right) = 100\left(10\,\mu\text{V} + \frac{36\,\text{mV}}{3226}\right) = 1\,\text{mV} + 1.12\,\text{mV} \qquad (4\text{-}35)$$

Consequently, the signal-to-common-mode interference ratio is equal to: 20 log(1/1.12)= -1 dB, which is clearly not acceptable in a practical application.

It should be noted that the ECG information content is contained in a frequency band between about DC and 100 Hz, thus including the mains frequency. Frequency filtering, therefore, cannot be applied. The coupling capacitance should be minimised. Reproducible detection of the ECG signals at a given C_c, therefore, relies solely on the CMRR performance, which should be maximized using well-matched components, whenever possible, and a high value for R_p/R_s.

4.4 Frequency dependence of the CMRR of the instr. ampl.

A remarkable and not-so-credible result of equation (4-21) is the infinitely high common-mode rejection of the differential-to-differential input stage in the instrumentation amplifier, $H_1 \rightarrow \infty$. This conclusion is the direct result of equation (4-20), which yields G_{cd}= 0. This rather unrealistic conclusion thus results from the implicit assumptions made with respect to the opamps applied. It was assumed that:

1. The common-mode impedance $Z_{ic} \rightarrow \infty$,
2. The effect of the finite and frequency-dependent value of the opamp CMRR can be disregarded and
3. The opamp open-loop gain is ideal: $A(\omega) \rightarrow \infty$.

The common-mode impedance is in parallel to any biasing resistor. The effect of assumption 1 is discussed in Section 4.3.3.

4.4.1 Frequency dependence of the CMRR of an OPAMP

The open-loop gain of an OPAMP cannot be considered infinitely large (assumption 2). A practical OPAMP should, therefore, be considered a first-order system with an open-loop gain described by: $A(\omega) = A_o/(1+j\omega\tau_v)$, with A_o as the DC gain and $1/\tau_v$ as the -3 dB cut-off frequency, as shown by the solid line in Fig. 4.14. Usually this frequency-dependent behaviour is specified in terms of the DC gain, A_o [dB], and the unity-gain frequency, $f_T = A_o/(2\pi\tau_v)$ [Hz]. This property directly implies that any transfer function to be realised using feedback should have a frequency response within this open-loop modulus plot (unless resonance occurs).

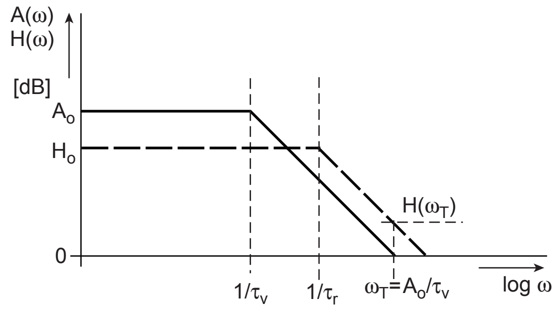

Figure 4.14, Specification of CMRR in an opamp relative to the open-loop gain.

Also the *CMRR* of an OPAMP cannot be considered infinitely large. As mentioned above, the major properties that limit common-mode rejection are component mismatch and differential input isolation. This conclusion also applies to the internal circuit of the opamp. The mismatch between the transistors that comprise the differential input stage and the finite output impedance of the biasing current source give rise to an opamp common-mode rejection $H_{OA}(\omega)$, which can be described by a first-order system: $H_{OA}(\omega)= H_o/(1+j\omega\tau_r)$ with the modulus transfer function shown by the dashed line in Fig. 4.14. Typical opamp specifications are: 80 dB < H_o< 100 dB and 50 Hz<$(2\pi\tau_r)^{-1}$< 500 Hz.

In a well-designed opamp the common-mode rejection is larger than unity for frequencies beyond unity-gain $\omega_T= A_o/\tau_v$. Therefore, the rejection-bandwidth is larger than the gain bandwidth, while $H_o< A_o$. As a consequence the common-mode rejection at the unity gain is:

$$CMRR(\omega_T)= 20 \log [H_{OA}(\omega_T)]= 20 \log [H_o/(1+j\omega_T\tau_r)]\sim$$
$$20 \log(H_o/(\omega_T\tau_r))= 20 \log(H_o\tau_v/(A_o\tau_r)) >1.$$

4.4.2 Effect of OPAMP A(ω) on the CMRR

The open-loop gain and CMRR of a practical OPAMP used in a the differential-to-differential pre-amplifier circuit, H_1 or H_2, are very large but not infinitely large (assumptions 2 and 3), which results in two operational constraints:

- The CMRR at DC has a finite value and
- The CMRR is frequency-dependent.

Firstly, the consequence of these practical constraints on the *CMRR* at very low frequencies (DC) is analysed, using Fig. 4.15.

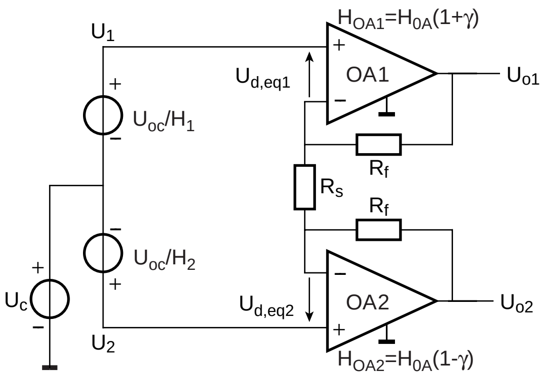

Figure 4.15, CMRR of the differential-to-differential voltage pre-amplifier in the case of mismatch in the CMRR of the opamps.

The effect of the opamp common-mode rejection on the common-mode rejection of the instrumentation amplifier depends on the nominal value of the opamp CMRR, $H_{OA}(\omega)$, and the matching between the CMRRs of the two OPAMPs comprising the differential-to-differential pre-amplifier. These have to be considered for a realistic estimate of the common-mode rejection at low frequencies

($\omega < 1/\tau_v$). From the definition of the CMRR follows that the effect of a common-mode signal at the output of an OPAMP, U_{oc}, can be represented by an equivalent differential input signal $U_{d,eq} = U_{oc}/H$.

Figure 4.15 shows that this signal is subtracted from the differential signal applied to the OPAMP. For OPAMP OA1 the effect is described by $U_{d,eq1} = U_{oc}/H_1$. Similarly, the effect on OA2 is described by $U_{d,eq2} = U_c/H_2$. Consider two OPAMPs, OA1 and OA2 with CMRR's that are not perfectly matched to the nominal value H_{OA}: $H_{OA1} = H_{OA}(1+\gamma)$ and $H_{OA2} = H_{OA}(1-\gamma)$. Since the circuit composed of OA1 and OA2 is a differential amplifier, the overall equivalent differential input signal representing the common-mode voltage is described by: $U_{d,eq} = U_{d,eq1} - U_{d,eq2}$, thus:

$$H_{eq} = \frac{1}{\dfrac{U_1 - U_2}{U_c}} = \frac{1}{\dfrac{1}{H_{OA2}} - \dfrac{1}{H_{OA1}}} = \frac{H_{OA}\left(1-\gamma^2\right)}{2\gamma} \approx \frac{H_{OA}}{2\gamma} \tag{4-36}$$

Therefore, it is not the finite values of the CMRR of the OPAMP's OA1 and OA2 in the pre-amplifier of the instrumentation amplifier that are limiting, but rather their CMRR mismatch.

The CMRR of the differential-to-differential voltage pre-amplifier in an instrumentation amplifier, H_1, is determined by the matching in *CMRR* of the opamps used.

Secondly, the consequence of the practical open-loop gain and CMRR of OA1 and OA2, as shown in Fig. 4.14, on the *CMRR* at higher frequencies is analysed using Fig. 4.16.

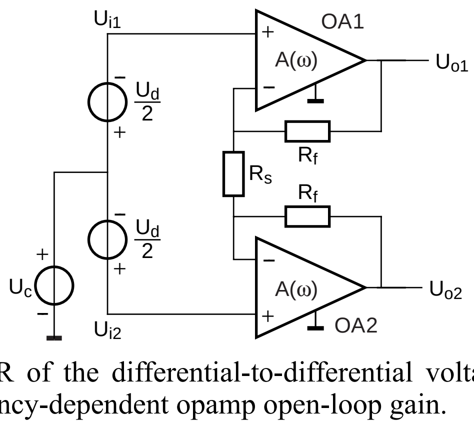

Figure 4.16, CMRR of the differential-to-differential voltage pre-amplifier in the case of a frequency-dependent opamp open-loop gain.

Note that only a feedback function with a modulus transfer function that fits within the open-loop of the OPAMP used can be realised. This constraint also applies to G_d and G_c, which makes the CMRR of the instrumentation amplifier frequency-dependent. Hence, the open-loop gain of the operational amplifiers, $A(\omega)= A_o/(1+j\omega\tau_v)$, used in the pre-amplifier has to be included in the calculation of both the common-mode rejection and the discrimination factor of the instrumentation amplifier.

This frequency dependence of the common-mode rejection of the instrumentation amplifier is derived by applying the same routine as used in the derivation of equations (4-20) through (4-22), but in this case with the non-ideal opamp properties. Hence, $U_o= A(\omega)(U_+-U_-)$, with $A(\omega)= A_o/(1+j\omega\tau_v)$, rather than stating that $U_+= U_-$ (which was based on the assumption that $A(\omega)\rightarrow \infty$). The following set of thee equations can be derived for the feedback circuit of OA1:

$$\begin{aligned}
&(a)\ \frac{U_{2-}-U_{1-}}{R_6}+\frac{U_{o1}-U_{1-}}{R_5}=0\\
&(b)\ \left.\begin{aligned} U_{o1} &= A(\omega)\left[U_{1+}-U_{1-}\right]\\ U_{1+} &= U_{i1}\end{aligned}\right\}\rightarrow U_{1-}=U_{i1}-\frac{U_{o1}}{A(\omega)}\\
&(c)\ \left.\begin{aligned} U_{o2} &= A(\omega)\left[U_{2+}-U_{2-}\right]\\ U_{2+} &= U_{i2}\end{aligned}\right\}\rightarrow U_{2-}=U_{i2}-\frac{U_{o2}}{A(\omega)},
\end{aligned} \tag{4-37}$$

where U_{1-} denotes the inverting input of OA1, etc.

Substituting (b) and (c) with (a) yields:

$$\frac{\left[U_{i2}-\dfrac{U_{o2}}{A(\omega)}\right]-\left[U_{i1}-\dfrac{U_{o1}}{A(\omega)}\right]}{R_6}+\frac{U_{o1}-\left[U_{i1}-\dfrac{U_{o1}}{A(\omega)}\right]}{R_5}=0 \tag{4-38}$$

Similarly for the feedback circuit of OA2:

$$\begin{aligned}
&\frac{U_{2-}-U_{1-}}{R_6}+\frac{U_{2-}-U_{o2}}{R_7}=0\rightarrow\\
&\frac{\left[U_{i2}-\dfrac{U_{o2}}{A(\omega)}\right]-\left[U_{i1}-\dfrac{U_{o1}}{A(\omega)}\right]}{R_6}+\frac{\left[U_{i2}-\dfrac{U_{o2}}{A(\omega)}\right]-U_{o2}}{R_7}=0
\end{aligned} \tag{4-39}$$

Subtracting equation (4-39) from (4-38) with $R_5= R_7= R_f$ yields for the differential gain G_{dd}:

$$G_{dd} = \frac{U_{o1}-U_{o2}}{U_{i1}-U_{i2}} = \frac{A(\omega)\left[R_6+2R_f\right]}{2R_f+R_6\left[1+A(\omega)\right]} = \frac{\left(R_6+2R_f\right)A_o}{R_6\left(1+A_o\right)+2R_f+j\omega\tau_v\left(R_6+2R_f\right)} \qquad \text{(4-40)}$$

Adding equations (4-39) and (4-38) with $R_5 = R_7 = R_f$ yields for the common-mode gain G_{cc}:

$$G_{cc} = \frac{U_{o1}+U_{o2}}{U_{i1}+U_{i2}} = \frac{A(\omega)}{1+A(\omega)} = \frac{\frac{A_o}{1+A_o}}{1+j\omega\frac{\tau_v}{1+A_o}} \approx \frac{1}{1+j\omega\frac{\tau_v}{A_o}} \qquad \text{(4-41)}$$

The discrimination factor, F_1, is by definition the ratio between G_{dd} and G_{cc} and follows from equations (4-40) and (4-41) as:

$$F = \frac{G_{dd}}{G_{cc}} = \frac{\left[R_6+2R_f\right]\left[1+A(\omega)\right]}{2R_f+R_6\left[1+A(\omega)\right]} = \frac{\left(\frac{\left(R_6+2R_f\right)\left(1+A_o\right)}{R_6\left(1+A_o\right)+2R_f}\right)\left(1+j\omega\frac{\tau_v}{A_o}\right)}{1+j\omega\tau_v\frac{R_6+2R_f}{R_6\left(1+A_o\right)+2R_f}} \approx \frac{A_{ddo}\left(1+j\omega\frac{\tau_v}{A_o}\right)}{1+j\omega\tau_v\frac{A_{ddo}}{A_o}} \qquad \text{(4-42)}$$

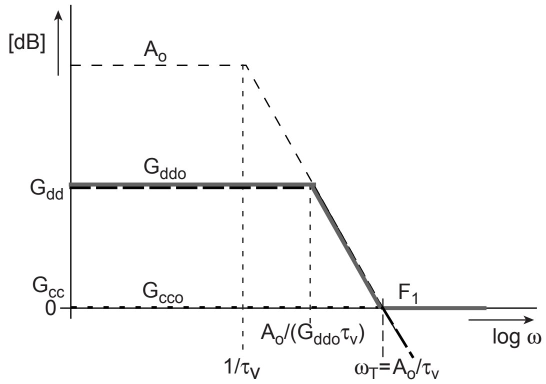

Figure 4.17, Modulus plot of the differential gain, $G_{dd}(\omega)$, and common-mode gain, $G_{cc}(\omega)$, in the differential-to-differential voltage pre-amplifier with reference to the opamp open-loop gain, A(ω).

Figure 4.17 shows the modulus plots of both the differential gain, G_{dd}, (line with long dashes) and the common-mode gain, G_{cc}, (line with short dashes) of the differential-to-differential pre-amplifier. The figure demonstrates one of the main problems. The discrimination factor F_1 is shown as a solid grey line. At low frequencies a high value of F_1 is based on a high value for G_{dd}, while G_{cc}= 1.

However, the gain in a feedback circuit cannot exceed the open-loop gain, which implies that G_{cc}= 1 up to the unity-gain frequency, ω_T, while G_{dd} is forced to follow the open-loop gain for frequencies where it would otherwise cross $A(\omega)$, which is at $\omega_{dd}= A_o/(\tau_v G_{dd})$. Consequently, F_1 is equal to G_{ddo} up to $\omega_{dd}= A_o/\tau_v G_{ddo}$. At larger frequencies F_1 reduces with frequency and remains constant and equal to F_1= 1 for frequencies beyond ω_T.

The advantage of the differential-to-differential pre-amplifier to the overall CMRR of the instrumentation amplifier ($H_{tot}\sim F_1 H_2$) is, therefore, limited for frequencies up to ω_{dd}. This property needs to be included in the system design by restricting the differential signal bandwidth to ω_{dd}.

4.4.3 Instrumentation amplifier for high-CMRR bridge readout

The instrumentation amplifier is very suitable for the read-out of resistive sensors, such as the strain gauge, when included in a Wheatstone bridge. Integrated circuits are available with all the components of the instrumentation amplifier included, with the exception of R_6. Either two pins are made available to connect an external gain-setting resistor, or a limited number of resistors are included on-chip, each with a pin for external gain selection to one common pin. On-chip component matching is sufficient to enable the fabrication of instrumentation amplifiers. The frequency dependence shown in Fig. 4.17 applies when *CMRR* > 100 dB typically for frequencies up to 100 Hz and reduces at higher frequencies.

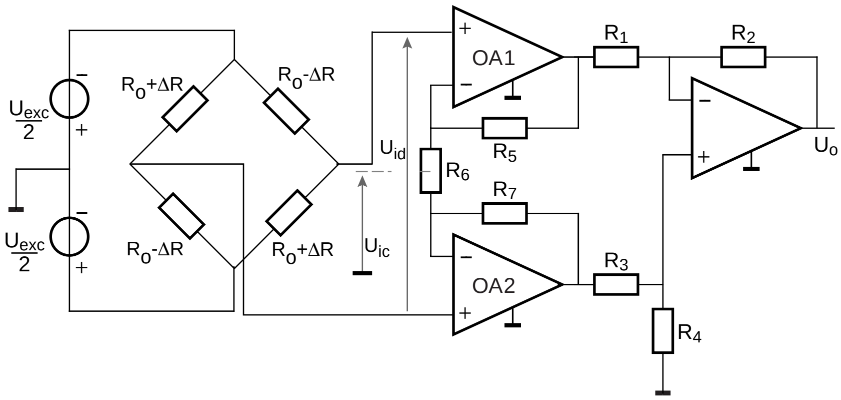

Figure 4.18, Instrumentation amplifier for the read-out of a Wheatstone bridge with strain gauges.

The considerations regarding offset, as presented in Chapter 3, and those related to excitation source balancing for reducing CMRR requirements, do apply. As a consequence the balanced AC excitation with balanced excitation, as shown in Fig. 4.18, is to be used for the best possible detectivity. The next detection limiting factor is noise, which is discussed in the next chapter.

4.5 Exercises

Figure 4.19 shows an instrumentation amplifier.

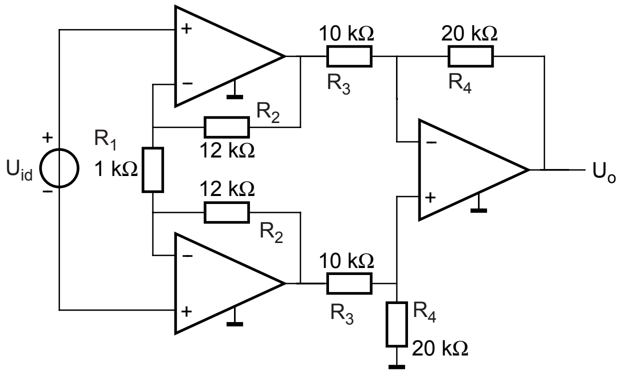

Figure 4.19, Instrumentation amplifier for a gain of 50.

4.1 What is the gain, U_o/U_{id}, in the case of the component values shown in the figure?

Solution:.

$$G_d = \frac{U_o}{U_{id}} = \frac{2R_2 + R_1}{R_1} \times \frac{R_4}{R_3} = \frac{24k\Omega + 1k\Omega}{1k\Omega} \times \frac{20k\Omega}{10k\Omega} = 50 \qquad (4\text{-}43)$$

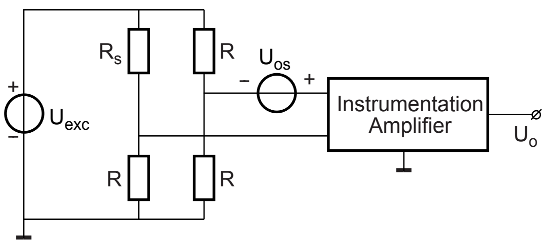

Figure 4.20, Instrumentation amplifier with offset and finite CMRR.

Figure 4.20 shows the application of an instrumentation amplifier for the readout of an impedance bridge.

4.2 Calculate U_o for DC excitation with U_{exc}= 10 V, R_s= 1.0002×R, U_{os}= 0.5 mV and *CMRR*= 80 dB. The gain of the instrumentation amplifier is set to G_d= 1.

Solution:.

$$U_o = G_d\left(\left|\frac{\Delta R}{4R_o}U_{exc}\right| + \left|\frac{U_{ic}}{H}\right| + \left|U_{os}\right|\right) = 1\times\left(\frac{2\times10^{-4}}{4}\times10 + \frac{5}{10^4} + 5\times10^{-4}\right) = 1.5mV \quad (4\text{-}44)$$

The circuit shown in Fig. 4.21 is used as a differential voltage amplifier. The OPAMPs can in a first approximation be considered ideal (no offset or bias, $A(\omega) = A_o \to \infty$, and infinitely large values for the *CMRR*).

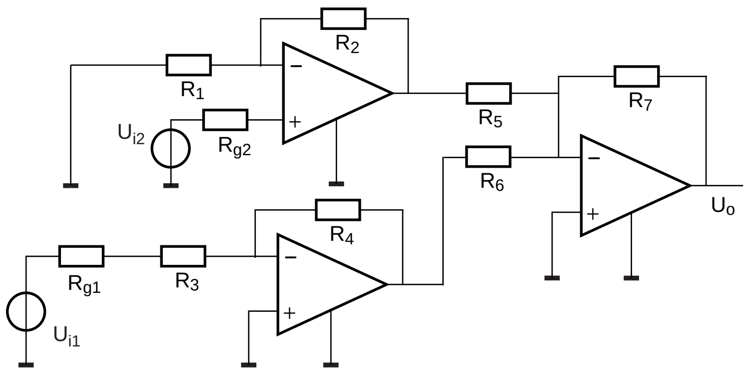

Figure 4.21, 3-OPAMP differential amplifier.

4.3 Derive an expression for $G = U_o/(U_{i1}-U_{i2})$ and dimension the circuit for a differential gain G= 20 for $R_2 = R_4 = R_7$= 10 kΩ (several solutions are possible). Assume at this stage that R_{g1} and R_{g2} can be disregarded.

Solution:

$$\begin{aligned} U_o &= (-)\frac{R_4}{R_3}\times(-)\frac{R_7}{R_6}U_{i1} + \frac{R_1+R_2}{R_1}\times(-)\frac{R_7}{R_5}U_{i2} = \\ &\frac{R_7}{R_6}\times\frac{R_2}{R_3}\left(\frac{R_4}{R_2}U_{i1} - \frac{R_6}{R_5}\times\frac{(R_1+R_2)R_3}{R_1R_2}U_{i2}\right) \end{aligned} \quad (4\text{-}45)$$

For R_1= 10 kΩ, R_3= 5 kΩ and $R_5 = R_6$= 1 kΩ the result is:
U_o= (10 kΩ/1 kΩ)×(10 kΩ/5 kΩ)[(10 kΩ/10 kΩ)×U_{i1}-(1 kΩ/1 kΩ)×(10 kΩ+10 kΩ)×5 kΩ)/(10 kΩ×10 kΩ)×U_{i2}]=
10×2[U_{i1}-U_{i2}]= 20(U_{i1}-U_{i2}).

4.4 Is the circuit in Fig. 4.21 sensitive to significant differences in the impedances R_{g1} and R_{g2} of the signal sources? Explain your answer.

Solution:
Yes, U_{i1} is connected to the input of an inverting amplifier with input impedance $Z_{ip}= R_3$, whereas U_{i2} is connected to the input of a non-inverting amplifier with a very high input impedance $Z_{is}\to\infty$ (inverting input is driven to the same -albeit virtual- potential). Hence, the difference in scale error due to source loading would be significant.

4.5 Derive an expression for the common-mode rejection (CMRR) of this differential amplifier when using the nominal component values calculated in problem 4.3 and considering tolerances in the resistors (assume: $R_{g1}= R_{g2}= 0$, $R_1= R_3$, $R_4= 2R_2(1+\delta)$, $R_5= R_6(1+\varepsilon)$, $G= 20$ and $|\varepsilon| = |\delta| \ll 1$). In a first approximation the OPAMPs can be considered ideal (no offset or bias and an infinitely large CMRR).

Solution:
In the solution to 4.3: $R_1= R_2$ and $R_7= 10R_6$.

$$U_o = \frac{G}{2}\left(2(1+\delta)U_{i1} - \frac{2}{1+\varepsilon}U_{i2}\right) \simeq G\left((1+\delta)U_{i1} - (1-\varepsilon)U_{i2}\right)$$

$$Worst-case: \delta = \varepsilon \to G\left(U_{i1} - U_{i2}\right) + 2\delta G\frac{U_{i1}+U_{i2}}{2} = G_d U_{id} + G_c U_{ic} \to \qquad (4\text{-}46)$$

$$H = \frac{G_d}{G_c} = \frac{G}{2\delta G} = \frac{1}{2\delta}$$

This circuit is outperformed by the instrumentation amplifier. Source loading is unbalanced. CMRR performance depends on component matching, which is due to the non-floating differential input circuit.