

3. *Underwater propagation*

3.1 *Basic principles of acoustics*

3.1.1 Introduction

Sound is a wave phenomenon. A wave in its turn is a disturbance of the equilibrium that spreads, or propagates, with time through space.

Waves appear in many contexts besides sound, and some examples of other contexts are:

- Light or electromagnetic waves in general;
- Waves on the water surface.

If, for example, we disturb the water surface at a certain point by throwing in a stone, the resulting disturbance will propagate in all directions along the surface. Here, the disturbance consists of the elevation of a point relative to the horizontal, flat, water surface. In the case of sound, the disturbance consists of a pressure disturbance.

Note that when considering propagating waves it is not the *material* of the medium through which the wave propagates that is transported. It is, however, the *disturbance* that is transported.

There are two types of elastic waves. (Elastic waves are called so because they deform the medium elastically - the medium returns to its original shape and position after the wave has passed through. An example of a non-elastic wave is a shock wave. This type of wave fundamentally changes the medium through which it propagates.):

- Longitudinal waves or compressional waves: the medium particles oscillate in the direction of the propagation
(examples: sound waves in gasses and liquids)
- Transverse waves: the medium particles oscillate in a direction perpendicular to the direction of the propagation
(examples: vibrating violin string, waves on the water surface)

Remark:

Earthquake waves are an example of both types.

For sound and light, the propagation speed of the wave is independent of the wavelength. This is, however, not the case for all wave phenomena (e.g. propagating waves on the water surface).

As we will see later in this chapter, sound can be understood by means of Newton's laws (classic mechanics).

The mathematical expression $\xi(x, t) = f(x - ct)$ describes a physical situation that propagates (without deforming) along the positive x -axis.

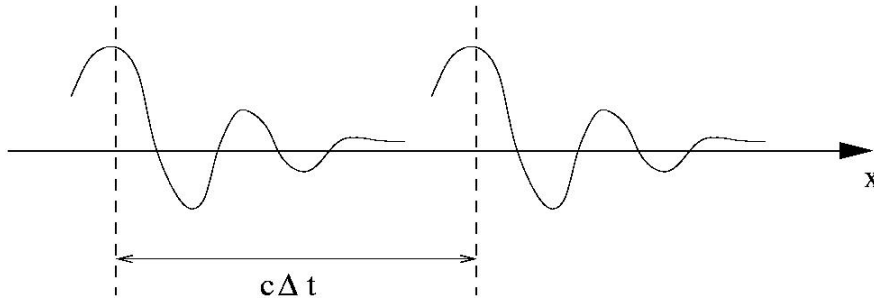
The expression $\xi(x, t) = f(x + ct)$ describes propagation along the negative x -axis.

The quantity $\xi(x, t)$ can be:

- The pressure in a gas or liquid (sound);
- An electromagnetic field (light).

If we now consider the figure plotted below, showing $\xi(x, t)$ as a function of position (x) for two moments in time with the two times differing by a time Δt , it is clear that the following has to be valid:

$$f(x - ct) = f(x + \Delta x - c(t + \Delta t))$$



This is indeed valid if $\Delta x = c\Delta t$. It can be concluded that c is the velocity (phase velocity) with which the disturbance $f(x)$ ‘spreads’ (propagates).

3.1.2 Plane harmonic waves

A plane wave can be written as $p(x, t)$, i.e., the pressure does not depend on y and z ; it only depends on x .

The expression for a plane *harmonic* wave is:

$$p(x, t) = p_0 \cos(kx - \omega t + \varphi)$$

Often an alternative expression is used (using complex numbers):

$$p(x, t) = p_0 e^{i(kx - \omega t + \varphi)} \quad \text{with } i^2 = -1$$

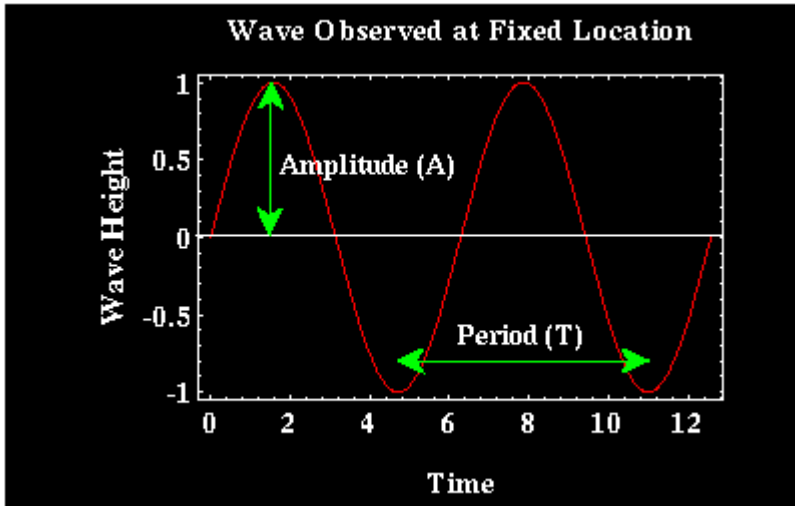
p_0 is the amplitude, ω the radial frequency (in rad/s) and k the wavenumber; $kx - \omega t + \varphi$ is denoted by the phase angle.

$f = \frac{\omega}{2\pi}$ is the frequency (unit: Hz or s^{-1}).

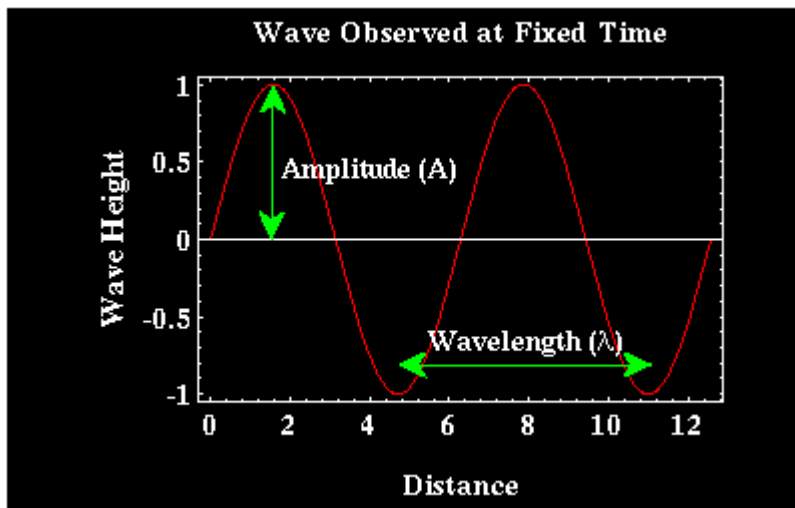
Remark:

The phase angle of the above wave is independent of y and z : at every moment in time the phase (angle) of the wave is equal at all points in a plane perpendicular to the x -axis. This property is the reason why these waves are denoted by *plane* waves: The wave front is a plane.

At a fixed position in space x , this function repeats itself in time intervals T (=period), see the figure below. The following is valid: $\omega T = 2\pi$. The frequency f is $f = 1/T$ (in Hz).



At a fixed moment in time t , the function repeats itself over the distance λ (see figure below). This distance is called the wavelength, with $k\lambda = 2\pi$.



The phase angle at moment $t = t_1$ and position $x = x_1$ is $kx_1 - \omega t_1 + \varphi$. Time Δt later the value of x , for which the phase angle is equal to the original phase angle, is $x_1 + \Delta x$. The following is valid:

$$k(x_1 + \Delta x) - \omega(t_1 + \Delta t) + \varphi = kx_1 - \omega t_1 + \varphi$$

This implies that the original phase (angle) moves at a velocity c :

$$c = \frac{\Delta x}{\Delta t} = \frac{\omega}{k} = \frac{\lambda}{T}$$

c is the phase speed (sound speed).

Some typical values:

In air the frequencies of audible sound are in the range between 20 Hz and 20 kHz.

The phase speed is about 343 m/s.

In water the phase speed of sound is about 1500 m/s. The frequencies typically used for sonar lie in between '0' Hz and a few hundreds kHz (see the table below).

| Frequency-range | Sonar |
|-------------------|-------------------------|
| '0' - 1000 Hz | Passive (military) |
| 5 kHz - 10 kHz | hull-mounted (military) |
| 10 - 200 kHz | echo-sounder |
| \approx 100 kHz | mine detection |
| \approx 400 kHz | mine classification |
| \approx 10 MHz | medical imaging |

3.1.3 Sound pressure level, the decibel and the effective value

The variations in pressure relative to the equilibrium pressure due to a sound wave are extremely small.

In air, the hearing threshold is 2×10^{-5} Pa ($1 \text{ Pa} = 1 \text{ N/m}^2$) and the pain threshold is 100 Pascal (Pa).

This corresponds to 2×10^{-10} atm and 10^{-3} atm, respectively.

The sound pressure level is defined as

$$L = 20^{10} \log \frac{P}{P_{\text{ref}}} \quad \text{decibel (dB)}$$

In air $p_{\text{ref}} = 2 \times 10^{-5}$ Pa, i.e., the hearing threshold.

Therefore the hearing threshold corresponds to 0 dB and the pain threshold to 134 dB.

In (sea) water the reference pressure p_{ref} is taken different from that in air, namely 10^{-6} Pa.

For the pressure p in the expression for L the effective value p_{rms} (rms = root-mean-square) needs to be taken. For a plane harmonic wave with amplitude p_0 , p_{rms} is

$$p_{\text{rms}}^2 = \langle p^2 \rangle = \langle p^2 \rangle = \frac{1}{\tau} \int_0^{\tau} p_0^2 \cos^2(kx - \omega t) dt = \frac{p_0^2}{2}$$

with $\langle \rangle$ denoting the mean; τ is the time of integration (should be large relative to the period T).

3.1.4 The wave equation for plane waves

Let us consider the propagation of sound in one dimension, for example due to the movement of an object (e.g. piston in tube) at a fixed position: the disturbance, viz. the change in pressure, propagates.

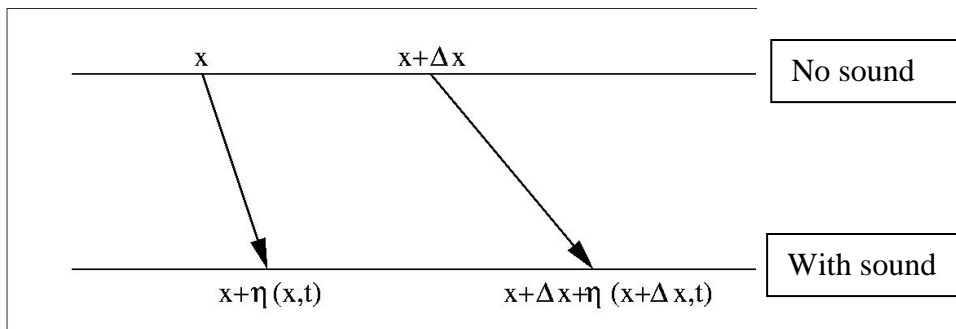
Which physical quantities are required?

- The particle displacement η
- The density ρ
- The pressure P
- The particle speed $v = \frac{\partial \eta}{\partial t}$
- The particle acceleration $a = \frac{\partial^2 \eta}{\partial t^2}$

All above quantities depend on x and t .

Note: In the mean sense the particles remain at the same position; the *disturbance* propagates.

Further: the behaviour of the medium through which the sound propagates is not described in terms of individual molecules, i.e., the particles mentioned above are not individual molecules, but are packages of the medium; classic mechanics is sufficient.



The following notation will be used (0 = equilibrium = undisturbed medium, i.e., no sound):

- $P = P_0 + p$
- $\rho = \rho_0 + \rho_1$
- $P = P(\rho)$

A. Movement of the medium gives rise to a change in the density

Consider a certain amount of the medium (air, water) per unit area:

Undisturbed this is $\rho_0 \Delta x$.

Due to the passing of a pressure wave this amount of the medium is displaced and the amount of medium becomes:

$$\rho (x + \Delta x + \eta(x + \Delta x, t) - x - \eta(x, t))$$

However, mass should remain equal and therefore both the undisturbed and disturbed amount of medium have to be equal.

Employing the Taylor series on $\eta(x + \Delta x, t)$ (using only the first order terms), conservation of mass gives:

$$\rho_0 \Delta x = (\rho_0 + \rho_1) \left[\Delta x + \Delta x \frac{\partial \eta}{\partial x} \right]$$

or

$$\rho_1 = -\rho_0 \frac{\partial \eta}{\partial x} - \rho_1 \frac{\partial \eta}{\partial x} \approx -\rho_0 \frac{\partial \eta}{\partial x}$$

where again a second order term is neglected

This last expression shows: if the displacement varies with x , the density will change.

B. Change in density gives rise to a change in pressure

Employing Taylor:

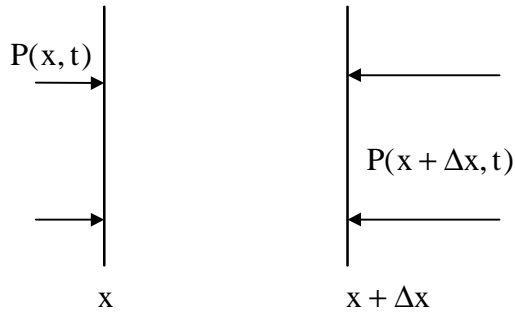
$$P_0 + p = f(\rho_0 + \rho_1) = f(\rho_0) + \rho_1 f'(\rho_0) = P_0 + \rho_1 \left(\frac{dP}{d\rho} \right)_0$$

The sound pressure is therefore given by

$$p = \rho_1 c^2 \text{ with } c^2 = \left(\frac{dP}{d\rho} \right)_0 \text{ a medium constant.}$$

C. Inequalities in the pressure result in a movement of the medium

Consider the following situation:



Employing Newton's third law (and using Taylor again):

$$\rho_0 \Delta x \frac{\partial^2 \eta(x, t)}{\partial t^2} = P(x, t) - P(x + \Delta x, t) = -\Delta x \frac{\partial P(x, t)}{\partial x}$$

Elimination of P by employing subsequently C, B and A:

$$\rho_0 \frac{\partial^2 \eta}{\partial t^2} = -\frac{\partial p}{\partial x} = -c^2 \frac{\partial \rho_1}{\partial x} = c^2 \rho_0 \frac{\partial}{\partial x} \frac{\partial \eta}{\partial x}$$

or (wave equation for η):

$$\frac{\partial^2 \eta}{\partial t^2} = c^2 \frac{\partial^2 \eta}{\partial x^2}$$

Elimination of η (second derivative with respect to t of B; employing A and then C):

Wave equation for the sound pressure:

$$\frac{\partial^2 p}{\partial t^2} = c^2 \frac{\partial^2 p}{\partial x^2}$$

Each plane wave $f(x - vt)$ (not necessarily harmonic) is a solution to this equation if $v = c$.

The wave equation is linear, i.e., if f_1 and f_2 are solutions to the wave equation, then also $f_1 + f_2$ is a solution (principle of superposition).

3.1.5 The harmonic solution to the wave equation

Consider a harmonic plane wave (for the quantity η):

$$\eta(x,t) = \eta_0 e^{i(kx - \omega t)}$$

According to (A) + (B) of the previous paragraph the following is valid:

$$p = -c^2 \rho_0 \frac{\partial \eta}{\partial x}$$

or (use $c = \frac{\omega}{k}$):

$$p = -i \rho_0 c \omega \eta$$

Also:

$$v = \frac{\partial \eta}{\partial t} = -i \omega \eta \quad \text{and} \quad a = \frac{\partial v}{\partial t} = -i \omega v$$

Important result:

For the harmonic plane wave it appears that the following is valid: $\frac{p}{v} = \rho_0 c$ for all x and t , i.e., a constant (p and v are in phase; this is not the case for p and η).

Example:

Consider a harmonic plane wave in air with a frequency of 1000 Hz and a sound pressure level of 100 dB.

$$\rho_0 = 1.21 \text{ kg/m}^3$$

$$c = 343 \text{ m/s}$$

$$p_{rms} = 10^5 \times p_{ref} = 2 \text{ Pa. Then } \eta_{rms} = \frac{p_{rms}}{\rho_0 c \omega} \approx 10^{-6} \text{ m} \ll \lambda = 0.34 \text{ m}$$

$$v_{rms} = \omega \eta_{rms} = 5 \times 10^{-3} \text{ m/s} \ll c. \text{ However, do not forget that } \eta_{rms} \text{ is very small.}$$

3.1.6 The sound speed

Sound is an adiabatic process, for which, according to the thermodynamics of gasses, the following holds:

$$PV^\gamma = \text{constant}$$

or

$$P = \text{constant } \rho^\gamma$$

with γ a gas constant (1.4 for air) and V the volume.

$$c^2 = \left(\frac{dP}{d\rho} \right)_{(0)} = \text{constant } \gamma \rho^{\gamma-1} = \frac{\gamma}{\rho} P$$

According to the gas law for ideal gasses (for 1 mol of gas):

$$PV = RT$$

with R the gas constant (8.31 Joule/Kelvin) and T the absolute temperature.

The mass of 1 mol of gas is $\rho V = M = mN$. m is the mass of 1 molecule and N is the Avogadro number (6×10^{23} molecules in 1 mol).

The sound speed then becomes:

$$c = \sqrt{\frac{\gamma RT}{M}}$$

For air (80 % nitrogen and 20 % oxygen, $M = 28.8 \times 10^{-3}$ kg) at 0°C (273 K) the sound speed becomes 332 m/s.

The influence of temperature on the sound speed in air is given by

$$c = c(0^\circ\text{C}) \sqrt{1 + 0.0037 T(^\circ\text{C})}$$

Using the Taylor approximation (only 1st order term) this becomes

$$c = 332 + 0.6 T(^\circ\text{C})$$

The sound speed in gasses and liquids can also be written as

$$c = \sqrt{\frac{K}{\rho_0}} \quad \text{with } K = \frac{1}{\kappa} \quad \text{the compressibility (bulk) modulus } (c = 343 \text{ m/s at } 20^\circ\text{C}).$$

For pure water $K = 2 \times 10^9 \text{ N/m}^2$ and $\rho_0 = 1000 \text{ kg/m}^3$, so $c = 1414 \text{ m/s}$.

An empirical formula for the sound speed in seawater is

$$c = 1449.2 + 4.6T - 0.055T^2 + 0.00029T^3 + (1.34 - 0.01T)(S - 35) + 0.017z$$

with

T the temperature in $^\circ\text{C}$

z the depth in m

S the salinity (salt content) in ppt

c the sound speed in m/s

The propagation speed of *longitudinal* waves in solid media (also transversal waves are possible) is given by

$$c = \sqrt{\frac{E}{\rho_0}} \quad \text{with } E \text{ Young's elasticity modulus.}$$

For steel $E = 2 \times 10^{11} \text{ N/m}^2$ and $\rho_0 = 7800 \text{ kg/m}^3$, so $c = 5100 \text{ m/s}$.

3.1.7 Acoustic intensity and acoustic impedance

The intensity I is the mean rate of flow of energy through a unit area normal to the direction of the propagation:

$$I = \left\langle \frac{F \partial \eta}{A \partial t} \right\rangle = \langle p v \rangle$$

with A the surface and F the force ($\langle \rangle$ denotes the mean).

For a harmonic plane wave this becomes (p and v in phase):

$$I = \frac{\langle p^2 \rangle}{\rho_0 c} = \frac{p_{rms}^2}{\rho_0 c}$$

The reference-intensity in air (hearing threshold) is $I_{ref} = 10^{-12} \text{ Watt/m}^2$.

The characteristic acoustic impedance is defined as

$$Z = \frac{p}{v}$$

For a harmonic plane wave this is real and equals $\rho_0 c$.

In general, the characteristic acoustic impedance is complex (see later: due to absorption, for spherical waves, for standing waves).

Example:

Consider a plane harmonic wave with a frequency of 1000 Hz in both air and water, with identical intensities. The sound pressure level of the wave in air is 100 dB.

The following holds:

$$\left(\frac{p_{rms}^2}{\rho_0 c} \right)_{\text{air}} = \left(\frac{p_{rms}^2}{\rho_0 c} \right)_{\text{water}} \quad \text{thus} \quad p_{rms, \text{water}} = 60 p_{rms, \text{air}} = 120 \text{ Pa}.$$

This corresponds to 162 dB in water.

In water $\eta_{rms} = 10^{-8} \text{ m}$ and $v_{rms} = 10^{-6} \text{ m/s}$ (very small!).

(Compare with the example in paragraph 3.1.5).

3.1.8 Spherical waves and cylindrical waves

In acoustics, spherical waves are at least as important as, but probably even more important than, plane waves. They are described, just as plane waves, in terms of one spatial variable.

The surface of a pulsating spherical object that expels and contracts radially around a mean value results in a disturbance that spreads in all directions through a spherical wave. Its wave fronts are spheres.

Examples: loudspeaker (at low enough frequencies) and a stone in the water (causes actually circular waves).

The expression for the wave equation in three dimensions (for $p(x,y,z,t)$) is:

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2}$$

(Compare with the 1-dimensional expression, as derived in paragraph 3.1.4).

The 3-dimensional wave equation in spherical coordinates, (r, θ, φ) , is at first sight rather complex (r is the radial distance towards the source):

$$\frac{1}{r} \frac{\partial^2}{\partial r^2} (rp) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta \frac{\partial p}{\partial \theta}) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 p}{\partial \varphi^2} = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2}$$

However, p is independent of θ and φ (uniform disturbance). Consequently, the second and third terms in the left hand side of the expression are zero:

$$\frac{1}{r} \frac{\partial^2}{\partial r^2} (rp) = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} \quad \text{or:} \quad \frac{\partial^2}{\partial r^2} (rp) = \frac{1}{c^2} \frac{\partial^2 (rp)}{\partial t^2}$$

The general solution of this latter equation (see the equation for the plane wave) is

$$rp(r,t) = f(r - ct) + g(r + ct), \text{ with } f \text{ and } g \text{ arbitrary functions.}$$

The second function g represents wave propagation towards the mid point. This is not possible and the solution therefore becomes:

$$p(r,t) = \frac{A}{r} f(r - ct), \text{ with } A \text{ being a constant.}$$

The *harmonic* solution is:

$$p(r,t) = \frac{A}{r} e^{i(kr - \omega t)}$$

For the spherical harmonic wave we are interested in the relation between pressure and particle speed. In the derivation for the plane wave equation:

$$\rho_0 \frac{\partial^2 \eta}{\partial t^2} = -\frac{\partial p}{\partial x}$$

For the spherical wave this becomes: $\rho_0 \frac{\partial v}{\partial t} = -\frac{\partial p}{\partial r}$

Now the following is valid $\frac{\partial v}{\partial t} = -i\omega v$ so $v = \frac{1}{i\omega\rho_0} \frac{\partial p}{\partial r}$

Resulting in: $v(r,t) = \frac{p(r,t)}{\omega\rho_0} \left[\frac{i}{r} + k \right]$

The spherical wave *impedance* therefore becomes:

$$Z = \frac{p(r,t)}{v(r,t)} = \rho_0 c \left[\frac{k^2 r^2}{1+k^2 r^2} - i \frac{kr}{1+k^2 r^2} \right]$$

This expression shows that for the spherical wave the particle velocity is, in general, not in phase with the pressure: the impedance is complex and not even constant. (This is in contrast to the plane wave impedance!)

An important parameter is $kr = \frac{2\pi r}{\lambda}$. Let's consider two limiting cases:

- $kr \gg 1$ ($r \gg \lambda$): $Z = \rho_0 c$, i.e. no phase difference between p and v : to a good approximation the spherical wave is a plane wave;
- $kr \ll 1$ ($r \ll \lambda$): $Z = -i\rho_0 ckr$, i.e. the phase difference between p and v is 90° .

Now about the *intensity* of a spherical wave:

At a large distance from the source this intensity is given by

$$I = \frac{p_{rms}^2}{\rho_0 c} \quad \text{with} \quad p_{rms}^2 = \frac{p_0^2}{2} = \frac{A^2}{2r^2} \quad \text{thus} \quad I \sim \frac{1}{r^2}$$

The mean power P passing through a sphere with radius r with the source in the mid point is:

$$P = 4\pi r^2 I = \frac{2\pi A^2}{\rho_0 c} \quad \text{and this is independent on the distance } r \text{ towards the source. This}$$

should indeed be the case according to the conservation of energy.

We will end this paragraph by considering *cylindrical waves*. For this type of waves the wave fronts are cylindrical surfaces.

Again the following is valid for the intensity (with large enough r):

$$I(r) = \frac{P_0^2}{2\rho_0 c}$$

The mean power P passing the cylindrical surface with radius r and height H is:

$$P = 2\pi r H \frac{P_0^2}{2\rho_0 c} \quad \text{and according to the conservation of energy law this should be}$$

constant, i.e., be independent of r . Consequently $p_0 \sim \frac{1}{\sqrt{r}}$

The harmonic cylindrical wave is then given by

$$p(r, t) = \frac{A}{\sqrt{r}} e^{i(kr - \omega t)}$$

3.1.9 Sound absorption and propagation loss

Absorption of sound, i.e., sound energy is transformed into heat, can be accounted for, e.g. for the plane wave, as follows

$$p(x, t) = e^{i(kx - \omega t) - \alpha x} \quad \text{with } \alpha \text{ the absorption coefficient (unit: m}^{-1}\text{)}$$

It is common practice to express the absorption coefficient in dB per unit distance: The loss in intensity in dB due to absorption is

$$-20^{10} \log e^{-\alpha x} = \alpha x (20^{10} \log e) = 8.686 \alpha x$$

Take $x = 1 \text{ km} = 1000 \text{ m}$, then α (in dB/km) = 8686α (m⁻¹)

For seawater several empirical expressions for α as a function of the frequency f exist. Despite the dependence of α on temperature, salinity, pH and depth, the following expression, denoted by Thorpe's expression, is often sufficiently accurate for practical use:

$$\alpha = \frac{0.11 f^2}{1 + f^2} + \frac{44 f^2}{4100 + f^2} + 0.0003 f^2 \quad \text{with } f \text{ in kHz and } \alpha \text{ in dB/km.}$$

In principle, this expression is only valid for the North Atlantic Ocean. Due to the smaller mean pH in the North Pacific (7.7 instead of 8.0 for the North Atlantic ocean), there the absorption (below 1 kHz) is about two times smaller. Due to the lower salinity of, for example, the Baltic Sea (8 ppt instead of 35 ppt for the Atlantic ocean),

there the absorption (above 1 kHz) is less than half of that of the open oceans. Also this expression is valid only for frequencies below 50 kHz.

Better attenuation formulas are available, having a wider applicability and accounting for effects of salinity and pH as mentioned above. An example of such a formula is the expression by Francois and Garrison (valid for the frequency range 0.2 - 1000 kHz). This expression is a function of temperature (T), salinity (S), depth (D) and pH (pH)

$$\alpha = \alpha_1 + \alpha_2 + \alpha_3$$

$$\alpha_1 = \frac{(A_1 \cdot P_1 \cdot f_1 \cdot f^2)}{(f^2 + f_1^2)} \quad \text{with}$$

$$A_1 = \left(\frac{8.86}{c} \right) 10^{0.78 pH - 5} \quad (c = 1412 + 3.21T + 1.19S + 0.0167D)$$

$$P_1 = 1$$

$$f_1 = 2.8 \sqrt{\frac{S}{35}} \cdot 10^{4 - \frac{1245}{\vartheta}} \quad \text{with } \vartheta = 273 + T$$

$$\alpha_2 = \frac{(A_2 \cdot P_2 \cdot f_2 \cdot f^2)}{(f^2 + f_2^2)} \quad \text{with}$$

$$A_2 = 21.44 \frac{S}{c} (1 + 0.025T)$$

$$P_2 = 1 - 1.37 \cdot 10^{-4} D + 6.2 \cdot 10^{-9} D^2$$

$$f_2 = \frac{8.17 \cdot 10^{\frac{8 - 1990}{\vartheta}}}{1 + 0.0018 (S - 35)}$$

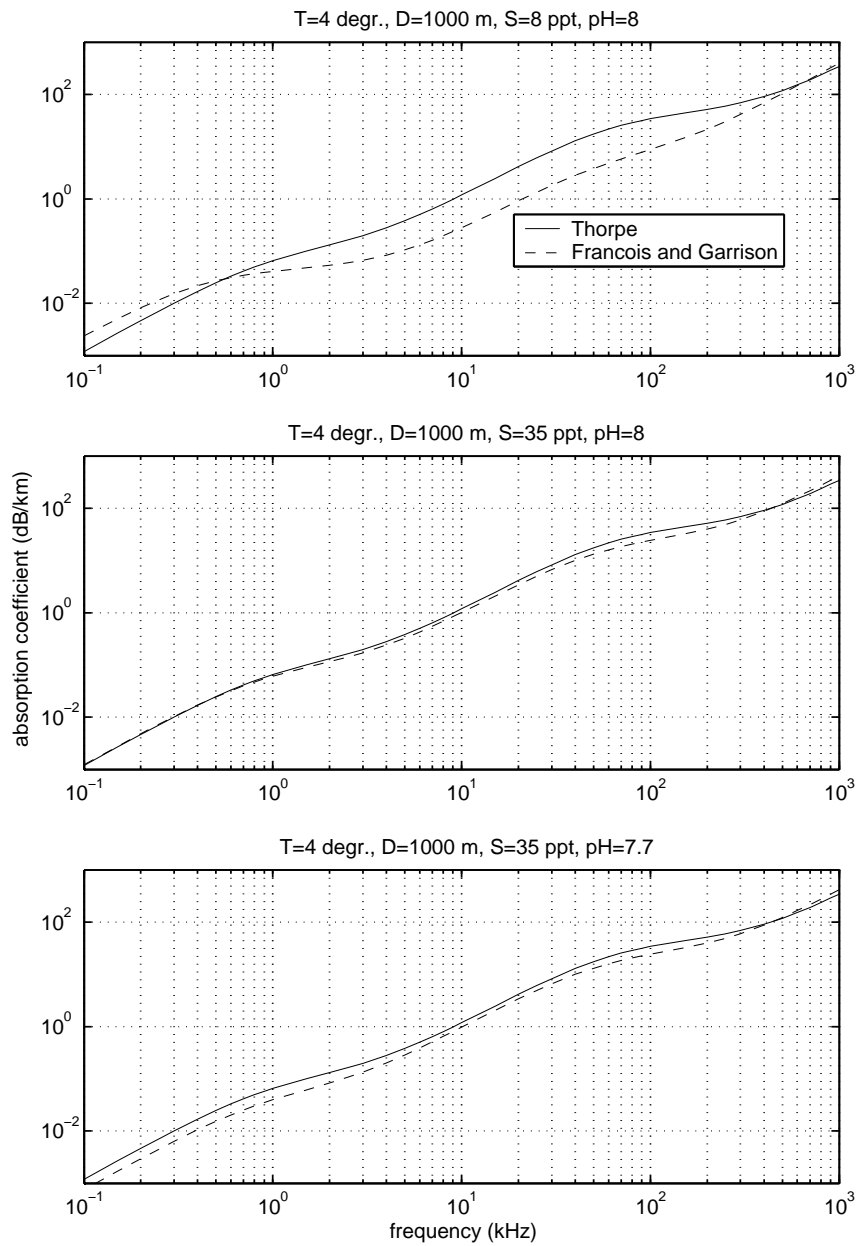
$$\alpha_3 = A_3 \cdot P_3 \cdot f^2 \quad \text{with}$$

$$\text{if } T \geq 20 : A_3 = 4.937 \cdot 10^{-4} - 2.59 \cdot 10^{-5} T + 9.11 \cdot 10^{-7} T^2 - 1.50 \cdot 10^{-8} T^3$$

$$\text{if } T < 20 : A_3 = 3.964 \cdot 10^{-4} - 1.146 \cdot 10^{-5} T + 1.45 \cdot 10^{-7} T^2 - 6.5 \cdot 10^{-10} T^3$$

$$P_3 = 1 - 3.83 \cdot 10^{-5} D + 4.9 \cdot 10^{-10} D^2$$

A comparison between the two here fore mentioned expressions is depicted graphically in the figure below.



Remarks:

In order to get a good feeling for the absorption of sound in seawater, we calculate for a set of frequencies the distance over which (for a plane wave) the sound intensity has decreased by a factor of 10 (10 dB loss):

| Frequency (kHz) | α (dB/km) (Thorpe) | r_{10dB} (km) |
|-----------------|---------------------------|-----------------|
| 0.1 | 0.0012 | 8333 |
| 1 | 0.07 | 143 |
| 10 | 1.2 | 8.3 |

This shows that the absorption is very small; no other form of radiation can compete with sound for long-distance propagation in the sea.

It is illustrative to compare the absorption coefficient of sound in seawater with the absorption coefficient in air (at 20 °C and 50 % humidity). For 2 kHz this is 0.02 m⁻¹, which is over a factor 1000 larger than in seawater at the same frequency (Thorpe: 1.4x10⁻⁵ m⁻¹).

We now come to the very important concept of *propagation loss* that is related to the loss in intensity with the sound moving away from the source.

The propagation loss *PL* is defined as

$$PL = 10^{10} \log \frac{I(1)}{I(r)} = 10^{10} \log \frac{p_{rms}^2(1)}{p_{rms}^2(r)} = 10^{10} \log \frac{p_0^2(1)}{p_0^2(r)}$$

where $I(1)$ and $I(r)$ are the intensities at 1 m and at r m distance from the source. The loss in intensity is due to geometrical spreading loss and absorption.

A simple 'propagation-model' is obtained by considering spherical spreading:

$$p_0 \sim \frac{e^{-\alpha r}}{r} \quad \text{then:}$$

$$PL = 20^{10} \log(re^{\alpha r}) = 20^{10} \log(r) + \alpha r 20^{10} \log(e)$$

or:

$$PL(dB) = 20^{10} \log[r(m)] + \alpha(dB/m)r(m) = 60 + 20^{10} \log[r(km)] + \alpha(dB/km)r(km)$$

Example: calculation of the propagation loss as a function of distance for a hull-mounted sonar ($f = 6$ kHz, $\alpha = 0.5$ dB/km (Thorpe)).

| r (km) | PL (dB) |
|----------|--------------------|
| 0.1 | 60-20+0.05 = 40.05 |
| 1 | 60+ 0+0.5 = 60.5 |
| 10 | 60+20+5 = 85 |
| 100 | 60+40+50 = 150 |

Remarks:

- More complex propagation is dealt with in a later chapter. However, the above simple 'model' is still very useful as a first guess for the loss. In general, the calculated loss is somewhat pessimistic, except for very short distances;
- The propagation loss for *cylindrical* spreading becomes

$$PL(dB) = 30 + 10^{10} \log[r(km)] + \alpha(dB/km)r(km)$$

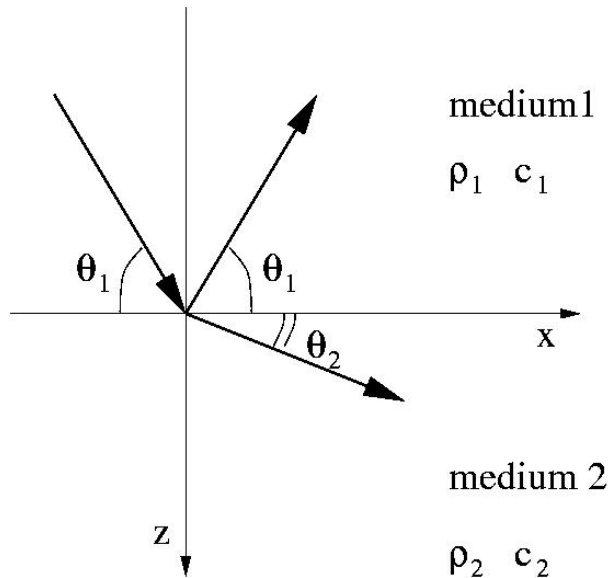
3.1.10 Reflection, refraction and transmission

We will consider the situation of a plane harmonic wave propagating through a medium and impinging on the interface of a second medium. Part of the energy of the wave will be reflected back into medium 1, part will be transmitted into medium 2. We will derive the (relative) intensities of the reflected and transmitted wave, and find that these depend on the impedance of the two media and the angle of the incoming wave.

At the same time we will derive the important Snell's law of refraction.

An important application of the situation considered in underwater acoustics is the interaction of the sound with the sea bottom.

The figure below shows the situation.



The expression for the 3D harmonic plane wave:

$$p(x, y, z, t) = p(\vec{r}, t) = e^{i(\vec{k} \cdot \vec{r} - \omega t)} \quad \text{with} \quad \vec{k} \cdot \vec{r} = k_x x + k_y y + k_z z$$

so the inner product of the two vectors. The wavenumber k is the absolute value of the wavevector \vec{k} : $\|\vec{k}\| = k$.

The incident (i), reflected (r) and transmitted (t) wave can be written as

$$p_i = e^{ik_1(x \cos \theta_1 + z \sin \theta_1)} \quad k_1 = \frac{\omega}{c_1} = \|\vec{k}_1\|$$

$$p_r = \text{Re} e^{ik_1(x \cos \theta_1 - z \sin \theta_1)}$$

$$p_t = T e^{ik_2(x \cos \theta_2 + z \sin \theta_2)} \quad k_2 = \frac{\omega}{c_2} = \|\vec{k}_2\|$$

Here θ_1 is the grazing angle of incidence (and angle of reflection) and θ_2 is the grazing angle of transmission (angle of refraction). The t -dependence of all expressions for the plane harmonic waves is omitted. R and T are the *amplitude* reflection coefficient and the *amplitude* transmission coefficient, respectively.

The following two boundary conditions are valid for $z = 0$:

(I) *continuity of pressure* $p_i + p_r = p_t$

(II) *continuity of the normal component of the particle velocity (i.e. in the z -direction)*:

$$\frac{1}{i\omega\rho_1} \frac{\partial(p_i + p_r)}{\partial z} = \frac{1}{i\omega\rho_2} \frac{\partial p_t}{\partial z}$$

Boundary condition (I) gives (use $z = 0$):

$$(1 + R) = T e^{i(k_2 \cos \theta_2 - k_1 \cos \theta_1)x}$$

This expression only holds if the right side is independent on x , so if

$$k_2 \cos \theta_2 - k_1 \cos \theta_1 = 0$$

This is the famous Snell's law of refraction that gives the relation between angles of incidence and transmission. Snell's law of refraction is often written as

$$\frac{\cos \theta_2}{c_2} = \frac{\cos \theta_1}{c_1}$$

Also we find: $(1 + R) = T$

Boundary condition (II) gives (use $z = 0$ and Snell's law):

$$1 - R = T \frac{\rho_1 c_1 \sin \theta_2}{\rho_2 c_2 \sin \theta_1}$$

We now have derived two equations for R and T . The solution is:

$$R = \frac{Z_2 - Z_1}{Z_2 + Z_1} \quad \text{and} \quad T = \frac{2Z_2}{Z_2 + Z_1} \quad \text{where}$$

$$Z_1 = \frac{\rho_1 c_1}{\sin \theta_1} \quad \text{and} \quad Z_2 = \frac{\rho_2 c_2}{\sin \theta_2} \quad (\text{'generalized' impedances}).$$

The incoming, reflected and transmitted *intensities* are, respectively:

$$I_i = \frac{1^2}{2\rho_1 c_1}, I_r = \frac{R^2}{2\rho_1 c_1}, I_t = \frac{T^2}{2\rho_2 c_2}$$

Now, for simplicity, let's consider $\theta_1=90^\circ$ (the wave comes in perpendicular). Then according to Snell's law $\theta_2=90^\circ$.

The 'reflectivity' is given by

$$\frac{I_r}{I_i} = R^2 = \frac{(\rho_2 c_2 - \rho_1 c_1)^2}{(\rho_2 c_2 + \rho_1 c_1)^2}$$

and

$$\frac{I_t}{I_i} = T^2 \frac{\rho_1 c_1}{\rho_2 c_2} = \frac{4\rho_2 c_2 \rho_1 c_1}{(\rho_2 c_2 + \rho_1 c_1)^2}$$

so that $I_i = I_r + I_t$ (note that according to the law expressing conservation of energy this should indeed be the case).

The critical angle (total reflection)

When $c_2 > c_1$ then an angle θ_c , exists, the *critical angle*, for which $|R| = 1$ for $0 < \theta < \theta_c$ (total reflection) and $R < 1$ for $\theta > \theta_c$.

θ_c is given by (use Snell's law)

$$\theta_c = \arccos\left(\frac{c_1}{c_2}\right)$$

If $0 < \theta < \theta_c$ then (Snell's law) $\cos \theta_2 > 1$, i.e., θ_2 is purely imaginary. Using the expressions $\sin i\theta = i \sinh \theta$ and $\cos i\theta = \cosh \theta$ it can be shown that $|R| = 1$. Note, however, that R is complex, so a shift of phase occurs. For $\theta > \theta_c$, $R < 1$ and real.

The angle of intromission

Situations exist for which $R = 0$. The corresponding incident angle is denoted as the angle of intromission θ_0 .

$R = 0$ if $\frac{\rho_2 c_2}{\sin \theta_2} = \frac{\rho_1 c_1}{\sin \theta_1}$ and together with Snell's law of refraction this gives

$$\theta_0 = \arctan \sqrt{\frac{1 - \left(\frac{c_2}{c_1}\right)^2}{\left(\frac{\rho_2 c_2}{\rho_1 c_1}\right)^2 - 1}}$$

θ_0 exists, i.e., is real, in the following two cases:

- $\rho_2 c_2 > \rho_1 c_1$ en $c_2 < c_1$. This occurs in muddy ocean bottoms;
- $\rho_2 c_2 < \rho_1 c_1$ en $c_2 > c_1$. This never occurs, i.e., is not a physical situation.

Typical numerical values for ocean bottoms

In table below indicative values for geo-acoustic parameters corresponding to a few different bottom types are given.

| Bottom type | ρ (g/cm ³) | c (m/s) |
|----------------|-----------------------------|-----------|
| clay | 1.2 | 1470 |
| very fine sand | 1.9 | 1680 |
| coarse sand | 2.0 | 1800 |
| basalt | 2.5 | 4800 |

Remark:

For ocean bottom materials a good approximation is that the absorption coefficient linearly increases with frequency. In this case it is useful to express the absorption coefficient in dB/ λ . For most bottom types the absorption coefficient has a value in between 0.1 and 1 dB/ λ .

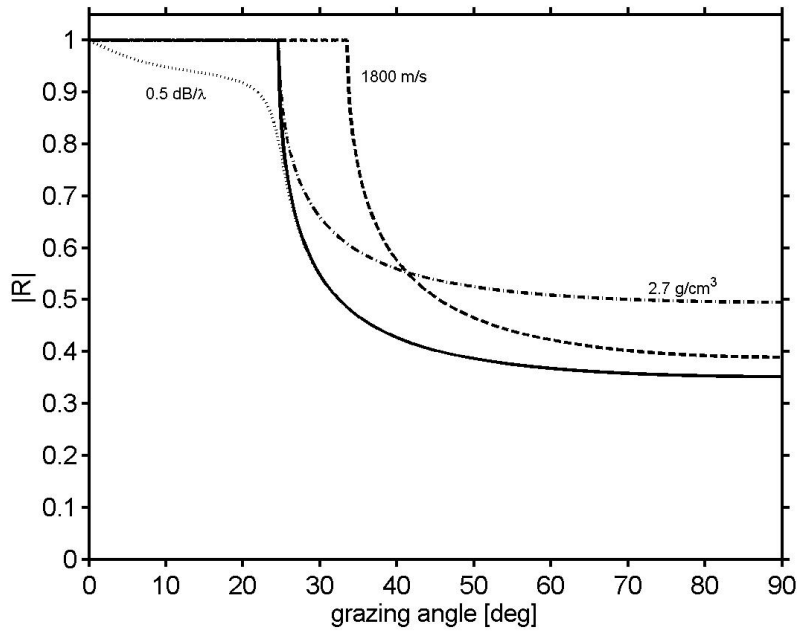
The following holds: $\alpha(\text{dB} / \lambda) = 8.686\lambda(m) \alpha(m^{-1})$

Examples

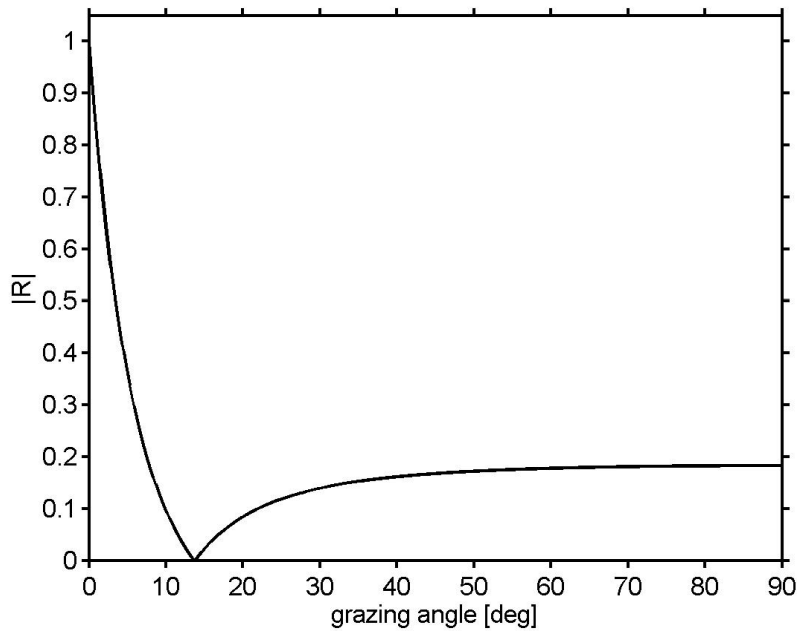
The figure below shows the absolute value of the amplitude reflection coefficient as a function of the incident angle for a number of situations.

Assuming a lossless (i.e. without absorption) sandy bottom (thick solid line) with $\rho_2 = 1.9 \text{ g/cm}^3$, $c_2 = 1650 \text{ m/s}$ ($\rho_1 = 1.0 \text{ g/cm}^3$, $c_1 = 1500 \text{ m/s}$)

It shows the effect of a change in c_2 , ρ_2 and absorption in the bottom (i.e. medium 2).



The figure below shows the occurrence of the 'angle of intromission' ($\rho_2 = 1.5 \text{ g/cm}^3, c_2 = 1450 \text{ m/s}$)



3.1.11 Diffraction and scattering

Here we consider shortly a set of concepts that relate to sound as a wave phenomenon in general.

Diffraction or bending of sound happens if the sound waves impact on an obstacle with a size that is similar (same order) to the sound wave length. The sound wave

passes around the obstacle and there is a bending (change of direction) due to the obstacle.

Reflection of sound at an object results when the object is large compared to the wavelength of the sound (at least a factor of ten).

Scattering of sound happens if the sound waves impact on an object that has a size, which is small relative to the wavelength. In principle, scattering occurs in all directions.

Finally: *Refraction* of sound occurs when there is a sudden change in sound speed in the medium through which the sound wave propagates (Snell's law of refraction). Refraction of sound also occurs when the sound speed varies smoothly.

3.1.12 The Doppler effect

Let's consider a sound source and an observer with speeds v_b and v_w (both in the direction of the line connecting the source and the receiver), respectively. $v_b > 0$ and $v_w > 0$ implies that the movement is towards each other (this is a choice).

The frequency of the stationary source is $f_0 = \frac{c}{\lambda_0}$.

(A) *The effect of a moving observer*

Let T be the time between observing two subsequent pressure maxima. The expression for T is

$$T = \frac{\lambda_0}{c + v_w}$$

since $c + v_w$ is the propagation speed of the pressure maxima. The observed frequency f now becomes

$$f = \frac{1}{T} = f_0 \left(1 + \frac{v_w}{c} \right)$$

(B) *The effect of a moving source*

A pressure maximum transmitted by the source will be at a distance cT_0 after a time T_0 . At that moment the source transmits another pressure maximum. However, in the mean time the source has moved a distance $v_b T_0$.

The (observed) wavelength λ , i.e. the distance between the two pressure maxima, is then

$$\lambda = cT_0 - v_b T_0$$

The observed frequency f now becomes

$$f = f_0 \frac{1}{1 - \frac{v_b}{c}}$$

(C) Both the source and the observer move

If both the source and the observer move, the observed frequency becomes

$$f = f_0 \frac{1 + \frac{v_w}{c}}{1 - \frac{v_b}{c}}$$

Remark:

- If the source and the observer do not move along the line connecting the source and observer, the velocities of the source and the observer (vectors) have to be projected on this connecting line.
- In practice often $v_w, v_b \ll c$. For this case a good approximation is (using Taylor)

$$f \approx f_0 \left(1 + \frac{v_w}{c}\right) \left(1 + \frac{v_b}{c}\right) \approx f_0 \left(1 + \frac{v_w + v_b}{c}\right)$$

The frequency shift is then (*passive* sonar)

$$\Delta f = f_0 \frac{v_w + v_b}{c}$$

- For *active* sonar the frequency shift is

$$\Delta f = 2f_0 \frac{v_d + v_e}{c} \quad \text{with } v_d \text{ and } v_e \text{ the target velocity and the own ship speed, respectively.}$$

Example: hull-mounted sonar $f_0 = 6$ kHz, $v_d = v_e = 5$ knots, then $\Delta f = 40$ Hz. This can be measured very well and is therefore used for measuring the target speed. (1 knot = 0.5144 m/s).

