

# **Traffic Flow Theory and Simulation**

vk4821

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# Introduction

Traffic flow theory pertains to *The Knowledge of Fundamental Traffic Flow Characteristics and Associated Analytical Techniques*. Understanding the basic traffic flow characteristics as well as these techniques is an essential requirement for planning, design, and operation of any transportation system. For example, planners assess traffic and environmental impacts of proposed modifications - either in the infrastructure or in terms of traffic management measures - of the transportation system, which can only be accomplished through demand-supply analysis, which in turn requires an understanding of the flow characteristics as well as their interrelations with the candidate modifications. A designer will for instance need to determine the number of lanes of a particular roadway segment in the network, and will hence need to carefully evaluate the trade-off between traffic flow levels and levels of service. Operators may need to identify locations and causes of problems in the existing infrastructure design and generate operational improvement plans as well as predicting the effect of the latter.

Some important examples of traffic flow characteristics:

- Trajectories of cars, pedestrians, bicycles, etc.
- Time and distance headways, vehicle speeds
- Traffic volumes, densities, time-mean and space-mean speeds

Amongst the analytical techniques are:

- Supply - demand modeling<sup>1</sup>
- Capacity and level-of-service analysis
- Shock wave analysis
- Macroscopic traffic flow modeling
- Queuing analysis
- Microscopic simulation models

## 0.1 Microscopic and macroscopic characteristics and tools

A key distinction that is made in the study of traffic systems is the distinction between microscopic and macroscopic characteristics and (mathematical and simulation) models. Microscopic characteristics pertain to the individual driver-vehicle unit (DVU). Examples of microscopic characteristics are time headways, individual speeds, and distance headways. Similarly, microscopic models describe the behavior of individual vehicles, for instance in relation to the infrastructure and other vehicles in the flow. That is, they predict the behavior in terms of

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<sup>1</sup>As will be argued in the ensuing, nearly all problems relating to the traffic system can be described within the context of a demand-supply framework. Thus, the demand-supply framework is very generic and will serve as the backbone for this reader.

microscopic characteristics. Typical analytical techniques that are based on microscopic traffic data and theory are headway distribution modelling, and microscopic simulation models. On the contrary, macroscopic characteristics pertain to the properties of the traffic flow as a whole (for instance at a cross-section, or at a time instant). Examples of macroscopic characteristics and flow, time mean speed, density, and space-mean speed. Macroscopic models describe traffic flow in terms of macroscopic characteristics of the flow. Macroscopic analytical techniques involve demand – supply analysis, shock wave theory, capacity analysis, and macroscopic traffic flow modelling.

At this point, let us clearly state that a distinction is made between mathematical models and simulation models. A mathematical (or theoretical) model consists of the mathematical equations describing the systems behavior, such as the partial differential equations describing the conservation of vehicles. A simulation model refers to the computer implementation of a mathematical model, as well as all additional simulation model features.

## 0.2 Aims and scope of the course

Gaining knowledge of fundamental traffic flow characteristics. Developing the ability to analyze traffic operations, focusing of car-based traffic. Gaining knowledge of techniques for traffic analysis and the ability to select and apply the appropriate technique for the problem at hand. Considered techniques include data analysis and statistics, traffic flow models, and macroscopic and microscopic traffic simulation. The course consist of recitations, exercises and two assignments. The first assignment shows how the microscopic simulation model FOSIM is used in a simulation study. The second assignment will either involve working with empirical data.

## 0.3 Contents of the reader

The reader is roughly divided into four parts. The first part presents the demand-supply framework. This framework is generic for analyzing traffic systems, either using a microscopic or a macroscopic representation of traffic flow. It is shown how basically any problem in traffic analysis can be described within this framework. The second part of the reader describes microscopic and macroscopic characteristics of traffic flow. The third part discusses several analytical techniques that use these empirical findings in order to answer all kind of research questions. Also for the analytical tools, distinction is made between microscopic and macroscopic tools and models: macroscopic models represent traffic flow in an aggregate manner, i.e. as a continuum. Although the representation of traffic may be macroscopic, the rules predicting the dynamics of the traffic flow may be based on driver behavior. Microscopic models represent all vehicles in the traffic flow individually. This does however not necessarily mean that the behavior of the vehicles is based on microscopic rules. Several models describe the behavior of individual vehicles as a function of macroscopic characteristics, such as the traffic density. The last part of the reader discusses simulation models and how to properly conduct a simulation study.