Regeltechniek

Lecture 4 – Basics of Feedback Control

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Lecture Outline

Previous lecture: Stability and transient response.

Today:

- Steady-state response.
- Feedforward vs. feedback control.
- Control design goals.
- System type.
- PID control.

Steady-State Response

Final value theorem

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) \quad \text{iff all poles of } sY(s) \text{ in the LHP}$$

If u is a unit step, $U(s) = \frac{1}{s}$,

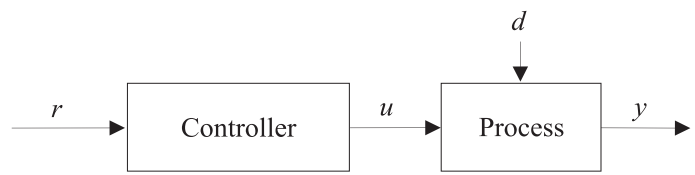
$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} s \cdot G(s) \cdot \frac{1}{s} = \lim_{s\to 0} G(s)$$

Consequence: DC (stationary) gain of $G(s)$

$$DC = \lim_{s\to 0} G(s)$$

Important: $G(s)$ must be stable (check stability first)!

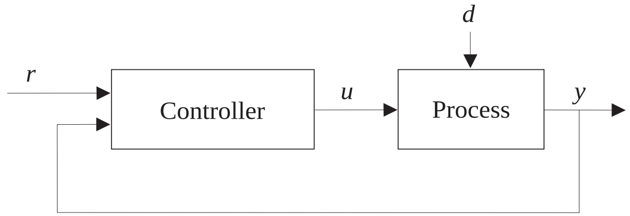
Feedforward Control



controller = inverse of process model

+ guaranteed stable for stable processes
– cannot stabilize unstable processes
– sensitive to disturbances
– sensitive to model uncertainty

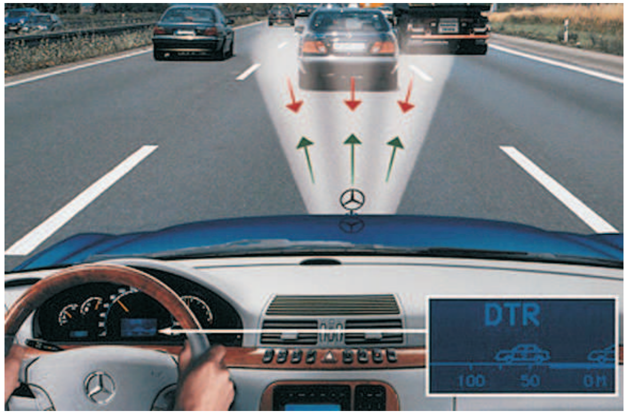
Feedback Control



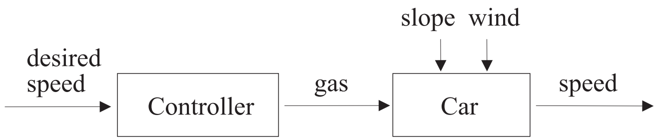
controller $\neq$ inverse of process model

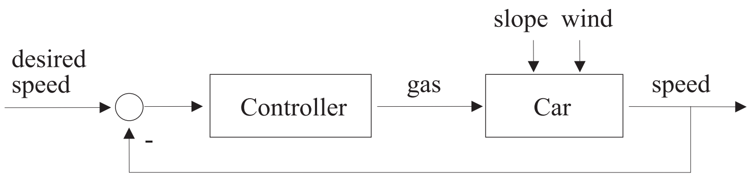
+ can stabilize unstable processes
+ less sensitive to disturbance
+ less sensitive to model uncertainty
– can potentially destabilize a stable process

Feedback vs. Feedforward: Cruise Control

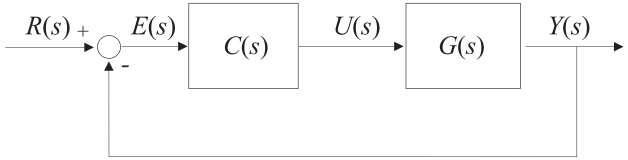


Feedback vs. Feedforward: Cruise Control





Closed-Loop Transfer Function



$$Y = GC\,(R - Y)$$

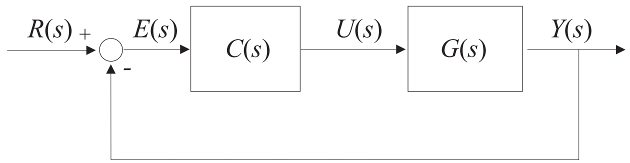
$$(1 + GC)\,Y = GCR$$

$$G_{cl} = \frac{Y}{R} = \frac{GC}{1 + GC}$$

Controller Design: Goals and Choices

- Different control goals, for instance:
  - stabilize an unstable process
  - track a specific type of reference signal
  - reduce influence of disturbances
  - improve performance (e.g., speed of response)
- Structure of the controller (number of poles and zeros)
- Parameters (location of poles and zeros, gain)

Reference Tracking: System Type



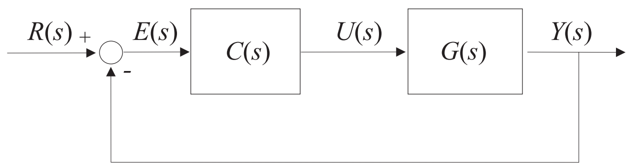
How well will the closed-loop system track a given reference signal?

Consider reference input: $R(s) = \frac{1}{s^k}$
$k = 1$ . . . step, $k = 2$ . . . ramp, etc.

Common Reference Signals

$R(s) = \frac{1}{s}$	$r(t) = 1(t)$	step (position)
$R(s) = \frac{1}{s^2}$	$r(t) = t$	ramp (velocity)
$R(s) = \frac{1}{s^3}$	$r(t) = \frac{t^2}{2}$	parabola (acceleration)

Steady-State Error



$$E(s) = \frac{1}{1+L(s)}R(s) \quad \text{with} \quad L(s) = G(s)C(s) \quad \text{(loop transfer)}$$

Steady-state error (final value theorem):

$$e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{s}{1+L(s)}R(s) = \lim_{s\to 0} \frac{s}{1+L(s)} \cdot \frac{1}{s^k}$$

Steady-State Error: Example 1

Consider the following loop transfer: $L(s) = \dfrac{K}{s+1}$

Steady-state error:

$$e_{ss} = \lim_{s\to 0} \frac{s}{1+L(s)} \cdot \frac{1}{s^k} = \lim_{s\to 0} \frac{s(s+1)}{s+1+K} \cdot \frac{1}{s^k}$$

Step $(R(s) = \frac{1}{s})$:

$$e_{ss} = \lim_{s\to 0} \frac{s(s+1)}{s+1+K} \cdot \frac{1}{s} = \frac{1}{1+K} = \text{finite constant}$$

Ramp $(R(s) = \frac{1}{s^2})$:

$$e_{ss} = \lim_{s\to 0} \frac{s(s+1)}{s+1+K} \cdot \frac{1}{s^2} = \infty$$

Steady-State Error: Example 2

Consider the following loop transfer: $L(s) = \dfrac{K}{s(s+1)}$

Steady state error:

$$e_{ss} = \lim_{s\to 0} \frac{s}{1+L(s)} \cdot \frac{1}{s^k} = \lim_{s\to 0} \frac{s^2(s+1)}{s(s+1)+K} \cdot \frac{1}{s^k}$$

Step $(R(s) = \frac{1}{s})$:

$$e_{ss} = \lim_{s\to 0} \frac{s^2(s+1)}{s(s+1)+K} \cdot \frac{1}{s} = 0$$

Ramp $(R(s) = \frac{1}{s^2})$:

$$e_{ss} = \lim_{s\to 0} \frac{s^2(s+1)}{s(s+1)+K} \cdot \frac{1}{s^2} = \frac{1}{K} = \text{finite constant}$$

Steady-State Error in General

Loop transfer: $L(s) = \dfrac{L_0(s)}{s^m}$

$$e_{ss} = \lim_{s\to 0} \frac{s}{1+\frac{L_0(s)}{s^m}} \cdot \frac{1}{s^k} = \lim_{s\to 0} \frac{s^m s}{s^m + L_0(s)} \cdot \frac{1}{s^k}$$

Zero steady-state error:

$$e_{ss} = 0 \quad \text{iff} \quad m \geq k$$

System type m = number of pure integrators in loop transfer

System Type → Controller Structure

If zero steady-state error required (for a given reference type)

and

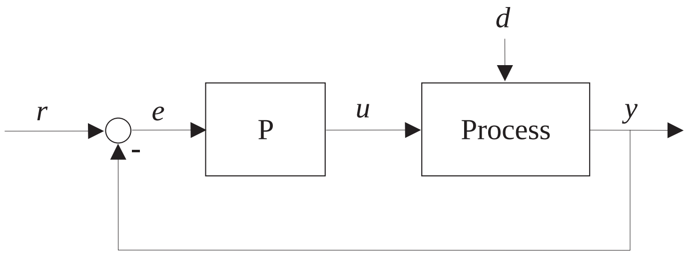
the loop transfer is not of sufficiently high type

then

add integrator(s) in the controller.

(see also PID control)

Proportional Control



Controller:

- static gain K_p: $u(t) = K_p e(t) = K_p \left(r(t) - y(t) \right)$

Closed-Loop Transfer With P Controller

Process (example): $G(s) = \dfrac{K}{s(s+a)}$

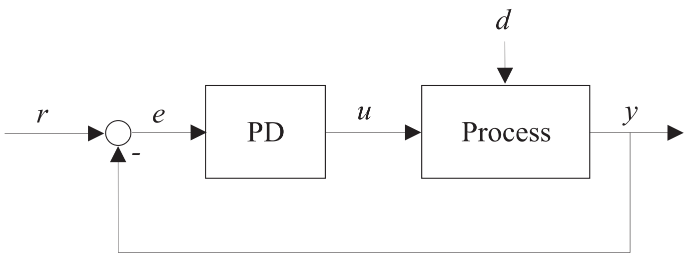
Proportional controller: $C(s) = K_p$

Closed-loop poles – solutions of: $1 + \dfrac{KK_p}{s(s+a)} = 0$

$$s^2 + as + KK_p = 0$$

K_p has some influence on the closed-loop poles
(does modify the stiffness, but not the damping)

Proportional-Derivative (PD) Control



Controller:

- dynamic: $u(t) = K_p e(t) + K_d \frac{de(t)}{dt}$
- K_p and K_d are the proportional, and derivative gains, respectively

Closed-Loop Transfer with PD Controller

Process (example): $G(s) = \dfrac{K}{s(s+a)}$

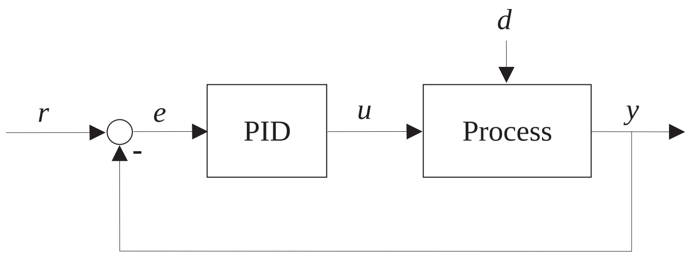
Proportional controller: $C(s) = K_p + K_d s$

Closed-loop poles – solutions of: $1 + \dfrac{K(K_p + K_d s)}{s(s+a)} = 0$

$$s^2 + (a + KK_d)s + KK_p = 0$$

we can choose K_p and K_d to completely determine the closed-loop poles (for this second-order process)

PID Control



Controller:

- dynamic: $u(t) = K_p e(t) + K_i \int_0^t e(\tau) d\tau + K_d \frac{de(t)}{dt}$
- K_p, K_i and K_d are the proportional, integral and derivative gains, respectively

When Should We Use Integral Action?

If zero steady-state error required (for a given reference type)

and the loop transfer is not of sufficiently high type

Example: $L(s) = G(s)K_p = \dfrac{KK_p}{\tau s + 1}$

Reference = step: $R(s) = \dfrac{1}{s}$

Required: zero steady-state error $e_{ss} = 0$

Conclusion: as system type is 0 (no integrator), use PI!

Integral Action in Differential Equation

Process (example): $y + b\dot{y} = u$

Proportional controller: $u = K_p(r - y)$

Closed-loop differential equation: $y + b\dot{y} = K_p r - K_p y$

In steady state ($\dot{y} = 0$): $y = \dfrac{K_p}{1 + K_p} r \quad \Rightarrow \quad y \approx r$ (for large K_p)

non-zero steady-state error! (system is of type 0)

With a PI Controller

Process (example): $y + b\dot{y} = u$

PI controller: $u = K_p(r - y) + K_i \displaystyle\int (r - y)$

for r =const: $\dot{u} = -K_p \dot{y} + K_i(r - y)$

Closed-loop differential equation: $\dot{y} + b\ddot{y} = -K_p \dot{y} + K_i r - K_i y$

In steady state ($\ddot{y} = \dot{y} = 0$): $0 = K_i r - K_i y \quad \Rightarrow \quad y = r$

no steady-state error!

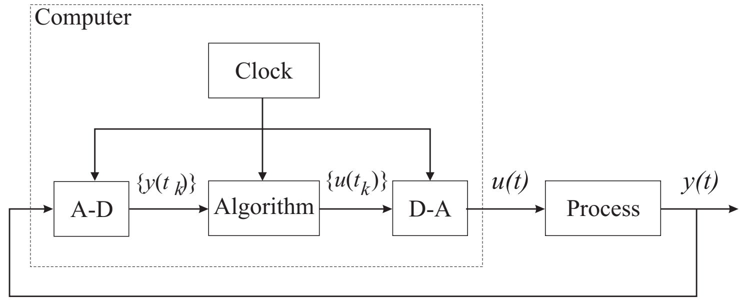
Influence of the PID Parameters

- K_p . . . stiffness (speed of response), but also oscillations
- K_d . . . damping (less oscillations), but sensitive to noise
- K_i . . . remove steady-state error, but more overshoot
  (re-tune K_p , K_i)

Tuning:

- Experimental tuning (often used in practice, sometimes computer-assisted)
- Model-based analysis and design (rest of our course)

Implementation: Computer Control



Controller implemented on a digital computer,
runs in discrete time and on discrete-valued data.