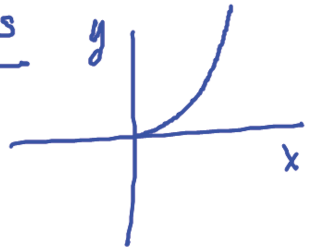
Locus



$x = t$

$y = t^2$

$t \in (0, \infty)$

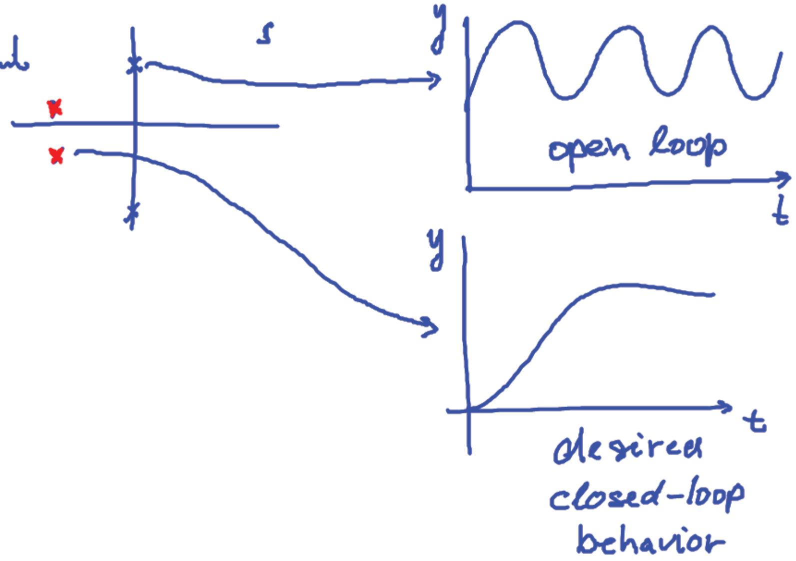
root locus

$$\frac{G(s)\cdot C(s)}{1 + G(s)\, C(s)}$$

$\uparrow$

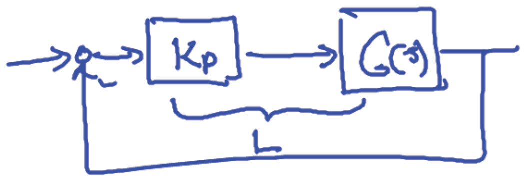
K

pole placement



$$J = \int_0^\infty \underbrace{x^T Q x}_{\uparrow\ \mathrm{PD}} + \underbrace{u^T R u}$$

linear quadratic control



$$1 + \underbrace{K_p}_{K}\ \underbrace{G(s)}_{L} = 0$$

characteristic equation in root-locus form !

PD: $C(s) = K_p + K_D s = K_p\left(1+\underbrace{\frac{K_D}{K_p}}_{T_D} s\right)$

$= K_p(1+T_D s)$

variable parameter

$1 + \underbrace{K_p}_{K}\underbrace{(1+T_D s)\,G(s)}_{\bar{L}} = 0$

PD: $1 + K_p(1+T_D s)\,G(s) = 0$

$\underbrace{1+K_p G(s)} + K_p T_D s\, G(s) = 0 \;/ : 1+K_p G(s)$

$1 + K\,\bar{L}(s) = 0$

$1 + \underbrace{T_D}_{K}\,\underbrace{\frac{K_p s\, G(s)}{1+K_p G(s)}}_{\bar{L}} = 0$ → Matlab

$G(s) = \frac{1}{s(s+1)}$, $C(s) = K$ $\quad 1+\frac{K}{s(s+1)} = 0$

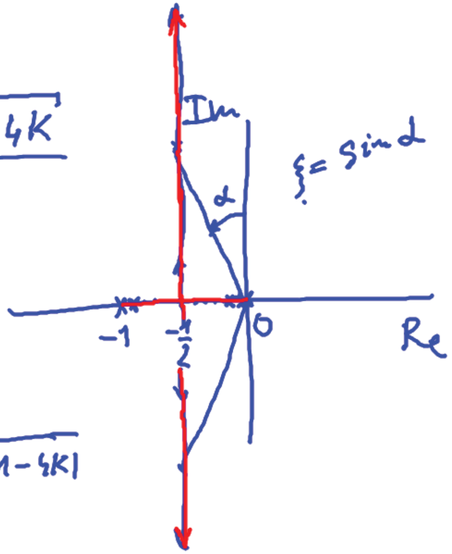
$s^2+s+K = 0$

$s_{1,2} = \frac{-1\pm\sqrt{1-4K}}{2}$

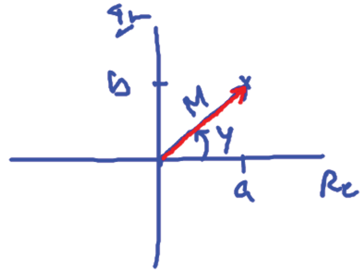
$K=0 \begin{cases} \frac{-1-1}{2} = -1 \\ \frac{-1+1}{2} = 0 \end{cases}$

$K = \frac{1}{4} \quad = -\frac{1}{2}$

$K > \frac{1}{4} \quad \ldots -\frac{1}{2} \pm j\sqrt{|1-4K|}$



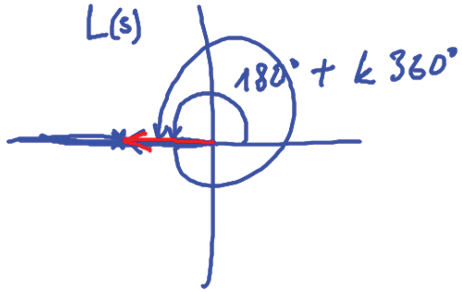
$\xi = \sin\alpha$



$a + jb$

$\boxed{M e^{j\varphi}}$!

$L(s)$



$$1 + K\,L(s) = 0 \qquad L(s) = \frac{B(s)}{A(s)}$$

$$1 + K\frac{B(s)}{A(s)} = 0 \;\rightarrow\; \frac{B(s)}{A(s)} = -\frac{1}{K}$$

$$A(s) + K\,B(s) = 0$$

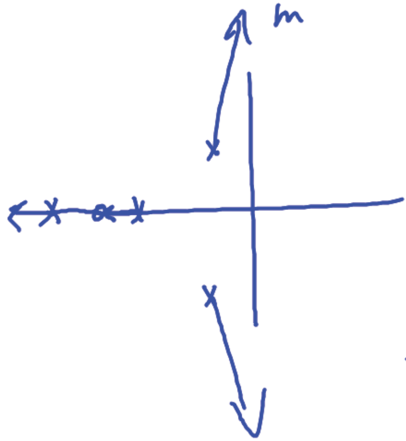
$$K = 0 \qquad A(s) = 0$$

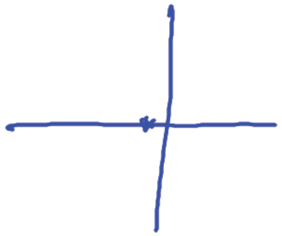
1) $B(s) = 0$

2) $A(s) \to \infty$, $s \to \infty$

$$\frac{\overset{m}{s+1}}{\underbrace{(s+3)(s+10)}_{n}} \qquad n-m>0$$

$$\frac{\cancel{(s-z_1)}\,\cancel{(s-z_2)}}{\underbrace{\cancel{(s-p_1)}\,\cancel{(s-p_2)}}_{m}\,\underbrace{(s-p_3)\cdots(s-p_n)}_{n-m}} \qquad 1 \qquad s\to\infty$$





$$\frac{1}{(s-\alpha)^{n-m}} = -\frac{1}{K}$$

$$(s-\alpha)^{n-m} = -K$$

$$a_1 = M_1 e^{j\varphi_1} \qquad a_1 a_2 = M_1 M_2 e^{j\varphi_1}\cdot e^{j\varphi_2}$$

$$a_2 = M_2 e^{j\varphi_2}$$

$$\underbrace{M_1 M_2}_{M}\, e^{j\underbrace{(\varphi_1+\varphi_2)}_{\varphi}}$$

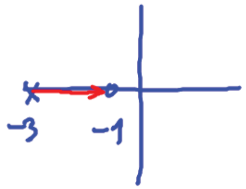
Fase

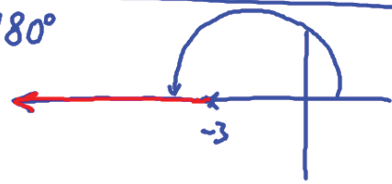
$$\angle\left((s-\alpha)^{n-m}\right) = 180 + k\,360^\circ$$

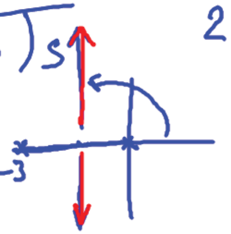
$$(n-m)\cdot\underbrace{\angle(s-\alpha)}_{\varphi_k} = 180^\circ + (k-1)\,360^\circ$$

$$\varphi_k = \frac{180^\circ + (k-1)\,360^\circ}{n-m}$$

	$n-m$	φ_n
$\frac{s+1}{s+3}$	0	—
$\frac{1}{s+3}$	1	180°
$\frac{1}{(s+3)s}$	2	$\varphi_1 = \frac{180^\circ + 0\cdot 360^\circ}{2} = 90^\circ$, $\varphi_2 = \frac{180^\circ + 1\cdot 360^\circ}{2} = 270^\circ$







$$\frac{(s+1)}{(s+2)(s+3)\,..}$$

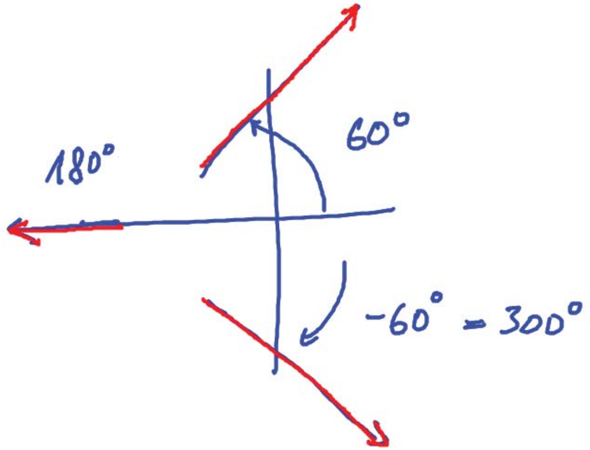
$n-m$

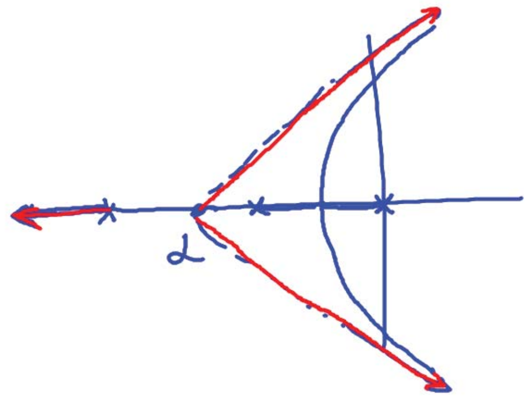
3

$$\frac{180^\circ}{3} = 60^\circ$$

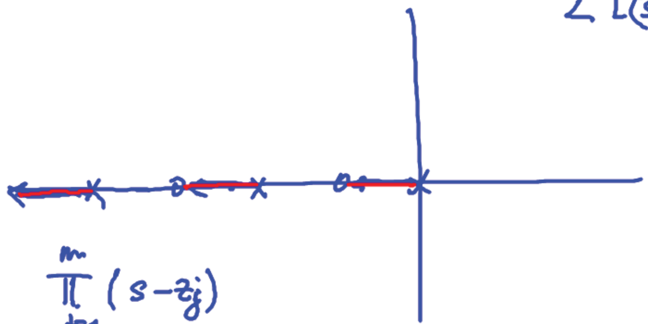
$$\frac{180^\circ + 360^\circ}{3} = 180^\circ$$

$$\frac{180^\circ + 720^\circ}{3} = 300^\circ$$





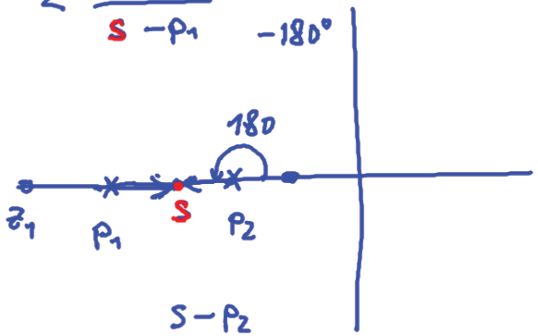
$\angle L(s) = 180^\circ + k360^\circ$



$$\frac{\prod_{j=1}^{m}(s-z_j)}{\prod_{i=1}^{n}(s-p_i)}$$

$$\angle\left(\frac{\prod M_j \cdot e^{j\sum \psi_i}}{\prod N_i \, e^{j\sum \varphi_i}}\right) = \underbrace{\sum \psi_i}_{\text{zeros}} - \underbrace{\sum \varphi_i}_{\text{poles}}$$

$$\angle \frac{s - z_1}{s - p_1} \quad \begin{matrix} +180^\circ \\ -180^\circ \end{matrix}$$

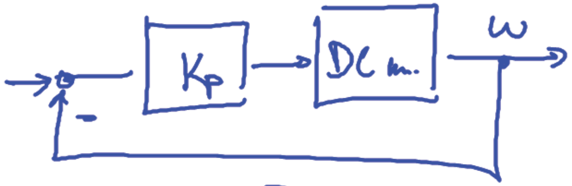


$180^\circ + k\,360^\circ$

$\Downarrow$

uneven #

poles & zeros



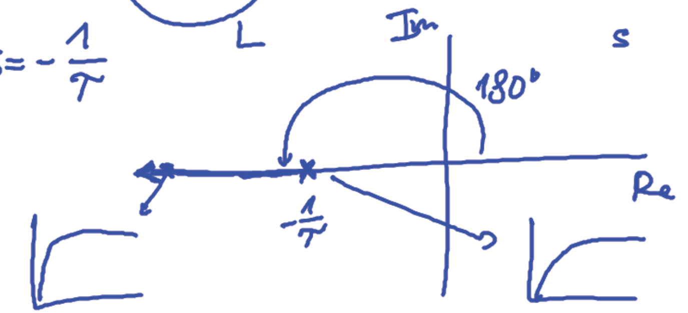
$$\overset{\approx 0}{L}\dot{i} + Ri = V - k\omega$$

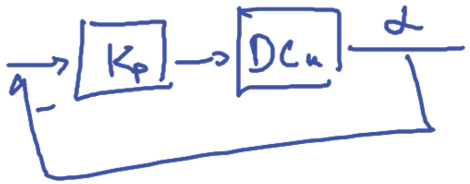
$$J\dot{\omega} + b\omega = ki$$

...

$$1 + K_p \frac{K}{\tau s+1} = 0 \qquad L = \frac{K}{\tau s+1}$$

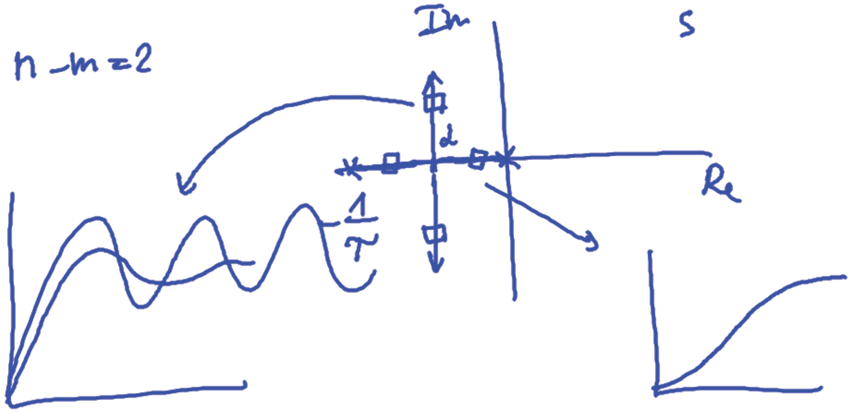
$$s = -\frac{1}{\tau}$$





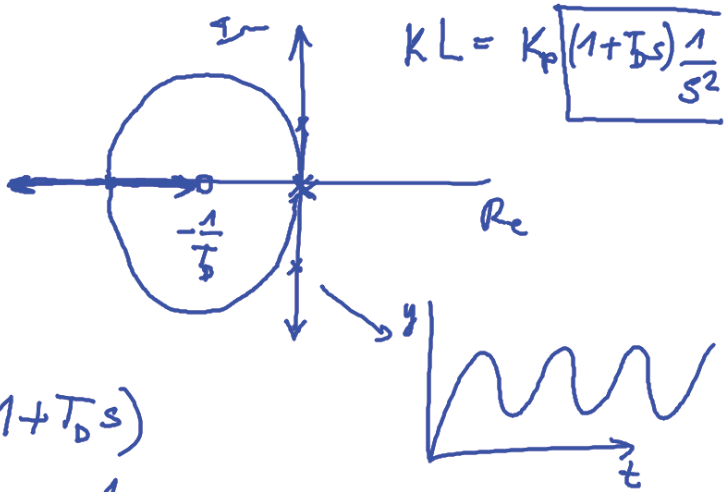
$$L = \frac{K}{s(\tau s+1)}$$

$n - m = 2$



$$\boxed{\frac{1}{s^2}}$$

$$KL = K_p \boxed{(1+T_D s)\frac{1}{s^2}}$$



$n - m = 2$

PD:

$$K_p(1 + T_D s)$$

$$s = -\frac{1}{T_D}$$