# Regeltechniek

**Lecture 7 – Frequency Response, Bode Plots**

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# Lecture Outline

Previous lecture: Root locus, frequency response derivation.

Today:

- Handout for the remaining computer sessions.
- Bode plots.
- Non-minimum-phase systems.
- System type in Bode plots.



# Frequency Response

Periodic input:

$$u(t) = M \sin \omega t$$

Steady-state output:

$$y(t) = |G(j\omega)| \cdot M \sin\Big(\omega t + \angle G(j\omega)\Big)$$

$|G(j\omega)|$ ... magnitude (gain)

$\angle G(j\omega)$ ... phase



# Magnitude and Phase

Magnitude:

$$|G(j\omega)| = \sqrt{\{\mathrm{Re}[G(j\omega)]\}^2 + \{\mathrm{Im}[G(j\omega)]\}^2}$$

Phase:

$$\angle G(j\omega) = \tan^{-1}\left(\frac{\mathrm{Im}[G(j\omega)]}{\mathrm{Re}[G(j\omega)]}\right)$$

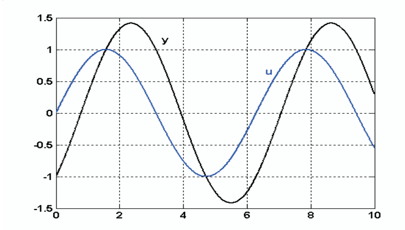
Both the magnitude and phase are generally functions of $\omega$!

Fully describe $G(s)$, can also can be measured experimentally.

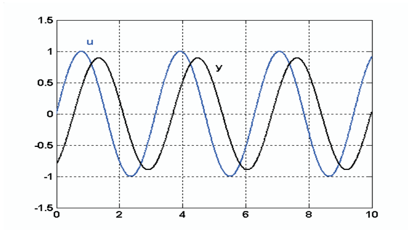
# Magnitude and Phase: Example


$$G(s) = \frac{2}{s+1}$$

$u = \sin \omega T \quad \omega = 1\,\text{rad/s}$

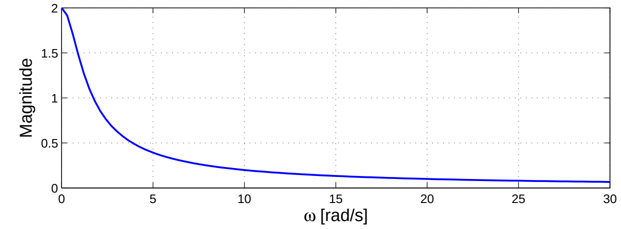


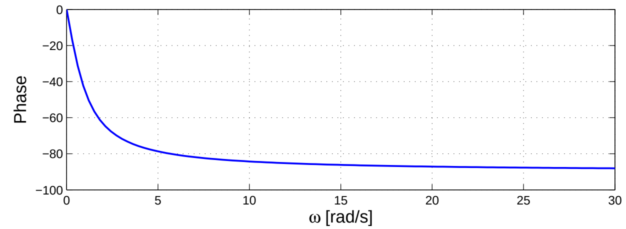

$u = \sin \omega T \quad \omega = 2\,\text{rad/s}$






# Magnitude and Phase Plot






# Bode Plot

Plotting on a linear scale is not so useful – plots are hard to interpret and cannot be easily drawn by hand.

If logarithmic scales are introduced, drawing becomes easier.

Such a logarithmic plot is called the Bode plot:

– frequency is plotted on a logarithmic scale ($\log_{10}$)

– amplitude is plotted using logarithmic units (decibels)

– phase is plotted on a linear scale (degrees)



# Logarithmic Scale: Decibels

The 10-base logarithm of a power gain is called a Bell (B):

$$x\ B = \log_{10} \frac{P_{\text{out}}}{P_{\text{in}}}$$

This unit appeared too large ($x$ was usually small) the decibel (dB) was introduced:

$$x\ dB = 10 \log_{10} \frac{P_{\text{out}}}{P_{\text{in}}}$$

# Logarithmic Scale: Decibels

Furthermore, power is proportional to the square of voltage:

$$x\ dB = 10 \log_{10} \frac{\alpha V_{\text{out}}^2}{\beta V_{\text{in}}^2} = 20 \log_{10} \frac{\alpha V_{\text{out}}}{\beta V_{\text{in}}}$$

Therefore for a gain $K$ the corresponding value in dB is:

$$x = 20 \log_{10}(K)$$

That is: $x$ is the value of $K$ expressed in $dB$.



# Decomposing Transfer Functions

Decompose a transfer function into:

$$G(s) = G_1(s)G_2(s)\cdots G_n(s)$$

Letting $s = j\omega$ we have:

$$G(j\omega) = |G(j\omega)|\, e^{j\angle G(j\omega)}$$

with

$$|G(j\omega)| = |G_1(j\omega)| \cdots |G_n(j\omega)|$$
$$\angle G(j\omega) = \angle G_1(j\omega) + \cdots + \angle G_n(j\omega)$$

In words: magnitudes multiply, phases add.



# Expressing Magnitude in Decibels

$$dB(|G(j\omega)|) = 20 \log_{10} |G(j\omega)|$$

which implies:

$$dB(|G(j\omega)|) = dB(|G_1(j\omega)|) + \cdots + dB(|G_n(j\omega)|)$$

In words: in dB, we can add magnitudes too.



# Bode Form of Transfer Function

$$G(s) = K \cdot s^k \cdot \frac{\prod_i (\tau_i s + 1) \cdot \prod_i [(\frac{s}{\omega_{n,i}})^2 + 2\zeta(\frac{s}{\omega_{n,i}}) + 1]}{\prod_j (\tau_j s + 1) \cdot \prod_j [(\frac{s}{\omega_{n,j}})^2 + 2\zeta(\frac{s}{\omega_{n,j}}) + 1]}$$

Example:

$$G(s) = \frac{2000(s + 0.5)}{s(s + 10)(s + 50)} = \frac{2(s/0.5 + 1)}{s(s/10 + 1)(s/50 + 1)}$$

$$G(j\omega) = \frac{2(j\omega/0.5 + 1)}{j\omega(j\omega/10 + 1)(j\omega/50 + 1)}$$

# Bode Plots: Basic Transfer Functions

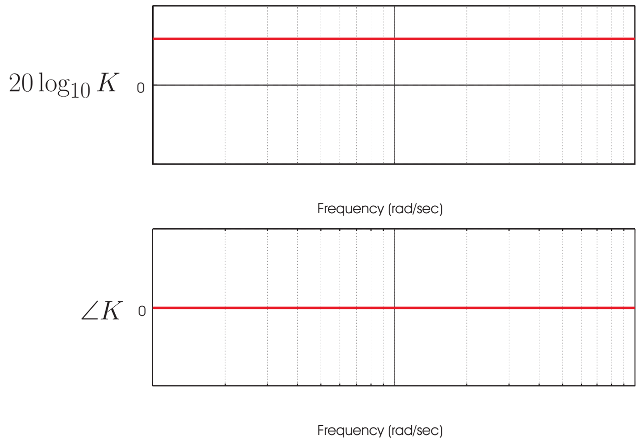
Any transfer function $G(s)$ can be represented as a product of (some of) the following terms:

- $K$
- $(s)^{\pm 1}$
- $(\tau s + 1)^{\pm 1}$
- $[(\frac{s}{\omega_n})^2 + 2\zeta(\frac{s}{\omega_n}) + 1]^{\pm 1}$

We can draw the magnitudes and phases of these basic terms and add them up graphically.

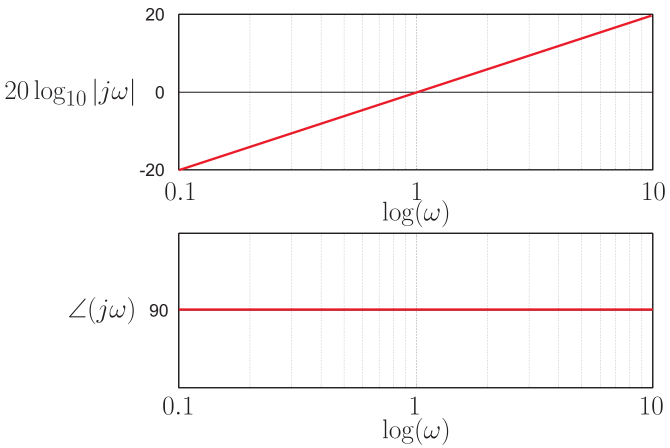


# Bode Plots: $G(j\omega) = K, \quad K > 0$



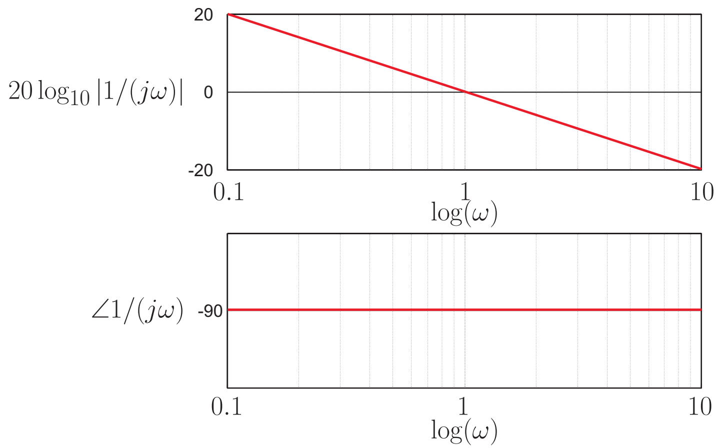



# Bode Plots: $G(j\omega) = j\omega$

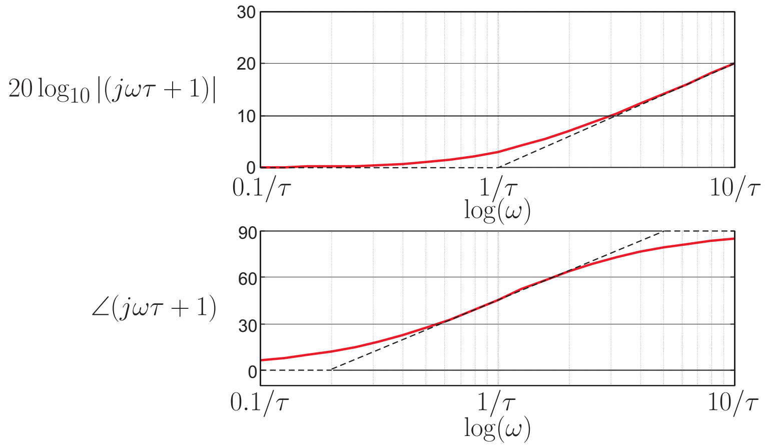





# Bode Plots: $G(j\omega) = \frac{1}{j\omega}$

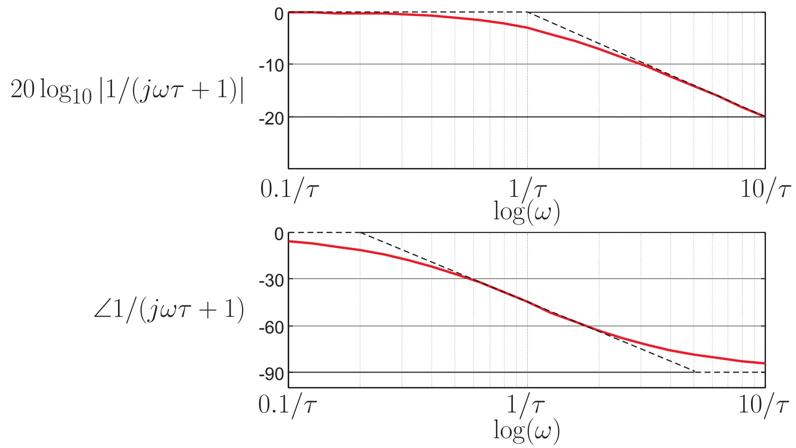


# Bode Plots: $G(j\omega) = j\omega\tau + 1$



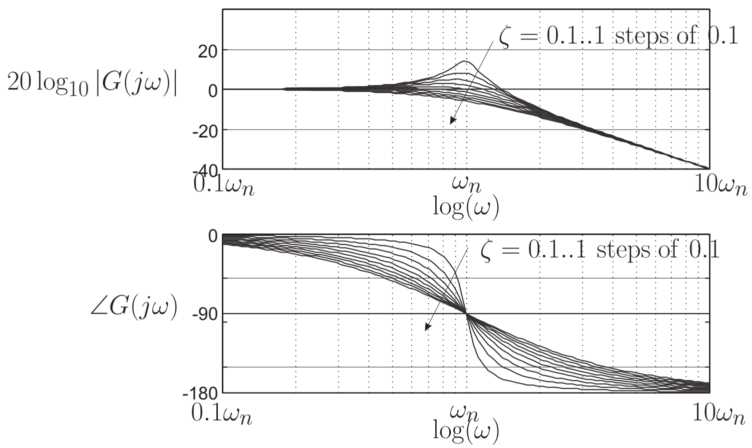



# Bode Plots: $G(j\omega) = \frac{1}{j\omega\tau+1}$






# Bode Plots: $G(j\omega) = [(\frac{s}{\omega_n})^2 + 2\zeta(\frac{s}{\omega_n}) + 1]^{-1}$






# Non-Minimum-Phase Systems

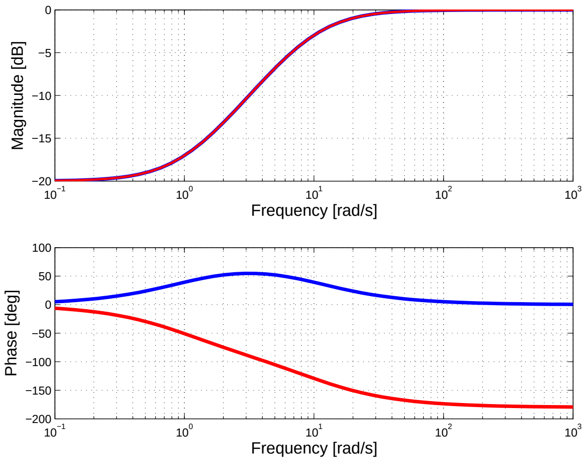
A system with zeros $z_i$ such that $Re\{z_i\} > 0$ is called non-minimum phase system.

Example:

$$G_1(s) = \frac{s+1}{s+10} \qquad G_2(s) = \frac{-s+1}{s+10}$$

System $G_2(s)$ undergoes a larger net change in phase than $G_1(s)$, i.e., $G_2(s)$ is called non-minimum phase.

# MP vs. NMP System: Bode Plots






# Type of System

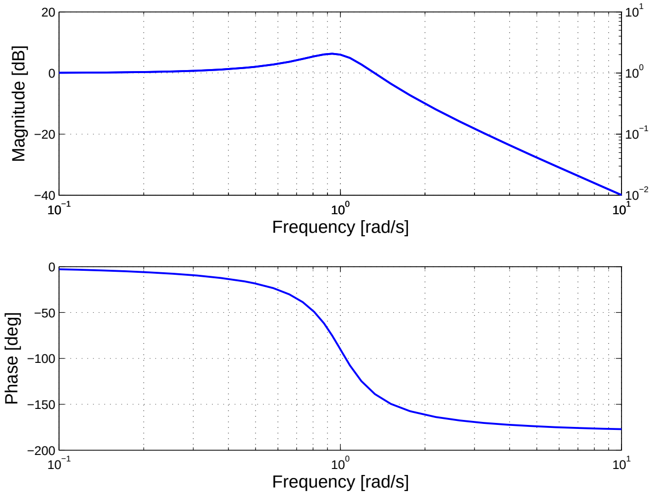
From the bode diagram of the open loop system $G(s)$ it is possible to see of what type the loop transfer $L(s)$ is, if proportional controller is used.

- If the slope of the magnitude on the extreme left of the Bode plot is $0 \Rightarrow$ no pure integrator $\Rightarrow$ Type 0.

- If the slope of the magnitude on the extreme left of the Bode plot is $-20n$ dB/decade $\Rightarrow$ $n$ integrators $\Rightarrow$ Type $n$.



# Type 0 System: Example






# Type 1 System: Example