Systeem- en Regeltechniek II

Lecture 8 – Frequency Domain Design

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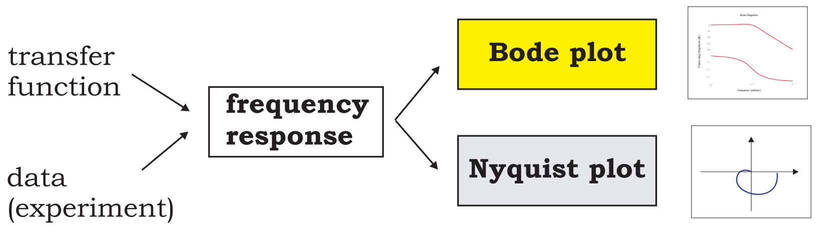
Lecture Outline

Previous lecture: Bode plots, non-minimum-phase systems.

Today:

- Bode's gain-phase relation.
- Neutral stability.
- Gain and phase margin, performance specs.
- Controller design.

Frequency Domain Methods



- Now we now how to sketch and plot Bode diagrams.
- The next step is analysis of system properties and design.

Bode's Gain-Phase Relation

For any stable minimum-phase system, phase $\angle G(j\omega)$ is uniquely related to magnitude $|G(j\omega)|$:

$$\angle G(j\omega_0) = \frac{1}{\pi}\int_{\infty}^{\infty} \frac{dM}{du} W(u) du$$

where $M = \ln|G(j\omega)|$, $u = \ln \omega/\omega_0$, $W(u) = \operatorname{ctanh}|u/2|$.

For a constant slope, we can approximate the above by:

$$\angle G(j\omega_0) \simeq n\frac{\pi}{2}$$

where n is the slope (1 for $20\ dB$/dec, 2 for $40\ dB$/dec, etc).

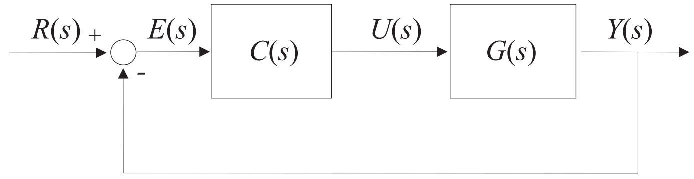
Consequence of the Gain-Phase Relation

For open loop stable minimum-phase system, it is sometimes sufficient to look at the magnitude only.

This property can be used to derive a simple design rule for control.

But first, we must be able determine, from the Bode plot, whether the system is stable!

Bode Plot: Closed-Loop Stability



$$L(s) = G(s)C(s)$$

Can we infer closed-loop stability from a Bode plot of the loop transfer function $L(s)$?

Proportional Controller: Loop Transfer

$$L(s) = \frac{Y(s)}{E(s)} = K\,G(s)$$

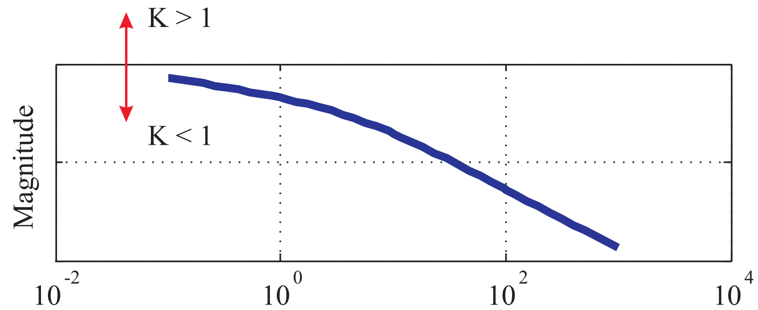
For the Bode plot, the following holds:

$$\angle G(j\omega) = \angle\left(KG(j\omega)\right) \qquad (K \text{ is a real number})$$

$$|G(j\omega)| = |K| \cdot |G(j\omega)| \qquad \text{(multiplication by a gain)}$$

$$|G(j\omega)|\ \mathrm{dB} = |K|\ \mathrm{dB} + |G(j\omega)|\ \mathrm{dB}$$

Proportional Controller: Loop Transfer



shift the magnitude response of $G(j\omega)$ by $20\log(K)$
phase does not change

Example: DC Motor

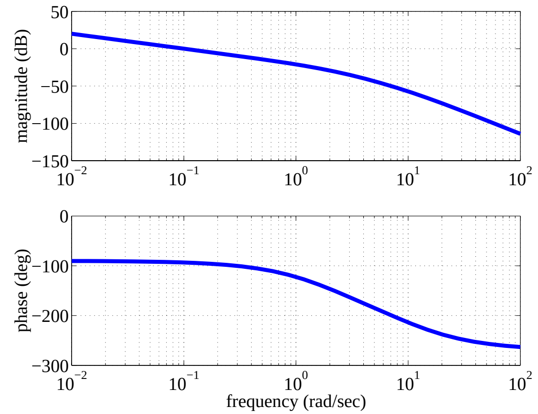
Transfer function:

$$G(s) = \frac{\theta(s)}{V(s)} = \frac{K_t}{s[(Ls+R)(Js+b)+K_t^2]}$$

inertia of the rotor	J	$= 0.01\,\mathrm{kg}\cdot\mathrm{m}^2$
damping (friction)	b	$= 0.1\,\mathrm{Nms}$
back emf	K_t	$= 0.01\,\mathrm{Nm/A}$
resistance	R	$= 1\,\Omega$
inductance	L	$= 0.5\,\mathrm{H}$

DC Motor: Open-Loop Bode Plot

$$G(s) = \frac{\theta(s)}{V(s)} = \frac{2}{s(s+10)(s+2)}$$



Influence of Proportional Gain

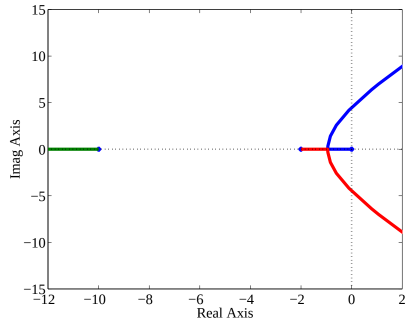
$$L(s) = KG(s) = \frac{K\cdot 2}{s(s+10)(s+2)}$$

Use Matlab: `sisotool('bode',G)`

OK, the magnitude moves up and down with the gain and the phase does not change . . .

. . . but, is there anything on the Bode plot that would hint on the stability of the closed loop?

Let's See Whether Root Locus Helps . . .

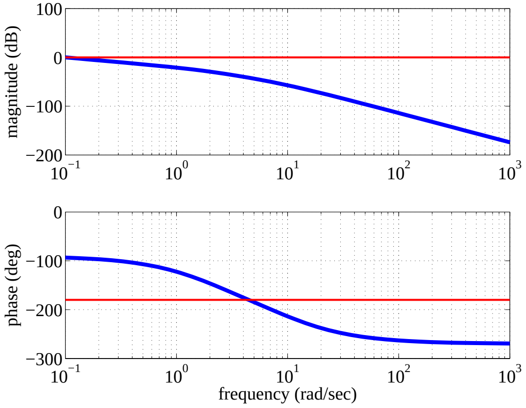


Basic properties of RL: $\angle G(s) = -180^\circ$ and $|KG(s)| = 1$

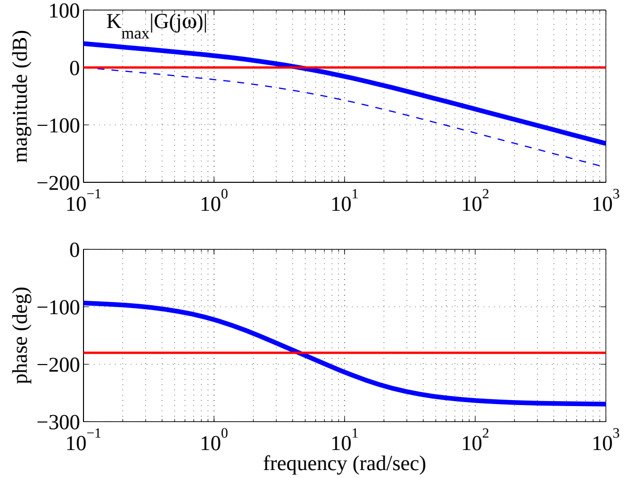
Neutral stability: $|K_{\max}G(j\omega)| = 0\,\mathrm{dB}$ and $\angle G(j\omega) = -180^\circ$

Back to the Bode Plot

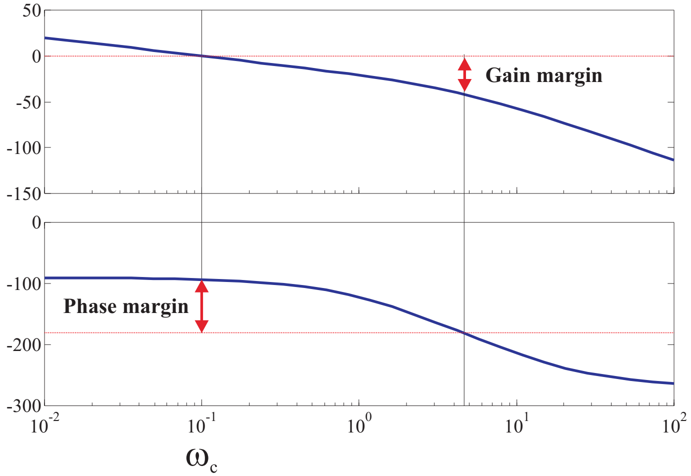
System is stable if: $|KG(j\omega)| < 0\,\mathrm{dB}$ at $\angle G(j\omega) = -180°$



Point of Neutral Stability



Crossover Frequency and Stability Margins



Crossover Frequency and Stability Margins

- The crossover frequency ω_c is the frequency for which the loop TF has gain 0 dB.
- The gain margin (GM) is the factor (or amount dB) by which the loop gain can be raised before instability occurs.
- The phase margin (PM) is the amount (in degrees) by which the phase exceeds 180° at ω_c.

Importance of Stability Margins

The margins tell us how far the closed-loop system is from the point of neutral stability. This indicates the robustness w.r.t. uncertainty in the plant model:

- Gain margin: by what factor the total process gain can increase.
- Phase margin: by how much the phase can decrease.

and performance:

- Phase margin: related to closed loop damping (overshoot).
- Crossover frequency: related to response speed (bandwidth).

Robustness: Example

Suppose our model is:

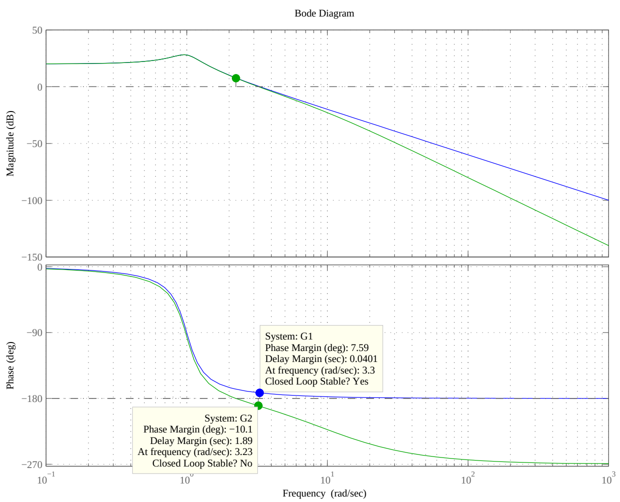
$$\hat{L}(s) = \frac{10}{s^2 + 0.4s + 1}$$

while the true plant is:

$$L(s) = \frac{10}{(s^2 + 0.4s + 1)(0.1s + 1)}$$

Relatively small mismatch in terms of step-response behavior, major difference in terms of closed-loop stability!

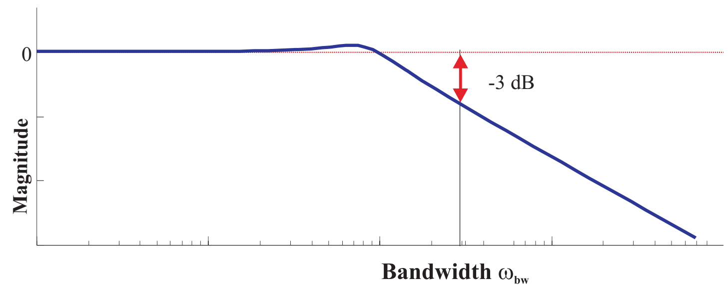
Bode Plot of Model and True Plant



Closed-Loop Bandwidth

Bandwidth = frequency up to which the input is "well reproduced" at the output of the closed-loop system.

Defined as frequency ω_{bw} at which the magnitude has an attenuation of 0.707 (3dB) – corresponds to 0.5 power gain.



Bandwidth and Crossover Frequency

Typically:

$$\omega_c \leq \omega_{bw} \leq 2\omega_c$$

The required speed of response (e.g., the settling time or rise time) can be expressed in terms of ω_c. Recall:

$$t_r = 1.8/\omega_n$$

(for second-order dominant response).

Phase Margin and Overshoot

The larger PM, the larger damping (less overshoot):

$$\zeta \approx \frac{\mathrm{PM}}{100}$$

this holds up to $\mathrm{PM} = 60°$

See the Franklin et al. for a graphical relationship between the overshoot and PM (page 357).

Recall Specs for Second-Order Systems

$$t_r = \frac{1.8}{\omega_n}$$

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$$

$$t_s = \frac{4.6}{\zeta\omega_n} \quad \text{for} \quad \pm 1\%$$

$$M_p = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}$$

More Complex Plants

- System unstable for small K and stable for large K, e.g.,:

$$L(s) = \frac{K(s+2)}{s^2-1}$$

- Conditionally stable systems (unstable for small and large K, stable for some intermediate values), e.g.,:

$$L(s) = \frac{K(s+2)}{(s+10)^2(s+1)(s-1)}$$

In the sequel, we consider systems with no poles in RHP.

The Basic Idea

- Adjust the proportional gain to get the required crossover frequency and/or steady-state tracking error.
- If needed, use the derivative action to add phase in the neighborhood of ω_c in order to increase the phase margin.
- If needed, use the integral action to increase the gain at low frequencies in order to guarantee the required steady-state tracking error.

Bode Plots: Homework Assignments

- Read Sections 6.1 through 6.6, except for the Nyquist criterion.
- Work out examples in these sections and verify the results by using Matlab.
- Reproduce the derivation of the frequency response as given on the overhead sheets.
- Work out a selection of problems 6.3 through 6.9 and verify your results by using Matlab.