Systeem- en Regeltechniek II

Lecture 10 – Nyquist plot and stability criterium

Robert Babuška

Delft Center for Systems and Control
Faculty of Mechanical Engineering
Delft University of Technology
The Netherlands

e-mail: r.babuska@dcsc.tudelft.nl
www.dcsc.tudelft.nl/~babuska
tel: 015-27 85117

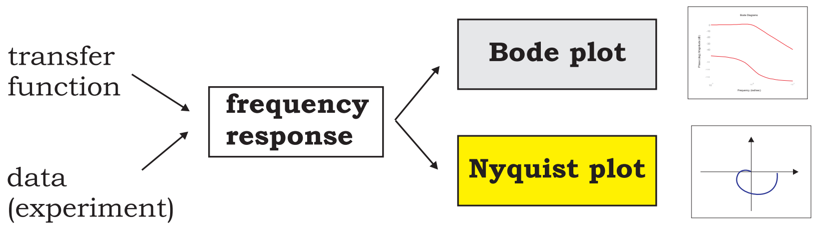
Lecture Outline

Previous lecture: PID controller design, lead and lag compensators.

Today:

- Nyquist plot.
- Nyquist stability criterion.

Frequency Domain Methods



Frequency Domain Methods

The key of frequency domain design:

provide sufficient phase at the crossover frequency

(= get the closed-loop far enough from the point of becoming unstable)

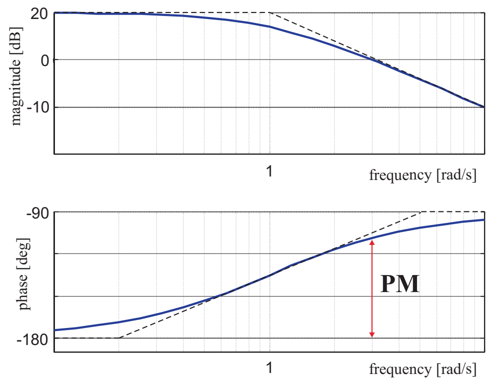
⇒ Bode plots are well suited as a design and analysis tool.

. . . , so, do we need yet another kind plot?

In fact, we do, let's have a look why . . .

Motivating Example

$G(s) = \dfrac{10}{s-1}$ sketch the bode plot, indicate PM



Deficiency of Bode Plots

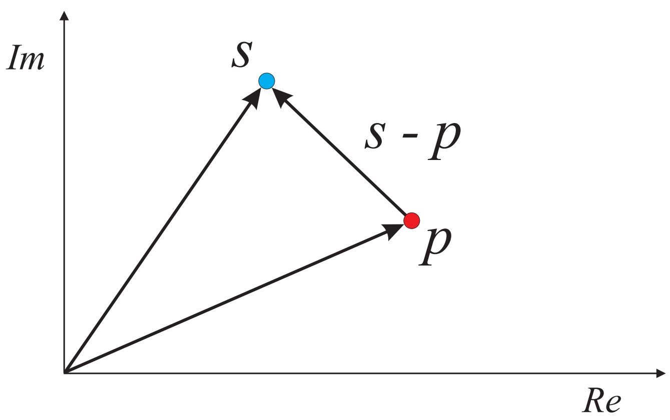
For systems with poles in right half-plane, the Bode plot alone does not provide any good indication of stability / instability.

→ In the above example, the phase will never cross $-180°$, and yet, for $K < 0.1$, the closed loop becomes unstable (check the root locus!).

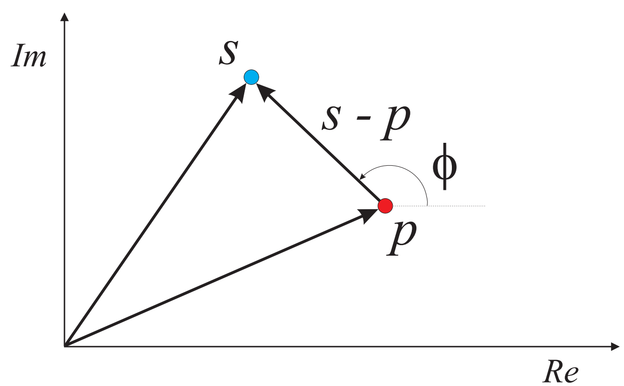
Is there a method for frequency domain design, considering stability for all kinds of systems?

Yes, the Nyquist plot and stability criterion.

Complex Numbers as Vectors



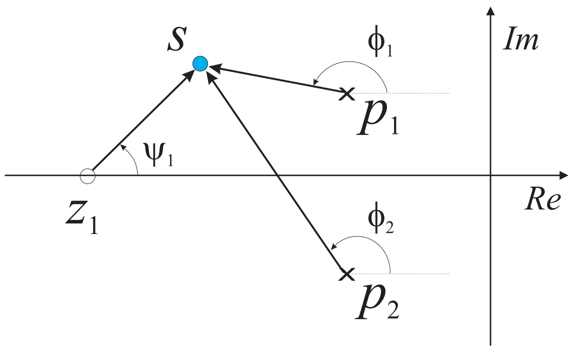
Euler Representation



$$s - p = |s - p| \, e^{j\phi}$$

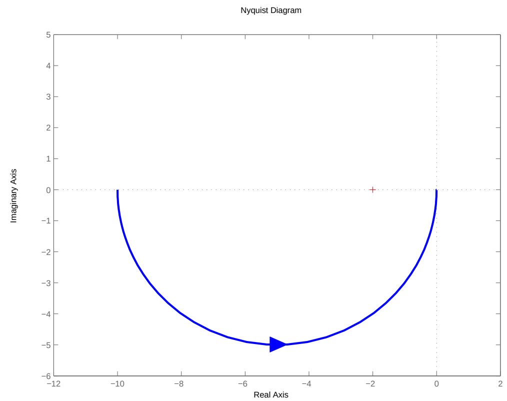
Transfer Function

$$G(s) = \frac{s - z_1}{(s - p_1)(s - p_2)} = \frac{|s - z_1|}{|s - p_1| \cdot |s - p_2|} e^{j(\psi_1 - \phi_1 - \phi_2)}$$



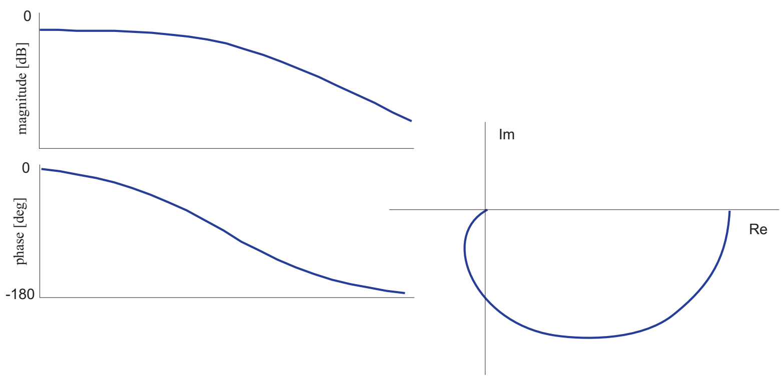
Nyquist (Polar) Plot

Let $s = j\omega$ for $\omega \in [0, \infty)$ and plot $\mathrm{Im}[G(j\omega)]$ against $\mathrm{Re}[G(j\omega)]$.

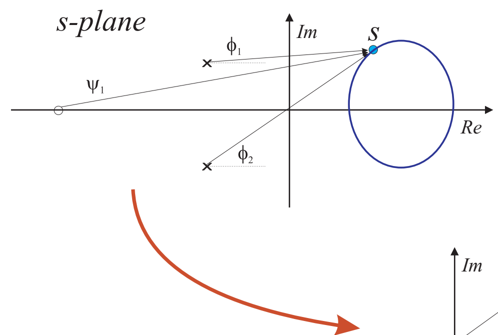


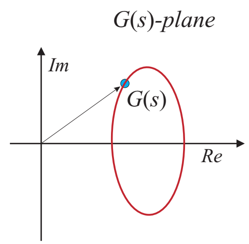
Relation Bode Plot – Nyquist Plot

$$G(s) = \frac{1}{(s+1)(s+5)}$$



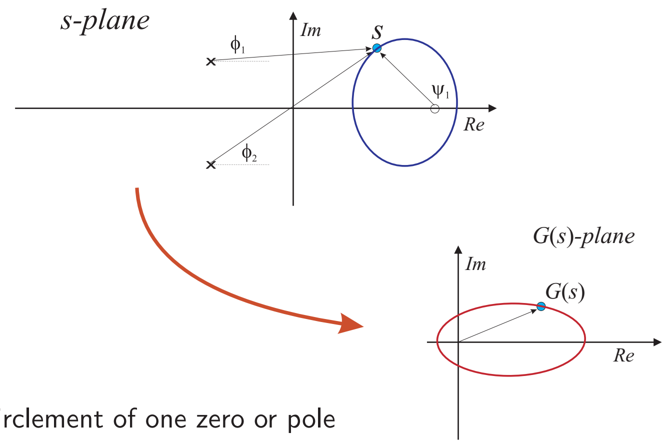
Argument Principle (I)





No zero/pole encirclement
$\Longrightarrow$ no origin encirclement

Argument Principle (II)



Encirclement of one zero or pole
$\Longrightarrow$ one encirclement of the origin

Argument Principle in General

For a clockwise contour in the s-plane, denote:

P number of poles encircled in the s-plane

Z number of zeros encircled in the s-plane

N number of clockwise encirclements of the origin by $G(s)$

N = Z - P

Recall:

$$\angle G(s) = \sum_i \angle(s - z_i) - \sum_j \angle(s - p_j) = \sum_i \psi_i - \sum_j \phi_j$$

Argument Principle for Stability Analysis

Given the Nyquist plot of $KG(s)$, we want to determine whether the closed loop:

$$G_{cl}(s) = \frac{Y(s)}{R(s)} = \frac{KG(s)}{1 + KG(s)}$$

is stable.

- Closed-loop stability $\Longleftrightarrow$ $G_{cl}(s)$ has no poles in RHP.
- Poles are given by $1 + KG(s)$, so let us study $1 + KG(s)$.

Argument Principle for Stability Analysis

$$G_{cl}(s) = \frac{Y(s)}{R(s)} = \frac{KG(s)}{1 + KG(s)}$$

Poles of $G_{cl}(s)$ are the solutions of $1 + KG(s) = 0$, i.e.:

poles of $G_{cl}(s)$ are the zeros of $(1 + KG(s))$

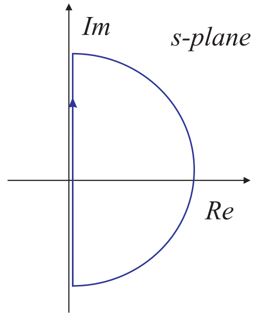
in addition, as:

$$1 + KG(s) = 0 \quad \rightarrow \quad 1 + K\frac{b(s)}{a(s)} = 0 \quad \rightarrow \quad \frac{a(s) + Kb(s)}{a(s)} = 0$$

poles of $G(s)$ are the poles of $(1 + KG(s))$

Argument Principle for Stability Analysis

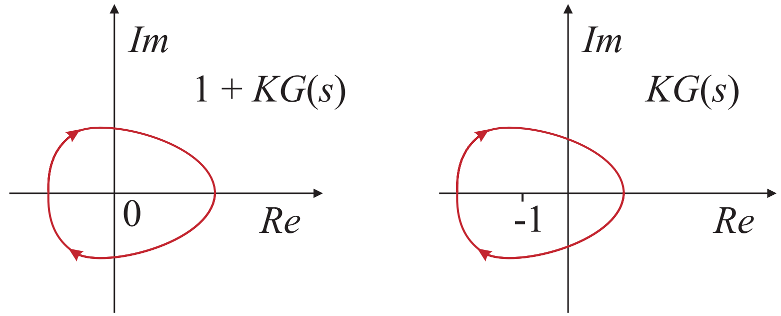
So, we want to find out, if $1 + KG(s)$ has no RHP zeros.



encircle the entire RHP

= draw the Nyquist diagram for frequencies $\omega \in (-\infty, \infty)$

Argument Principle for Stability Analysis



Draw the Nyquist diagram of the loop TF $L(s) = KG(s)$,
count clockwise encirclements of -1 : $N = Z - P$

Argument Principle for Stability Analysis

N = Z - P

Z = number of RHP zeros of $(1 + KG(s))$
P = number of RHP poles of $(1 + KG(s))$

Given that:

$$1 + KG(s) = 0 \quad \rightarrow \quad \frac{a(s) + Kb(s)}{a(s)} = 0$$

we have:

Z = number of RHP poles of $G_{cl}(s)$ ... CL poles
P = number of RHP poles of $G(s)$ ... OL poles

Nyquist Stability Criterion

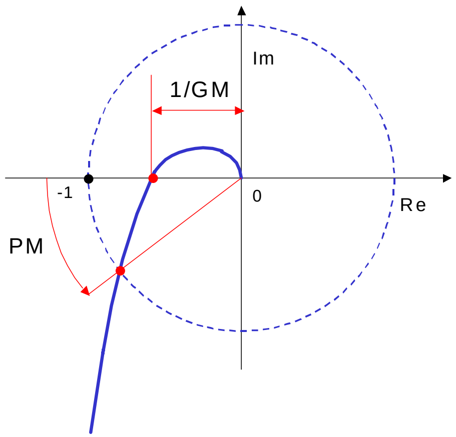
Z = N + P

In words:

number of RHP closed-loop poles =

clockwise encirclements + number of RHP open-loop poles

Stability Margins in Nyquist Plot



Why Don't We Ride These Bikes?

Simple Bicycle Models

Front steering:

$$G(s) = \frac{\phi(s)}{\delta(s)} = K\,\frac{s + \frac{v}{a}}{s^2 - \frac{g}{h}}$$

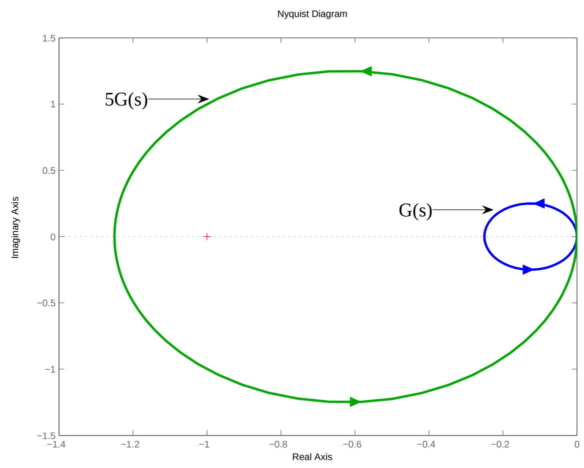
Rear steering:

$$G(s) = \frac{\phi(s)}{\delta(s)} = K\,\frac{-s + \frac{v}{a}}{s^2 - \frac{g}{h}}$$

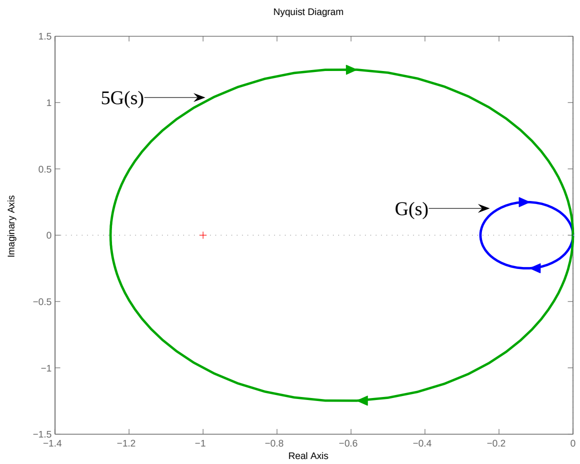
a – distance of COM to fixed wheel center

h – height of COM above ground

Front Steering: Nyquist Plot



Rear Steering: Nyquist Plot



Nyquist: Homework Assignments

- Read Section 6.3 (Nyquist stability criterion).
- Work out examples in this section and verify the results by using Matlab.
- Work out problems 6.18 – 6.22 and verify your results by using Matlab.