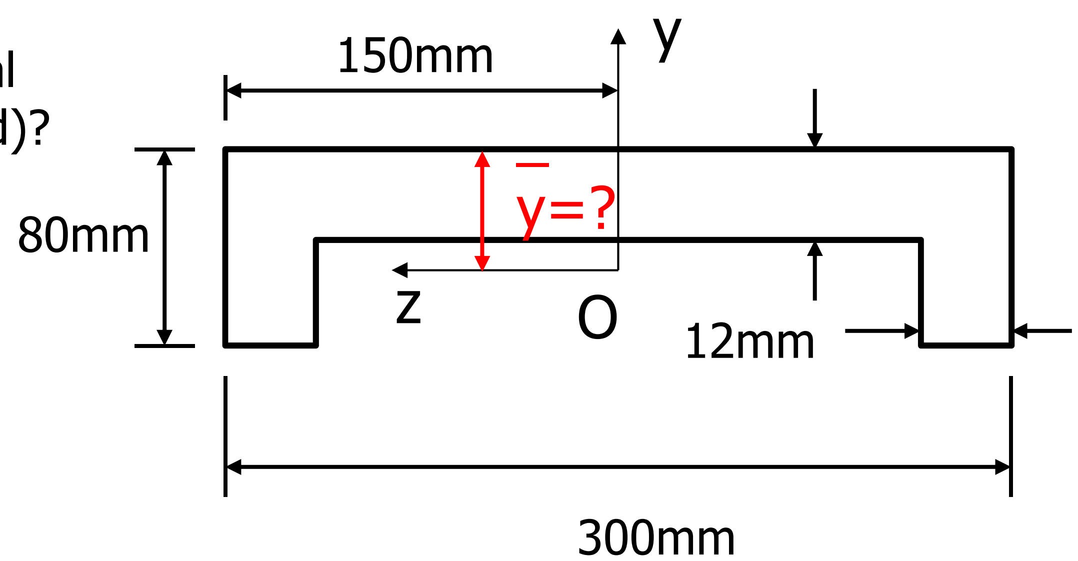
Example – Section Properties

Problem Statement

Determine the neutral axis location (z-axis) and moment of inertia for the following profile

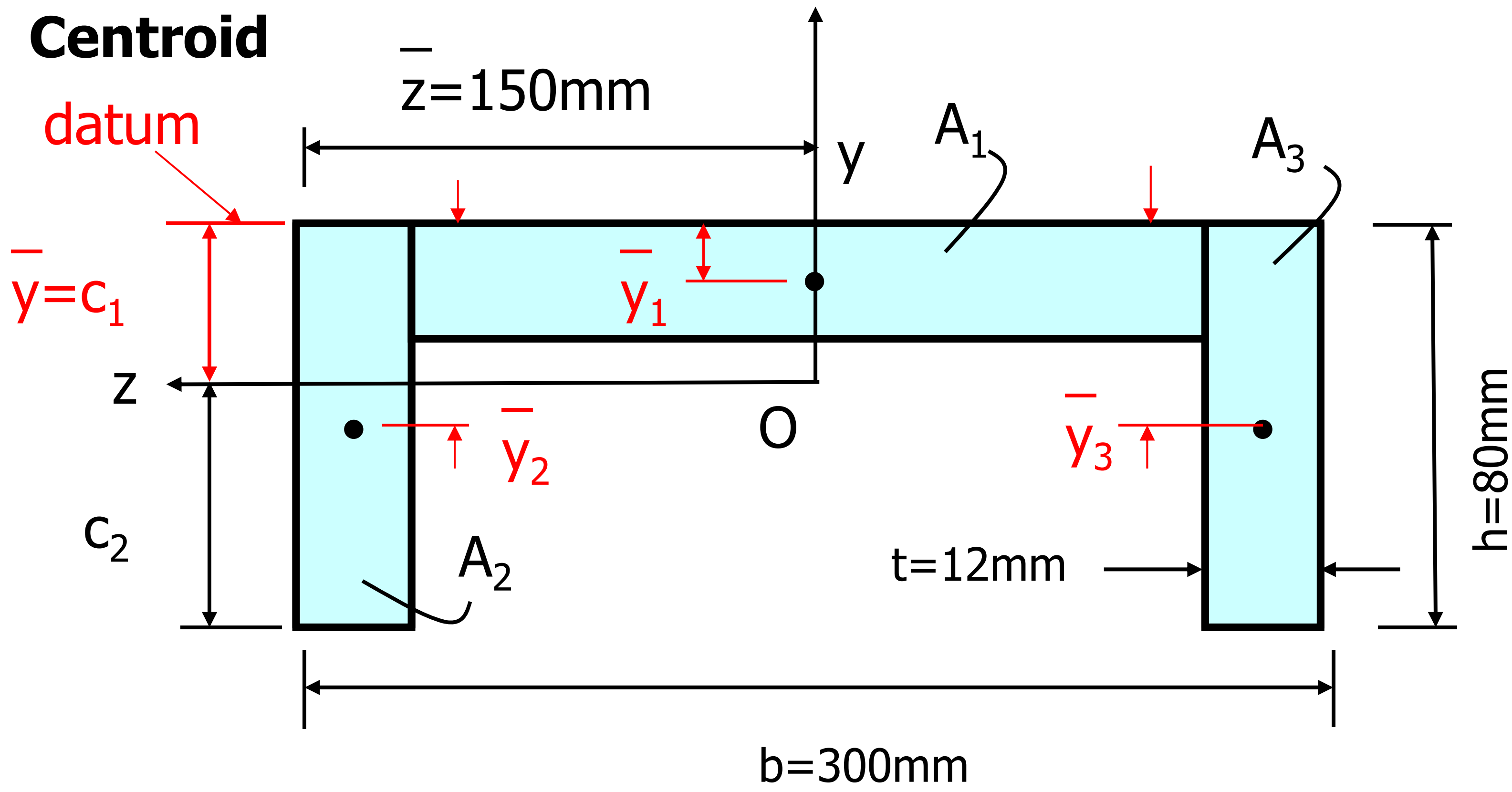
1) Where is the centroid (neutral axis passes through the centroid)?

- 1 axis of symmetry
- Location of y-axis known

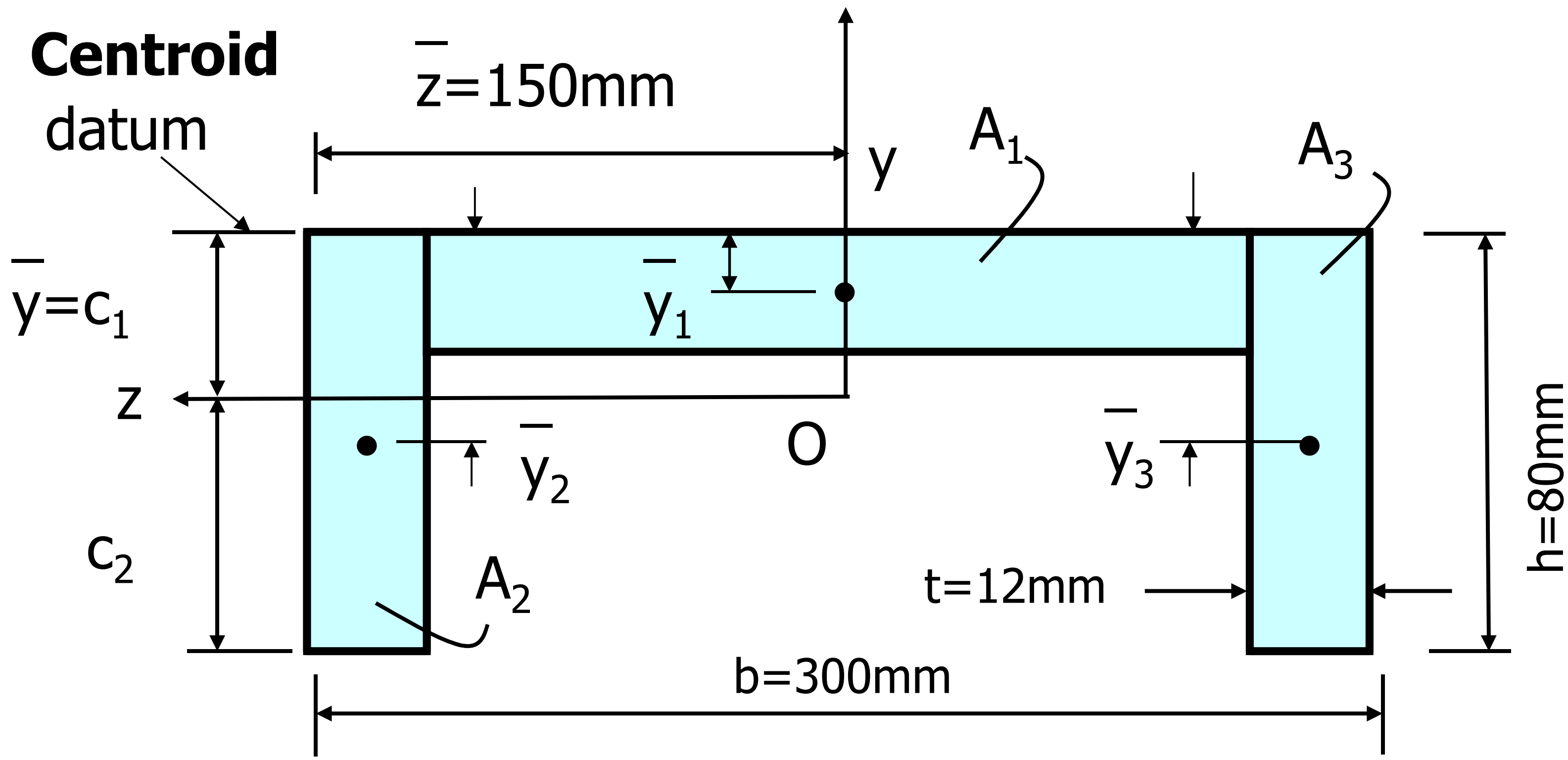


Divide into sub-areas

Centroid



$$\bar{y} = c_1 = \frac{\sum_{i=1}^{3} \bar{y}_i A_i}{\sum_{i=1}^{3} A_i} = \frac{\bar{y}_1 A_1 + \bar{y}_2 A_2 + \bar{y}_3 A_3}{A_1 + A_2 + A_3}$$

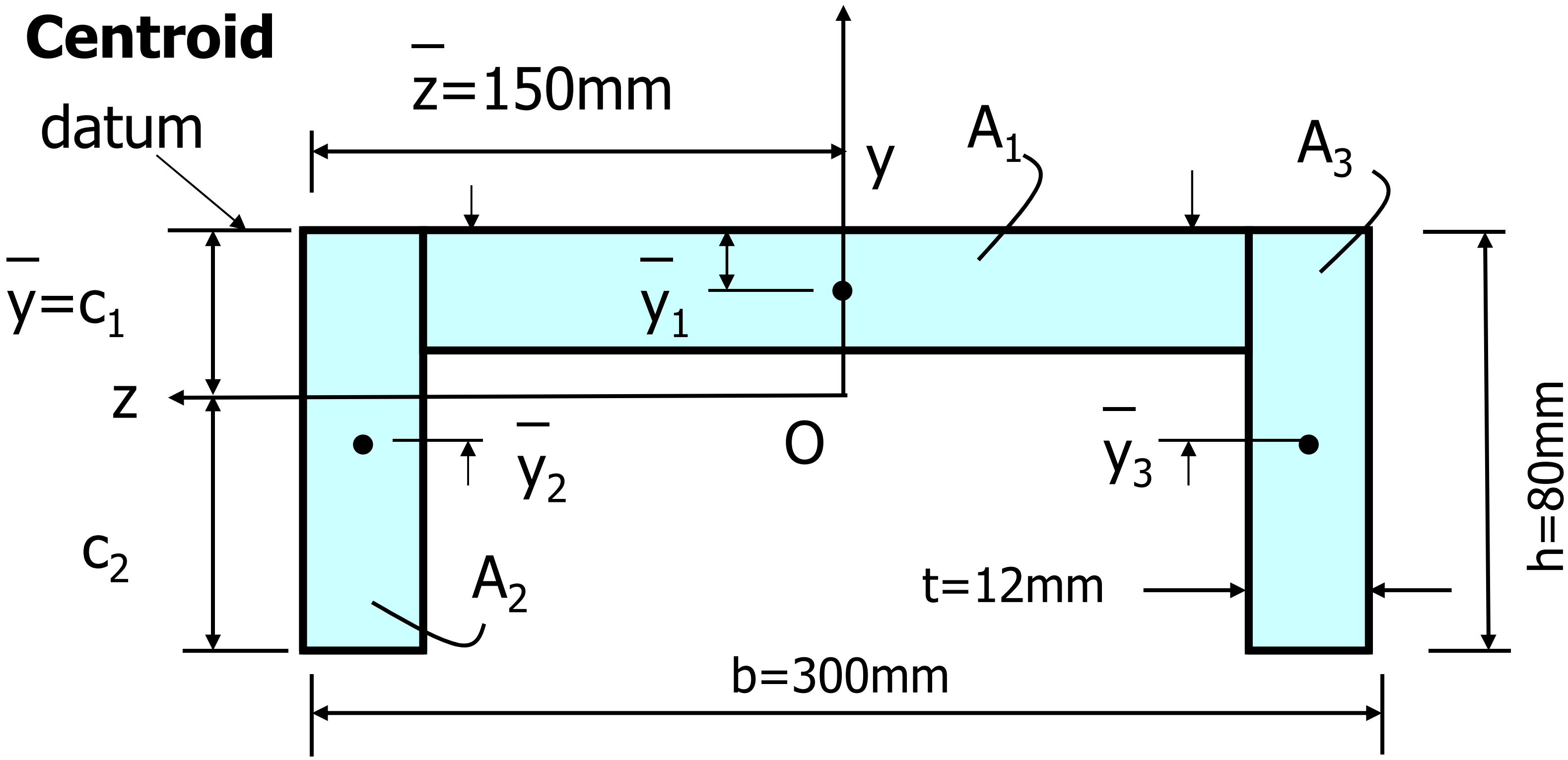


Area 1: $\bar{y}_1 = \dfrac{t}{2} = 6mm \qquad A_1 = t\left(b - 2t\right) = 3312mm^2$

Area 2: $\bar{y}_2 = \dfrac{h}{2} = 40mm \qquad A_2 = ht = 960mm^2$

Area 3: $\bar{y}_3 = \dfrac{h}{2} = 40mm \qquad A_3 = ht = 960mm^2$ Same as A_2!

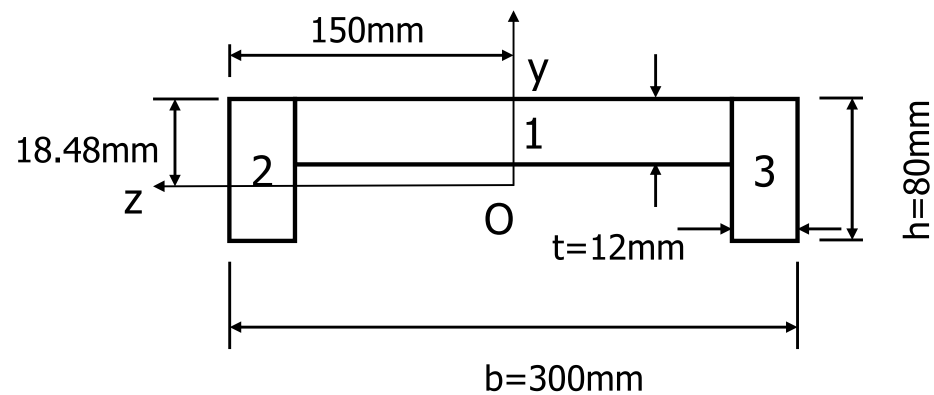
Centroid



$$c_1 = \frac{\bar{y}_1 A_1 + \bar{y}_2 A_2 + \bar{y}_3 A_3}{A_1 + A_2 + A_3} = 18.48mm$$

$$c_2 = h - c_1 = 61.52mm$$

Example – Section Properties



2) How to determine I?

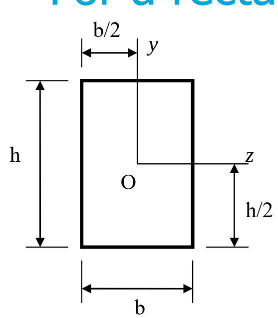
You can integrate: $I = \int_A y^2 dA$

or

Use parallel axis theorem

For a rectangle: $I_{centroid} = \dfrac{bh^3}{12}$

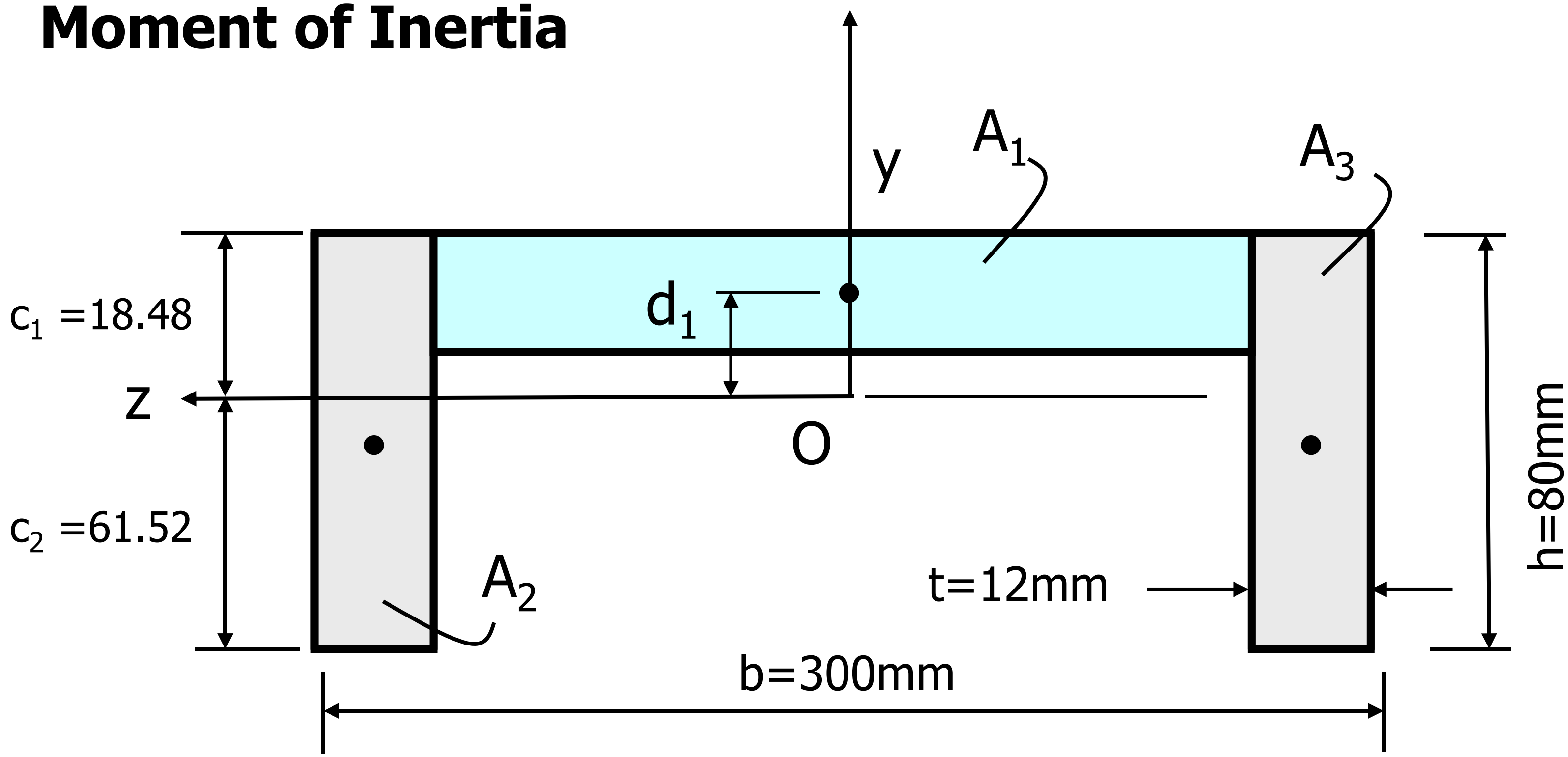
Parallel axis theorem: $I_{x'} = I_x + A \cdot d^2$



$$\therefore I_z = \sum_1^3 (I_{centroid_i} + d_i^2 A_i)$$

d = distance from centroid of A to centroid of overall section

Moment of Inertia



Area 1:

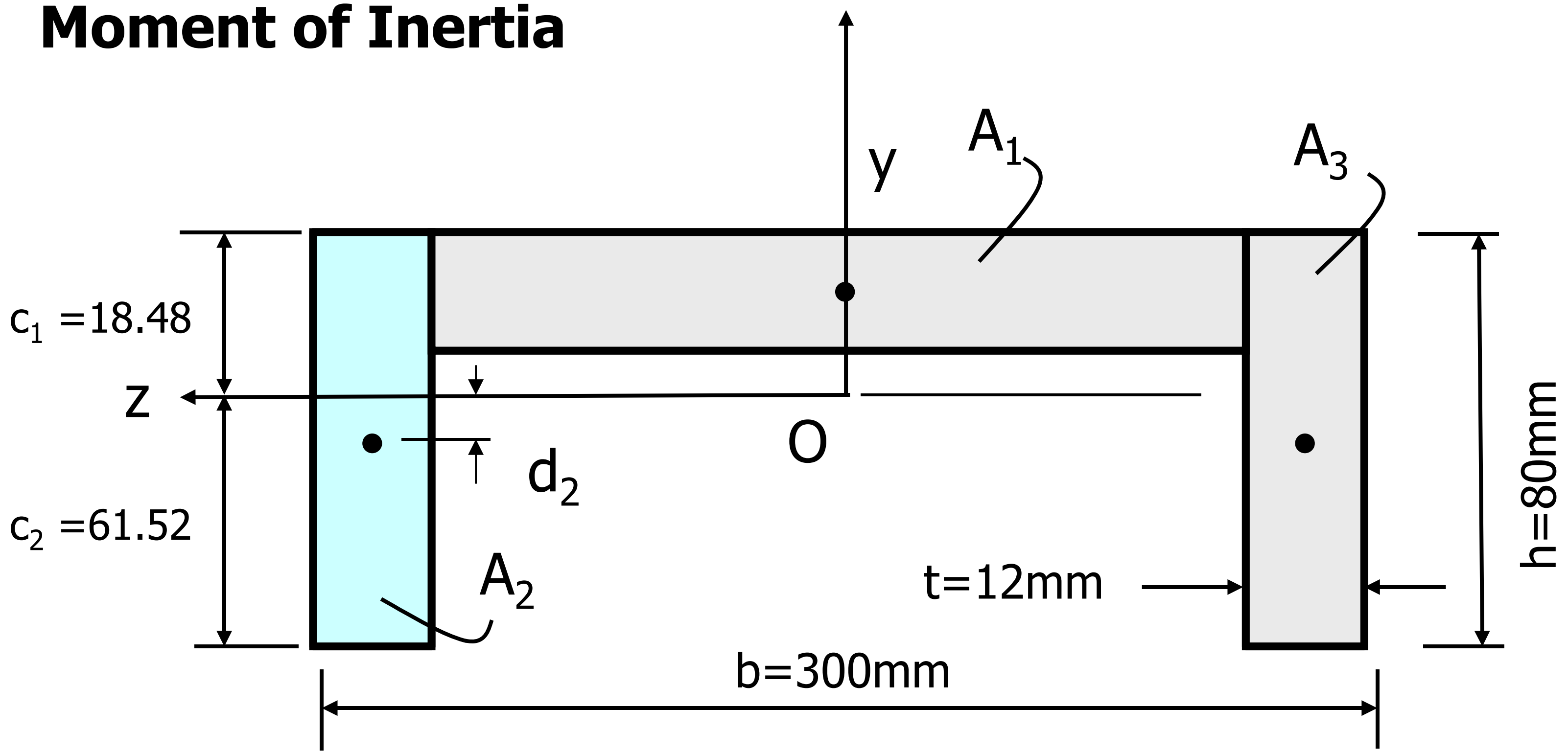
$$I_{c1} = \frac{(b-2t)\cdot t^3}{12} = 39744 mm^4$$

$$d_1 = c_1 - \frac{t}{2} = 12.48 mm$$

$$A_1 = 3312 mm^2$$ (Calculated previously)

$$I_{z1} = I_{c1} + A_1 d_1^2$$
$$= 555600 mm^4$$

Moment of Inertia



Area 2:

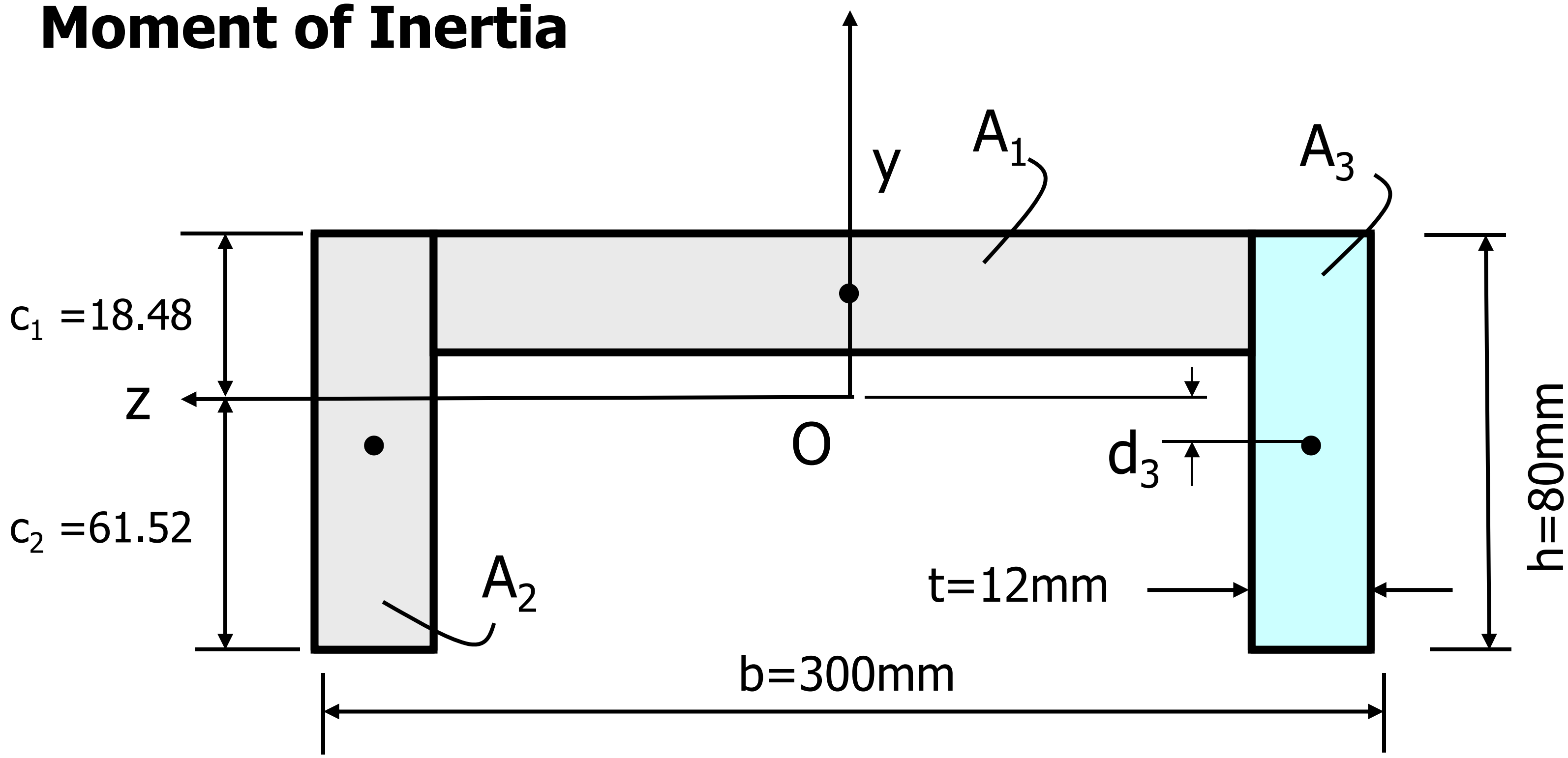
$$I_{c2} = \frac{t \cdot h^3}{12} = 512000mm^4$$

$$d_2 = \frac{h}{2} - c_1 = 21.52mm$$

$A_2 = 960mm^2$ (Calculated previously)

$$I_{z2} = I_{c2} + A_2 d_2^2$$
$$= 956600mm^4$$

Moment of Inertia



Area 3: Same as Area 2!

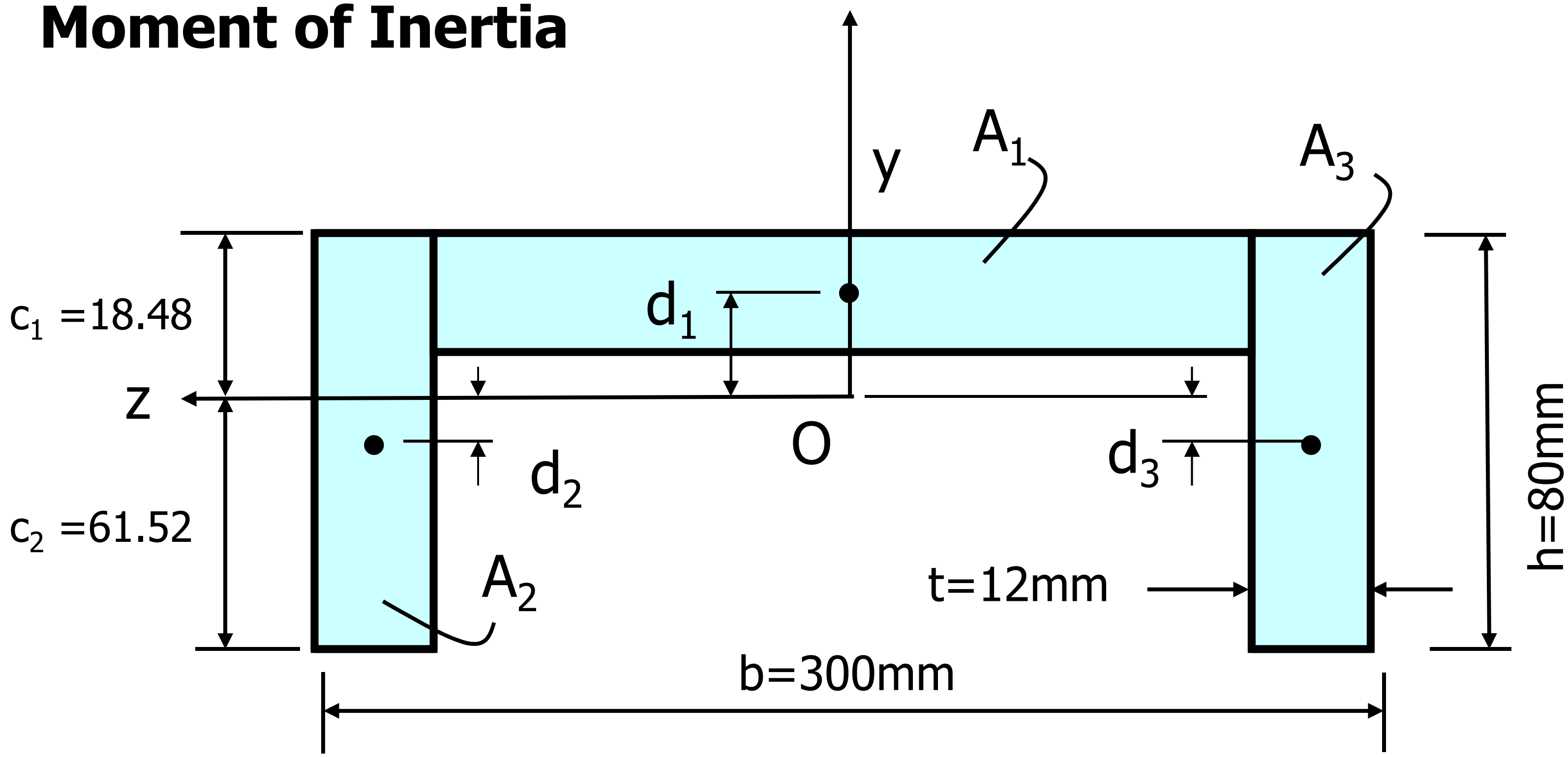
$$A_3 = 960mm^2$$

$$I_{c3} = \frac{t \cdot h^3}{12} = 512000mm^4$$

$$I_{z3} = I_{c3} + A_3 d_3^2$$
$$= 956600mm^4$$

$$d_3 = \frac{h}{2} - c_1 = 21.52mm$$

Moment of Inertia



Overall:

Area	A [mm²]	d [mm]	I_c [mm⁴]	$I_c + Ad^2$ [mm⁴]
1	3312	12.48	39,744	555,600
2	960	21.52	512,000	956,600
3	960	21.52	512,000	956,600
				2,468,800

I_z