Review Lecture

AE1108-II: Aerospace Mechanics of Materials

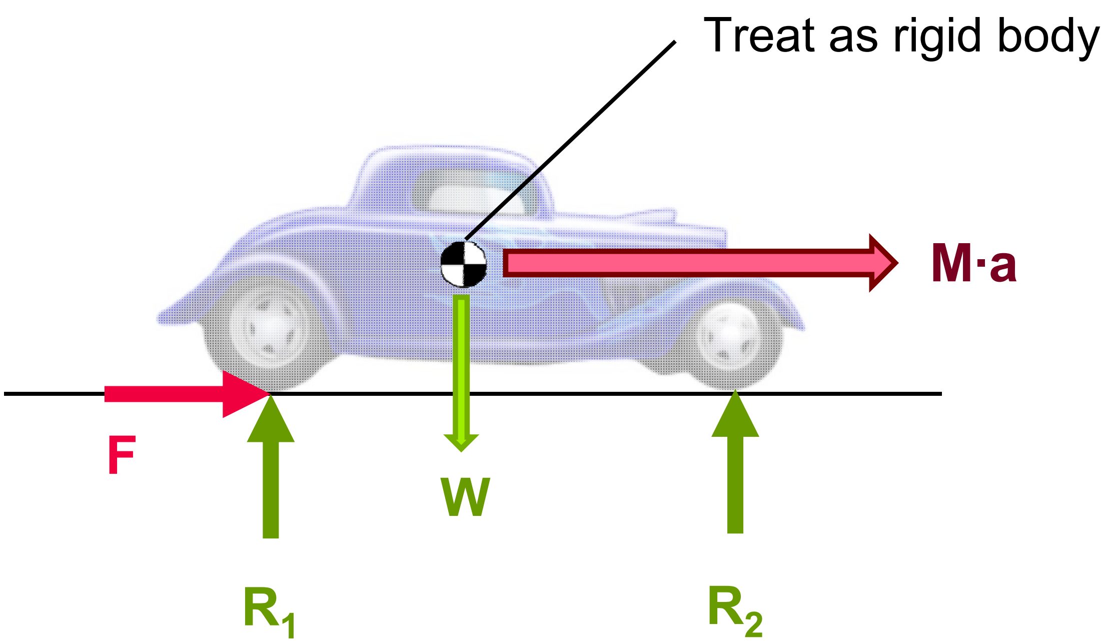
Dr. Calvin Rans
Dr. Sofia Teixeira De Freitas

Aerospace Structures & Materials
Faculty of Aerospace Engineering

TU Delft

Analysis of an Engineering System

Overview of Mechanics



Statics: $R_1 + R_2 = W$

Dynamics: $F = M \cdot a$

External analysis provides performance requirements of the system

Analysis of an Engineering System

Overview of Mechanics

External viewpoint

- Speed, acceleration, trajectory
- Weight, power, drag

Internal viewpoint

- Part interaction, deformation, failures
- Combustion, fuel flow, and many more

Engineering Systems are Not Rigid Blobs!

Overview of Mechanics

- What can happen to the car?
  - Suspension bottoms out
  - Chassis deforms and contact ground
  - Drive-shaft fails due to overload
  - Interference of moving parts
  - Vibration issues
  - Engine overheats
  - Lubricant issues

Solid Mechanics
(Mechanics of Materials)

Is the structure strong enough and stiff enough?

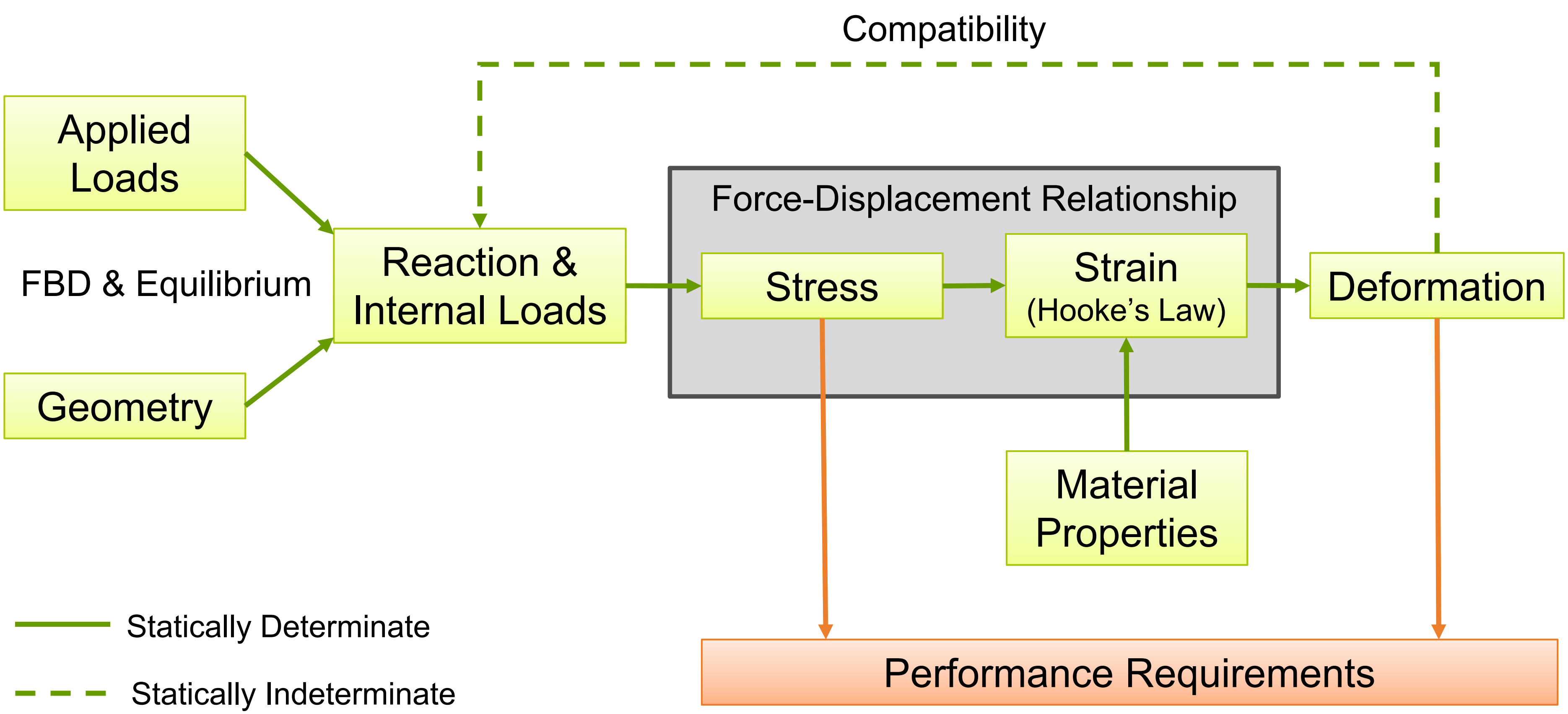
TU Delft

Overview of Engineering Mechanics

	Static Analysis ($\Sigma F = 0$)		**Dynamic Analysis** ($\Sigma F = ma$)
External Analysis: • Particle • Rigid Body	Statics (AE1130-I)		Dynamics (AE1130-II)
Internal Analysis: • Solids	Solid Mechanics (AE1108-II)	Structural Analysis (AE2135-I)	Vibrations (AE2106)
• Fluids	Aerodynamics (AE1110, AE2130)		
• Energy	Physics I, Power & Propulsion (AE1240, AE2203)		

TU Delft

Anatomy of a Solid Mechanics Problem

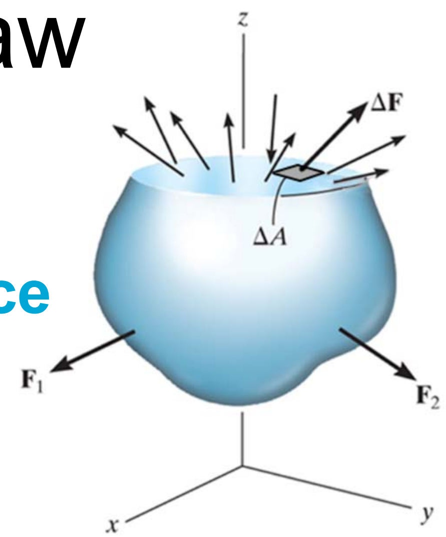


TU Delft

Stress, Strain, & Hooke's Law

Definition of Stress

- Stress is defined as the **intensity of a force**



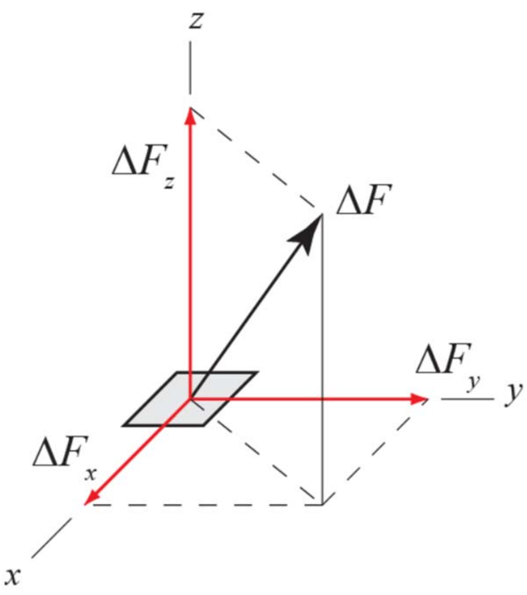
Normal Stress:

$$\sigma_z = \lim_{\Delta A \to 0} \frac{\Delta F_z}{\Delta A}$$

Shear Stress:

$$\tau_{zx} = \lim_{\Delta A \to 0} \frac{\Delta F_x}{\Delta A}$$

$$\tau_{zy} = \lim_{\Delta A \to 0} \frac{\Delta F_y}{\Delta A}$$



TUDelft

Stress, Strain, & Hooke's Law

Definition of Strain

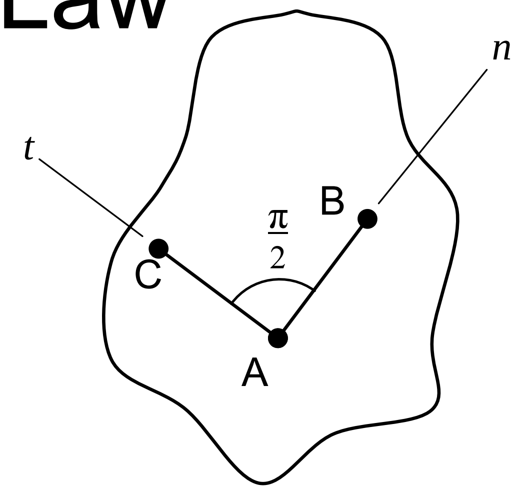
- Strain is defined as the **intensity of a deformation**

Normal Strain:

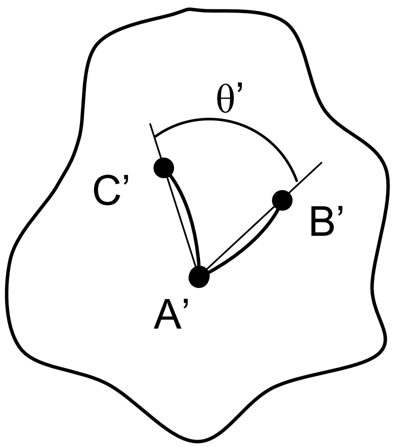
$$\varepsilon_n = \lim_{B \to A \text{ along n}} \frac{B'A' - BA}{BA}$$

Shear Strain:

$$\gamma_{nt} = \frac{\pi}{2} - \lim_{\substack{B \to A \text{ along n} \\ C \to A \text{ along t}}} \theta'$$



Undeformed body



Deformed body

TU Delft

Stress, Strain, & Hooke's Law

Generalized Hooke's Law

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E} + \alpha \cdot \Delta T$$

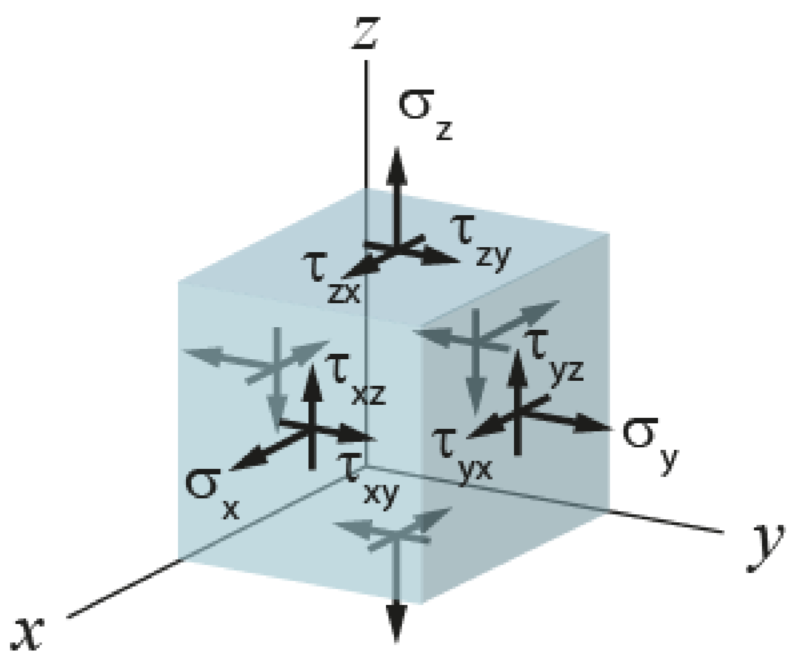
$$\varepsilon_y = \frac{\sigma_y}{E} - \nu\frac{\sigma_x}{E} - \nu\frac{\sigma_z}{E} + \alpha \cdot \Delta T$$

$$\varepsilon_z = \frac{\sigma_z}{E} - \nu\frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} + \alpha \cdot \Delta T$$

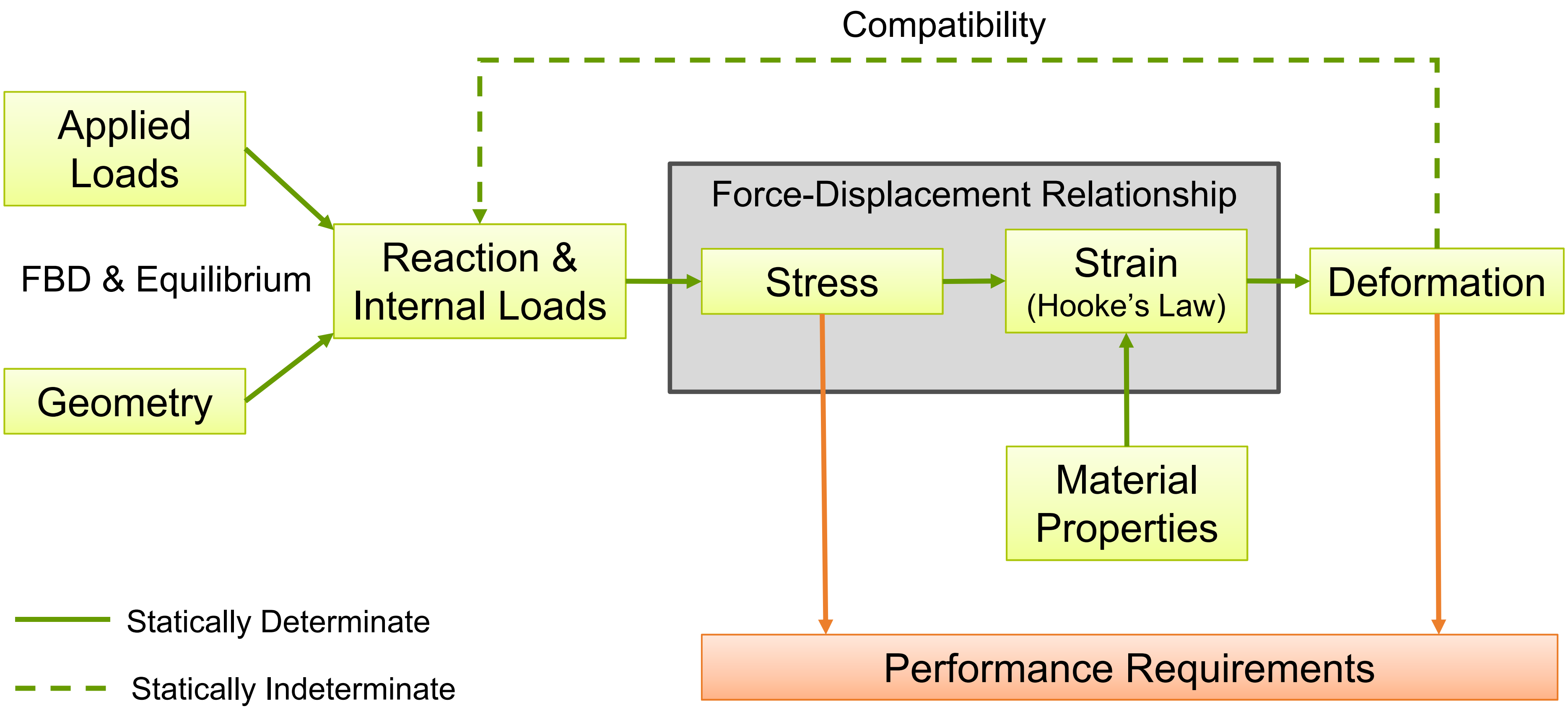
$$\gamma_{xy} = \frac{\tau_{xy}}{G}$$

$$\gamma_{xz} = \frac{\tau_{xz}}{G}$$

$$\gamma_{yz} = \frac{\tau_{yz}}{G}$$

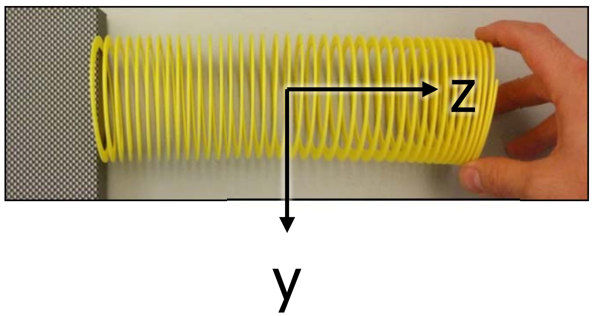


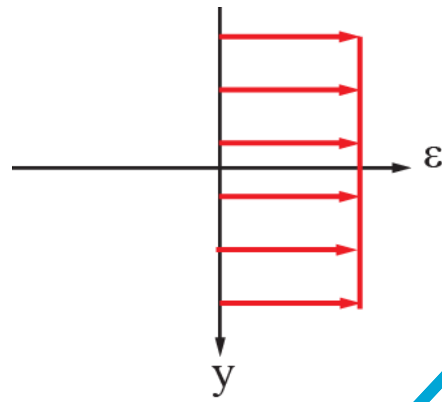
Anatomy of a Solid Mechanics Problem



TU Delft

Developing a Force-Displacement Relation





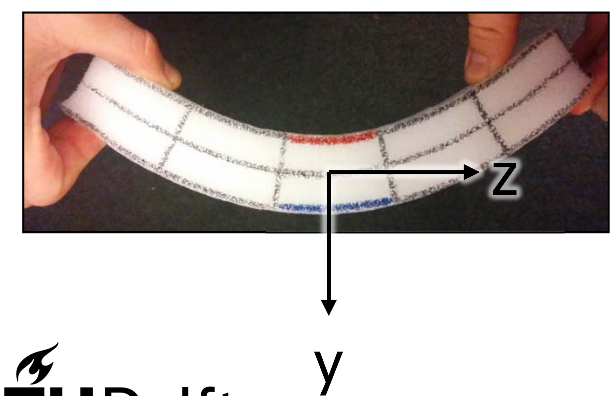
Displacement

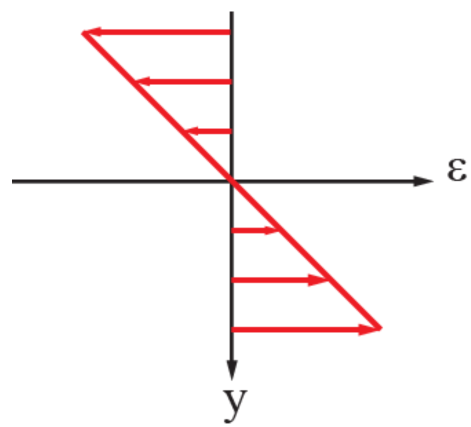
$\int$

Visualize Deformation → Compatible Strain → Hooke's Law → σ

$\int$

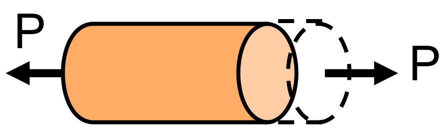
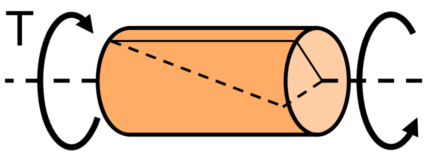
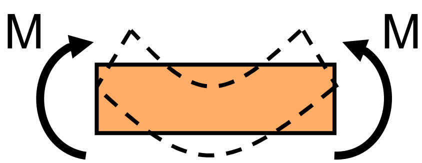
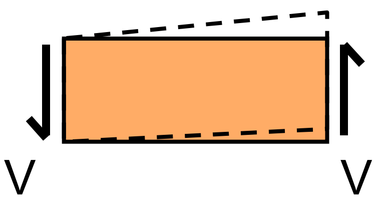
Force



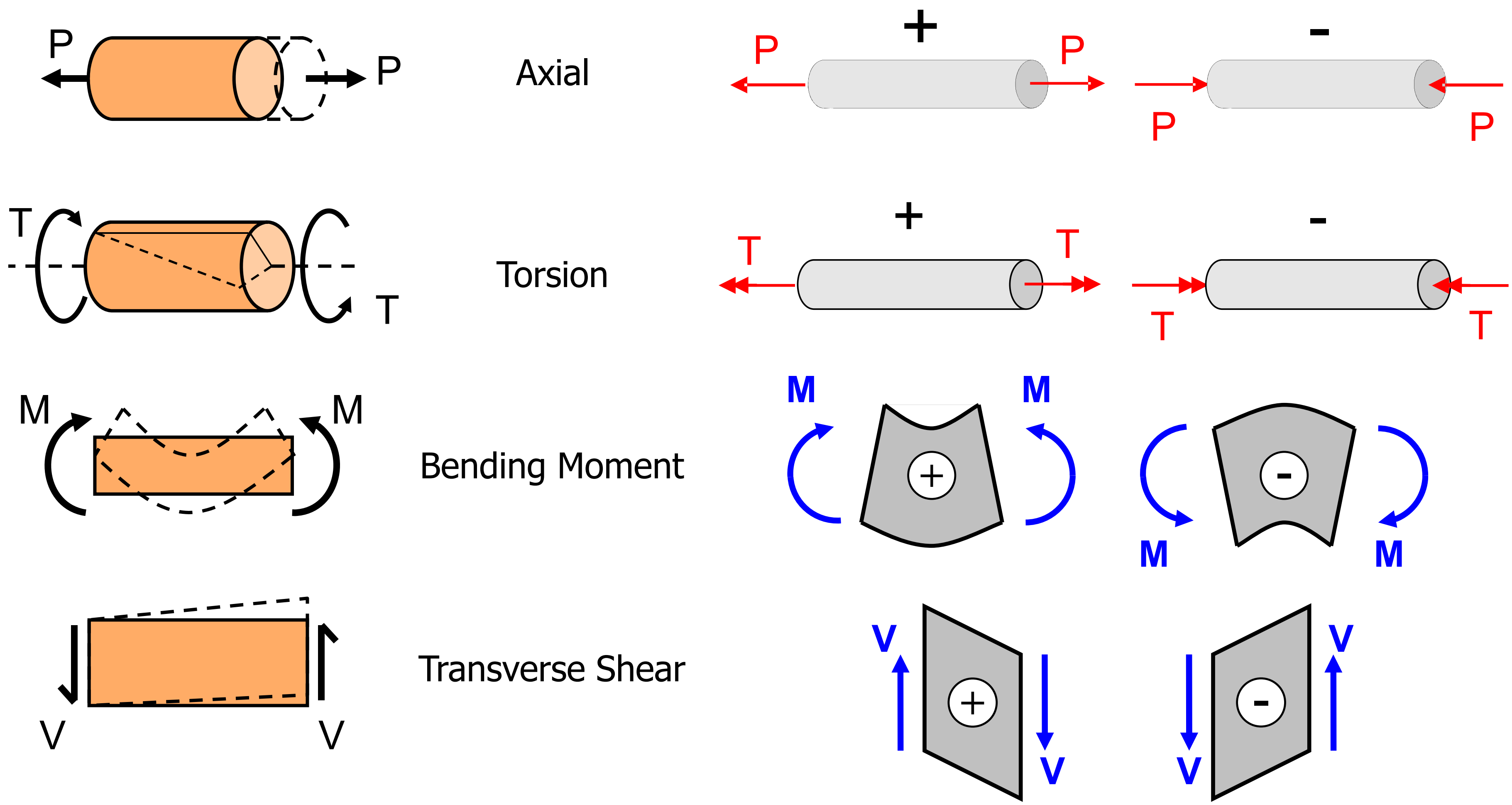


TUDelft

Force-Stress-Displacement Relations

		Stress	Deformation
	Axial	$\sigma = \frac{P}{A}$	$\delta = \frac{PL}{EA}$
	Torsion (circular)	$\tau = \frac{TR}{J}$	$\theta = \frac{TL}{GJ}$
	Torsion (thin-walled)	$\tau = \frac{T}{2tA_m}$	$\theta = \frac{TL}{4A_m^2 G} \cdot \int_0^{L_m} \frac{1}{t} ds$
	Bending Moment	$\sigma = \frac{My}{I}$	$EI \frac{d^2 v}{dz^2} = -M$
	Transverse Shear	$\tau = \frac{VQ}{It}$	Negligible for long beams (moment deformation dominates)

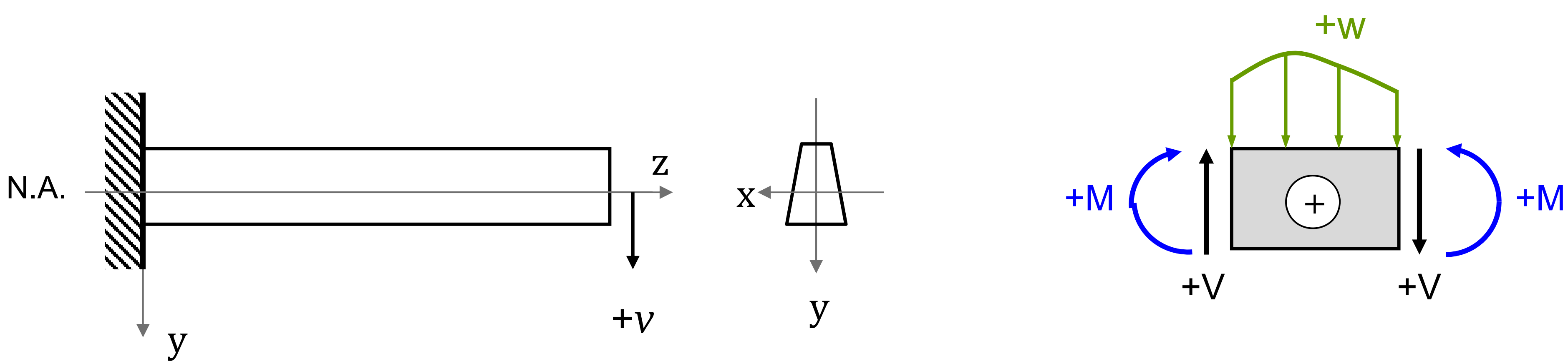
Sign Convention



Sign Convention

Beam Coordinate System

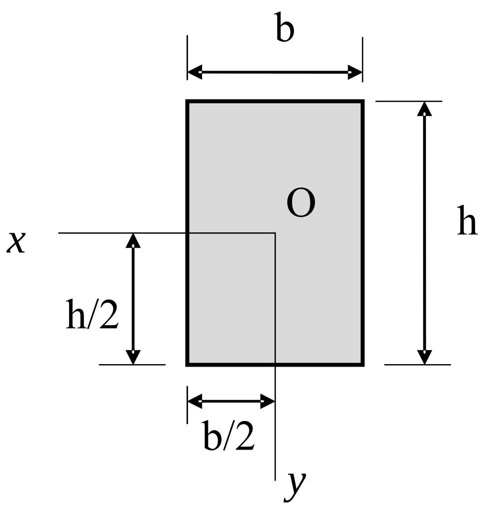
- The y-axis points downwards such that a positive moment results in positive (tensile) stress for positive y values
- Beam deflections are positive in the positive y-direction
- Positive distributed loads result in positive deflections



NOTE: The text book defines y and $+v$ as upwards!

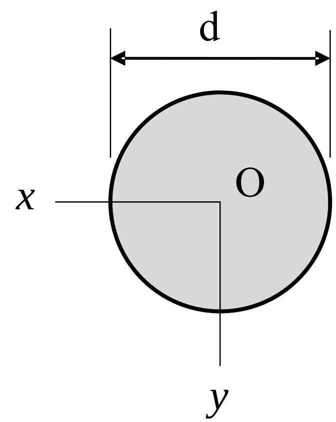
Section Properties

Solid Rectangular Section



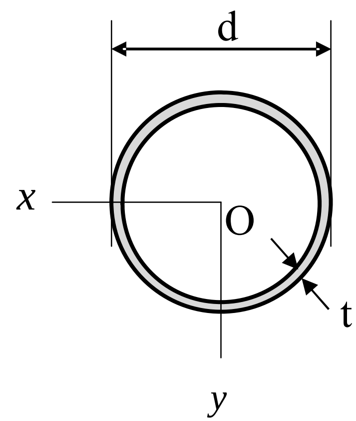
$$I_x = \frac{bh^3}{12}$$

Solid Circular Section



$$I_x = \frac{\pi d^4}{64} \qquad J = \frac{\pi d^4}{32}$$

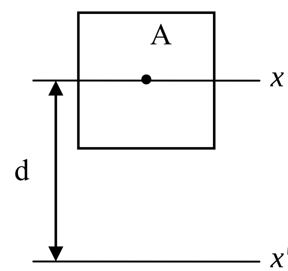
Thin-walled Circular Section



$$I_x = \frac{\pi t d^3}{8} \qquad J = \frac{\pi t d^3}{4}$$

$$I_{x'} = I_x + A \cdot d^2$$

Parallel axis theorem



Beam Deflections

Need to Integrate the Moment-Curvature Relationship

$$EI\frac{d^2v}{dz^2} = -M$$

Direct Integration

$$\iint \frac{-M}{EI}$$

Express M using
Macaulay Step Functions

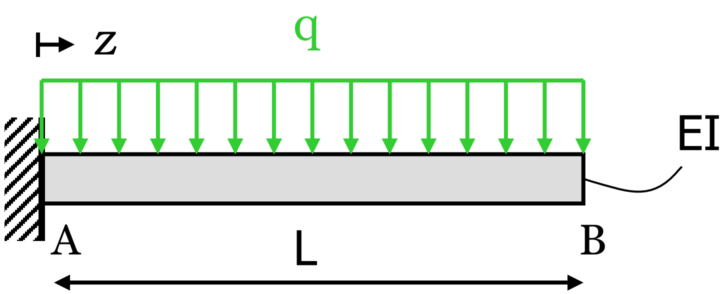
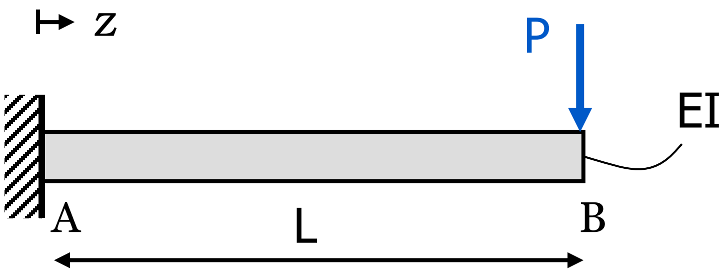
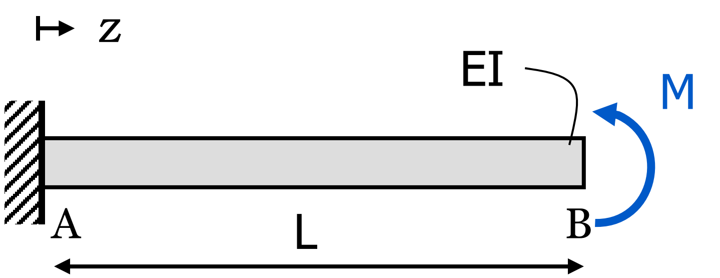
$$f_n(x) = [x-a]^n$$

$$\iint \frac{-M}{EI}$$

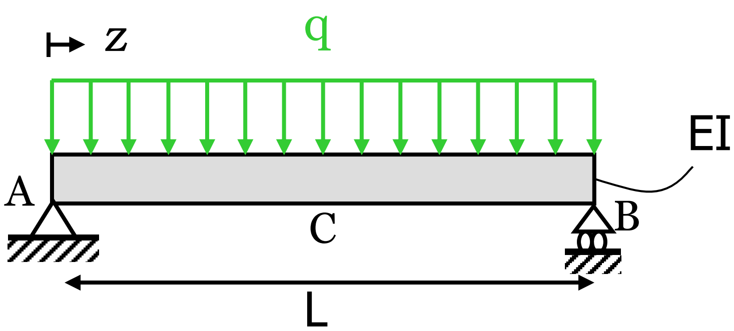
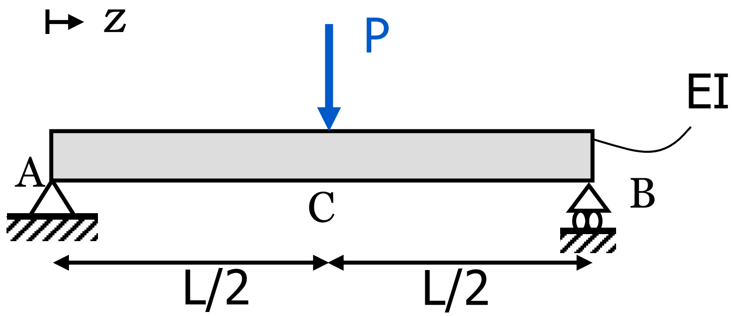
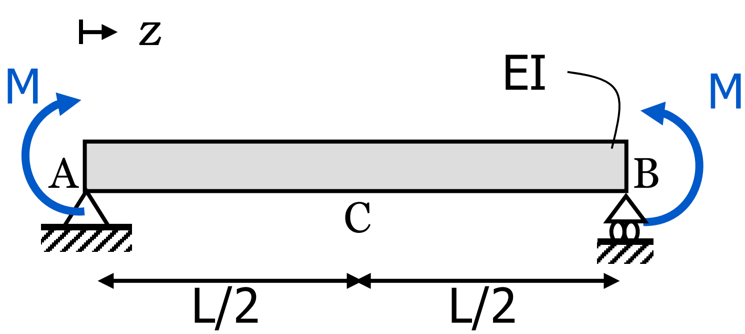
Divide Problem
using Superposition

Use standard/handbook
solutions

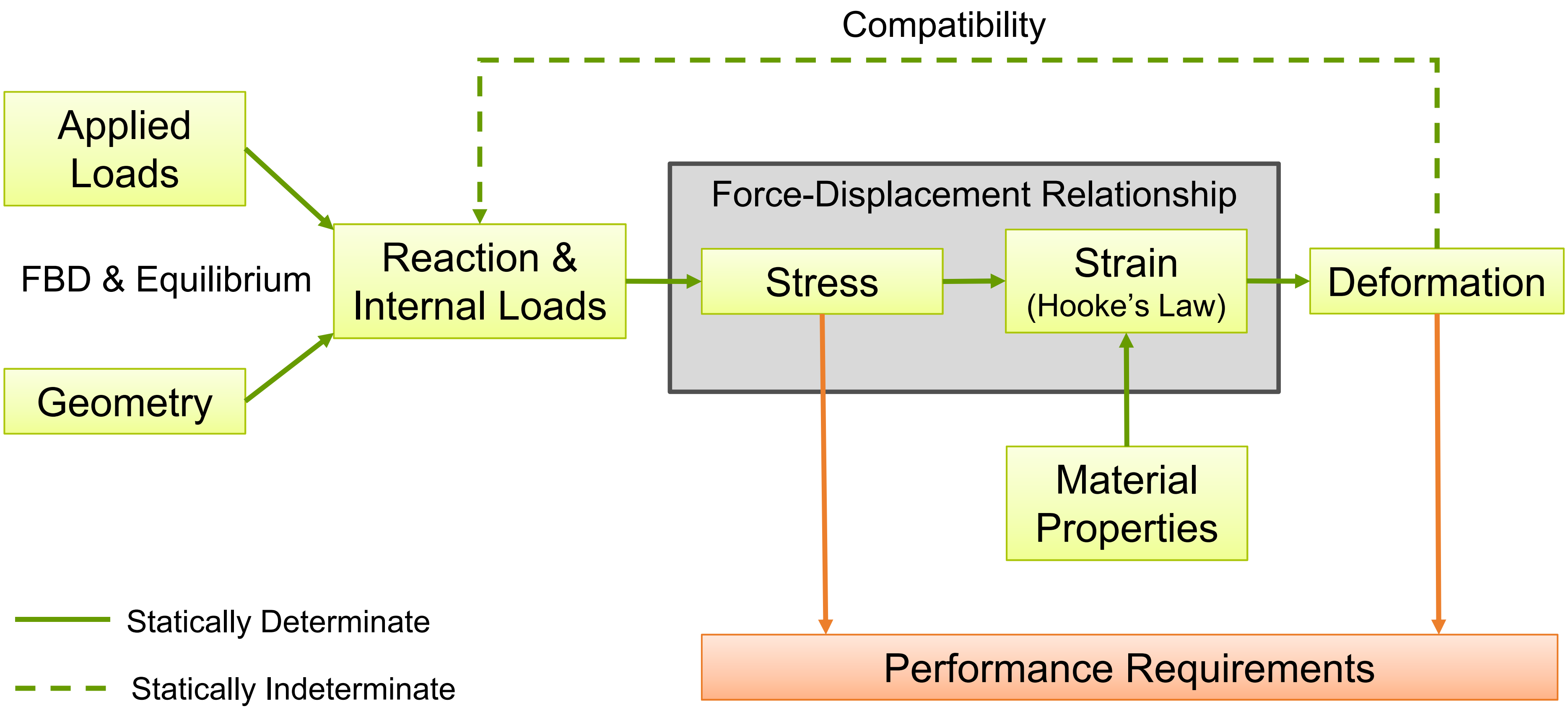
Cantilever Beam Standard Solutions

	Deflection (at B)	Slope (at B)
	$v_B = \dfrac{qL^4}{8EI}$	$\theta_B = \dfrac{qL^3}{6EI}$
	$v_B = \dfrac{PL^3}{3EI}$	$\theta_B = \dfrac{PL^2}{2EI}$
	$v_B = -\dfrac{ML^2}{2EI}$	$\theta_B = -\dfrac{ML}{EI}$

Simply Supported Beam Standard Solutions

	Deflection	Slope
	$v_C = \dfrac{5qL^4}{384EI}$	$\theta_{A,B} = \dfrac{qL^3}{24EI}$
	$v_C = \dfrac{PL^3}{48EI}$	$\theta_{A,B} = \pm\dfrac{PL^2}{16EI}$
	$v_C = \dfrac{ML^2}{8EI}$	$\theta_C = 0$

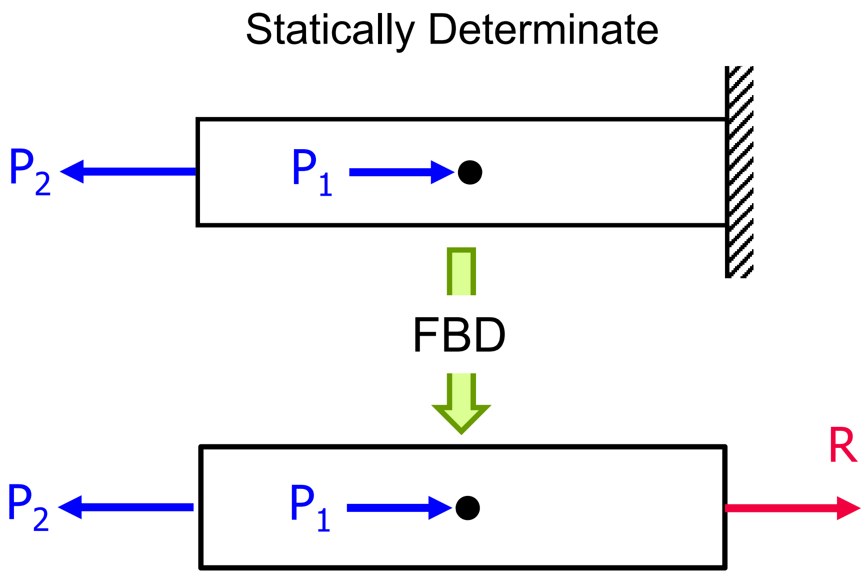
Anatomy of a Solid Mechanics Problem

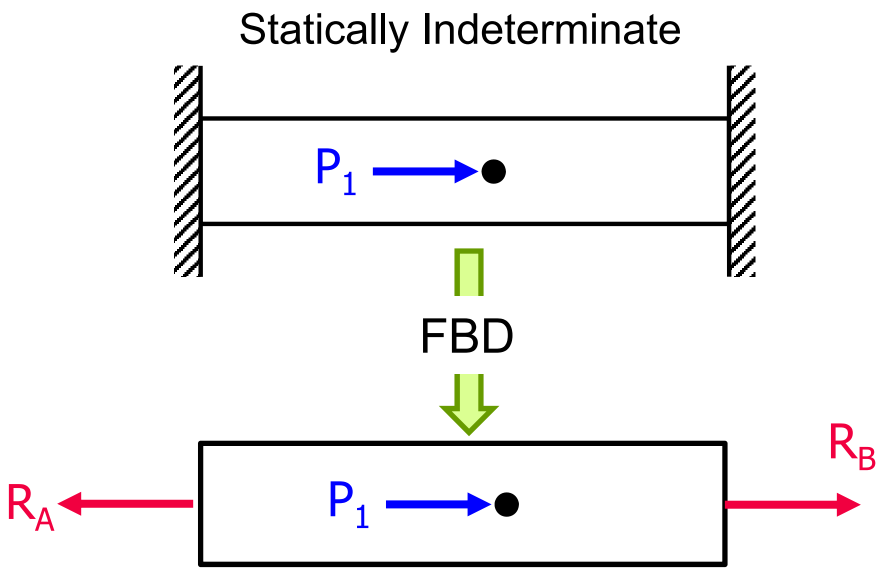


TU Delft

Compatibility

- Compatibility is needed for statically indeterminate problems
  - Too many supports/reaction forces to be determined by equilibrium

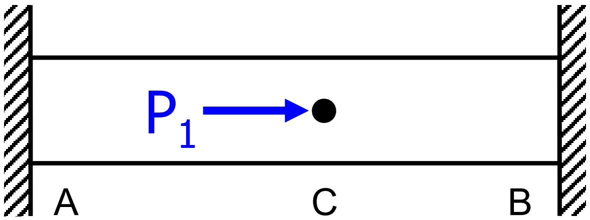




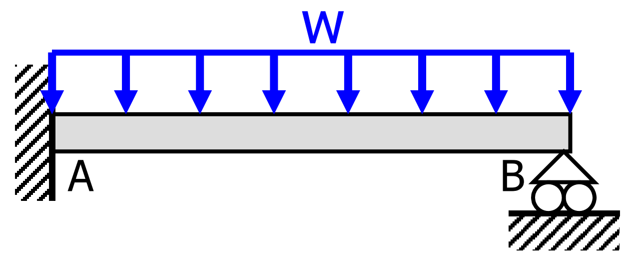
We remove support in FBD, but we need to maintain its constraint!

Compatibility

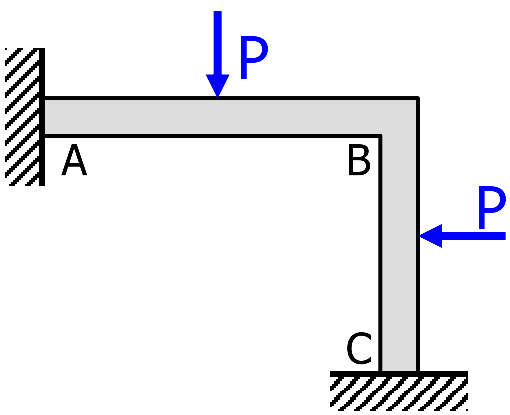
Some Examples



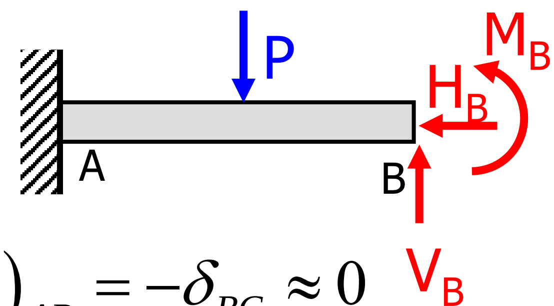
$$\delta_{AB} = 0 \Rightarrow \delta_{AC} + \delta_{CB} = 0$$



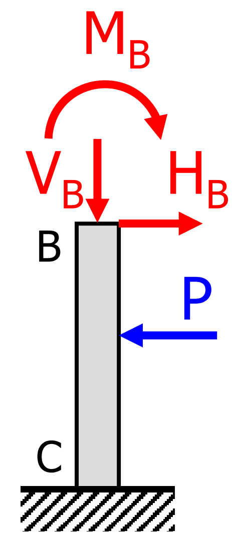
$$v_B = 0$$



=



+



$$\left(v_B\right)_{AB} = -\delta_{BC} \approx 0$$

$$\left(v_B\right)_{BC} = \delta_{AB} \approx 0$$

$$\left(\theta_B\right)_{AB} = \left(\theta_B\right)_{BC}$$

Compatibility

Real World Examples

TUDelft

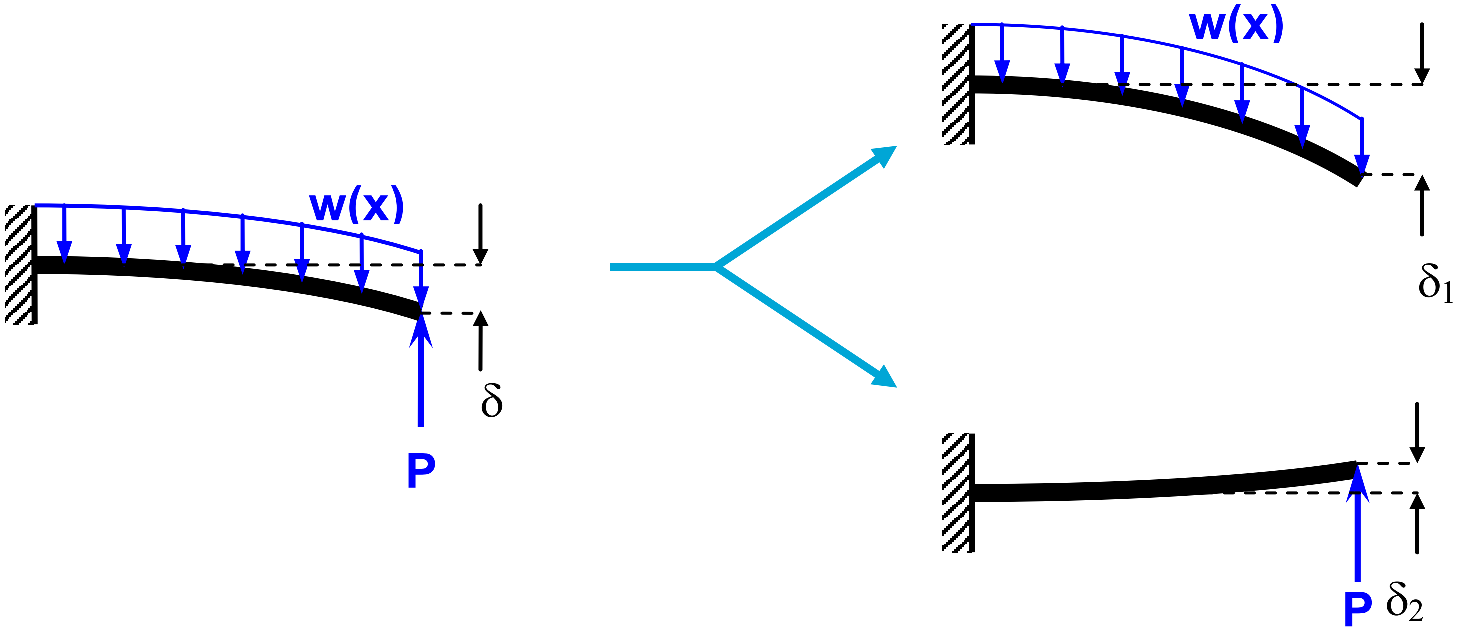
Compatibility

Real World Examples

Superposition

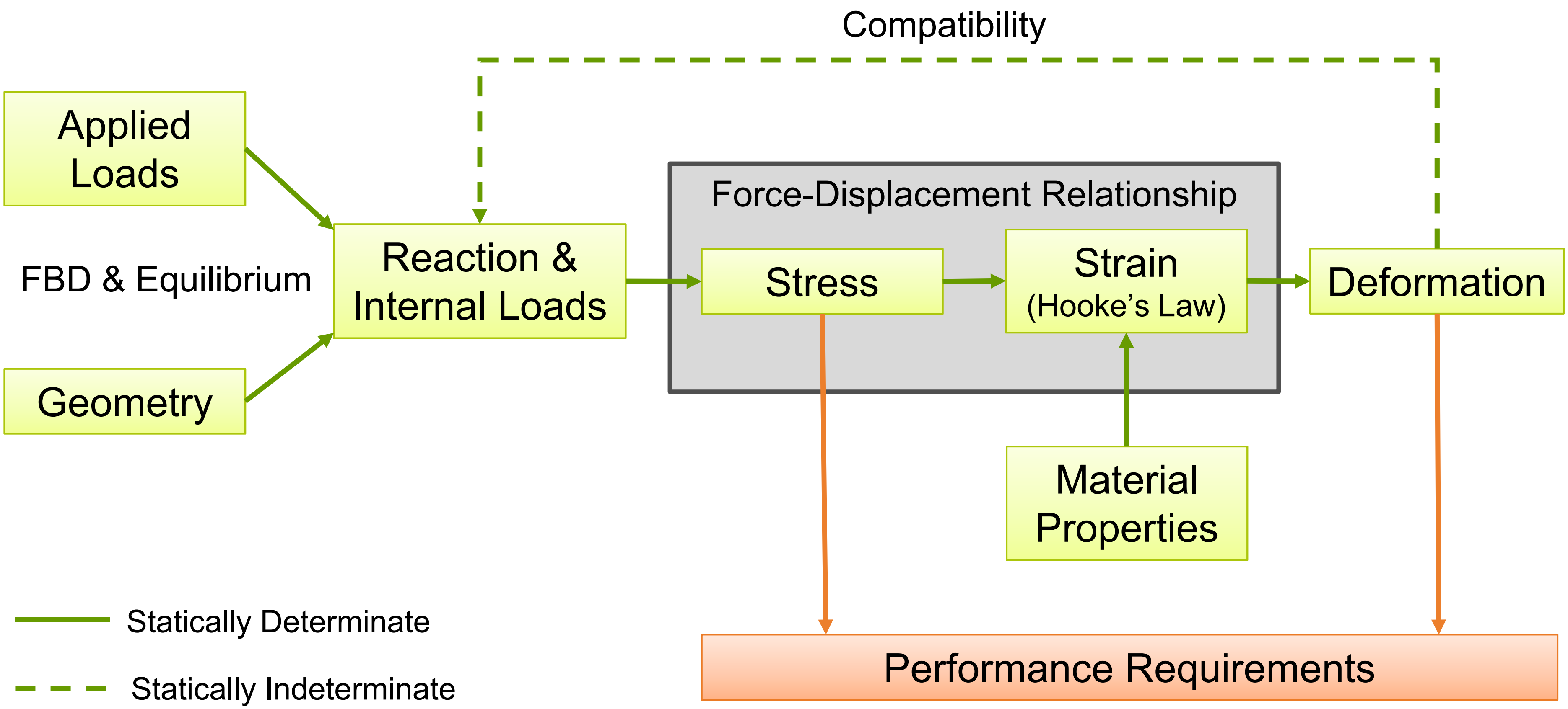
Difficult problem

Multiple simple problems



$$\delta = \delta_1 + \delta_2$$

Anatomy of a Solid Mechanics Problem

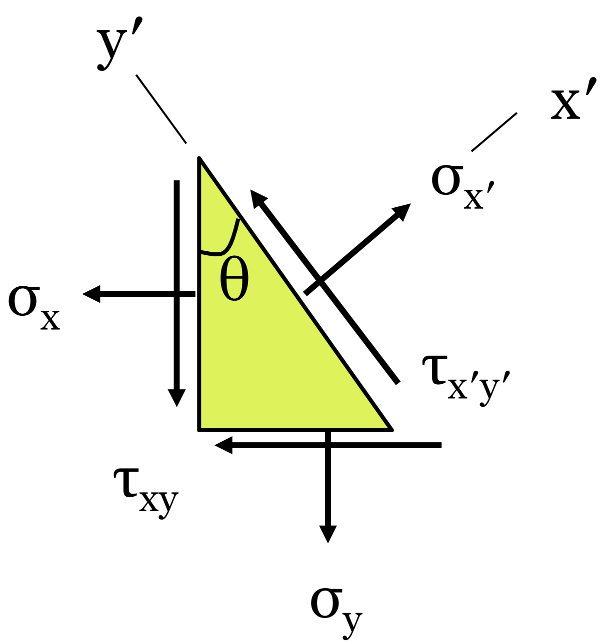


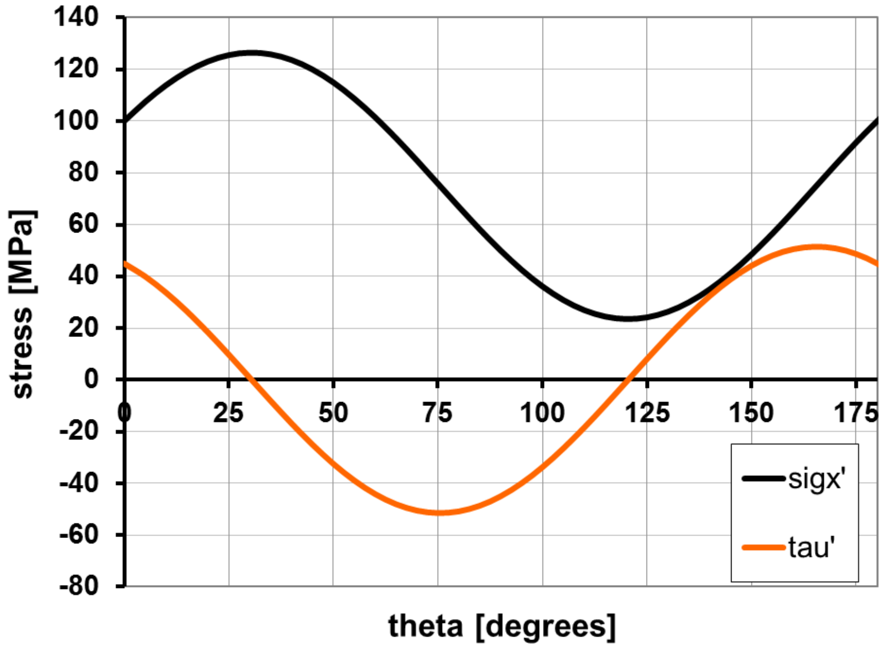
TU Delft

Stress Transformations

Towards Failure Analysis

- Complex stress state including many components of shear and normal stress possible
- Stress state at a point depends on orientation



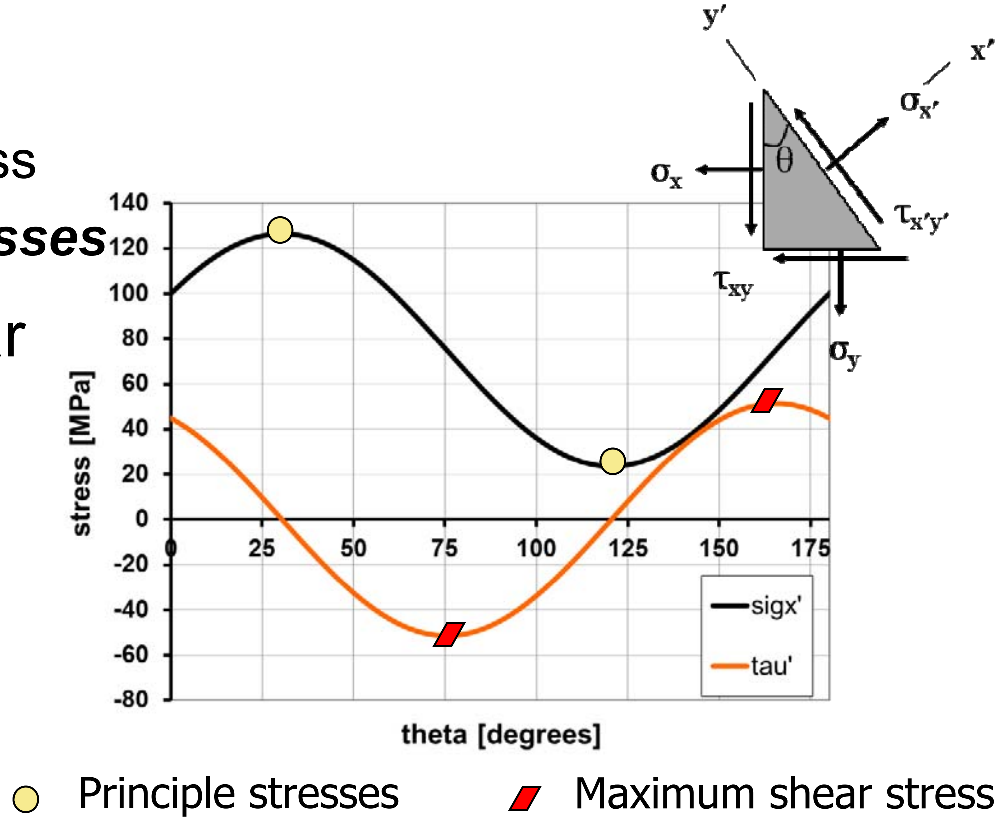


TUDelft

Stress Transformations

Towards Failure Analysis

- Orthogonal set of planes exist at every point where shear stress is zero
  - Principle stress planes
  - Max and min normal stress
  - Known as ***Principle Stresses***
- Plane of maximum shear stress
  - Inclined 45° from principle stress plane
  - Normal stresses can be non-zero



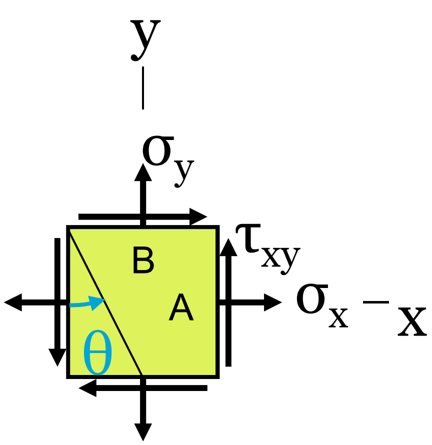
TUDelft

Stress Transformations

Towards Failure Analysis

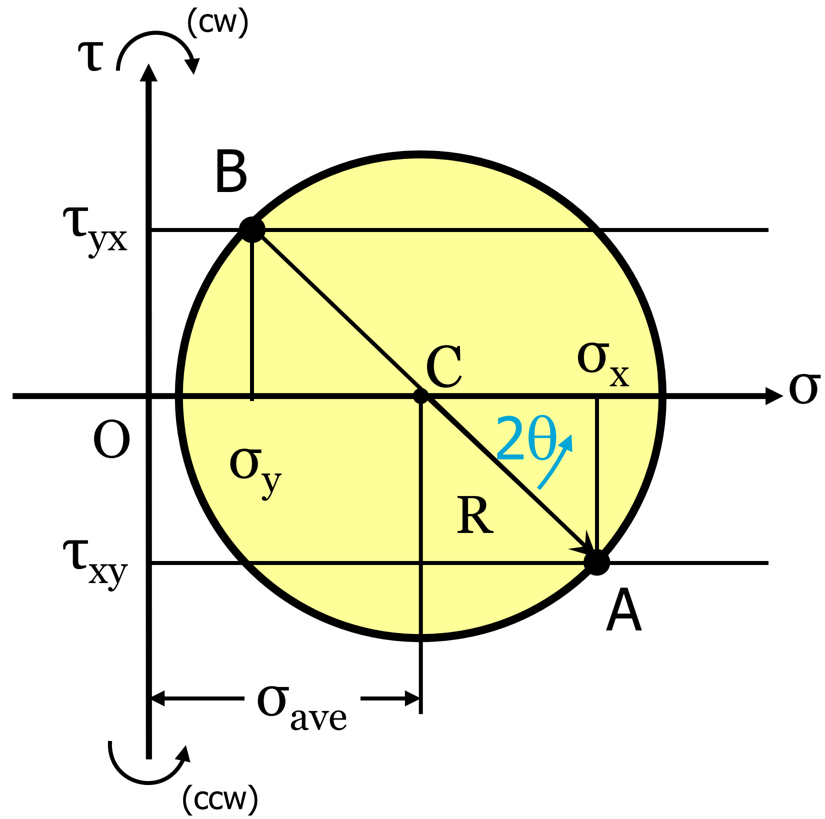
- Stress transformations can be described by equation of a circle
  - Mohr Circle

$$\left(\sigma - \sigma_{ave}\right)^2 + \tau^2 = R^2$$



$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2}$$

$$R = \sqrt{\left(\sigma_x - \sigma_{ave}\right)^2 + \tau_{xy}^2}$$



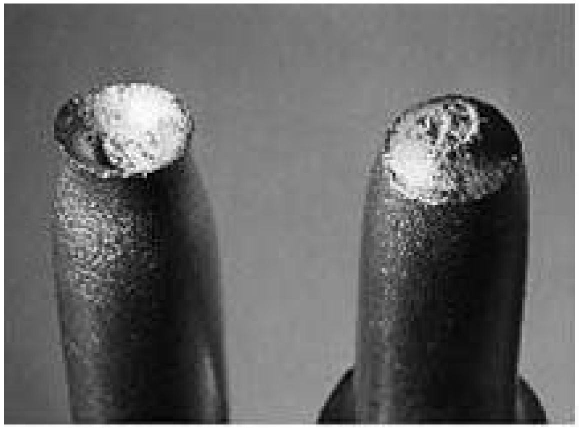
TUDelft

Stress Transformations

Towards Failure Analysis

Why do we care about stress transformations?

Ductile failure



- Dictated by maximum shear stresses

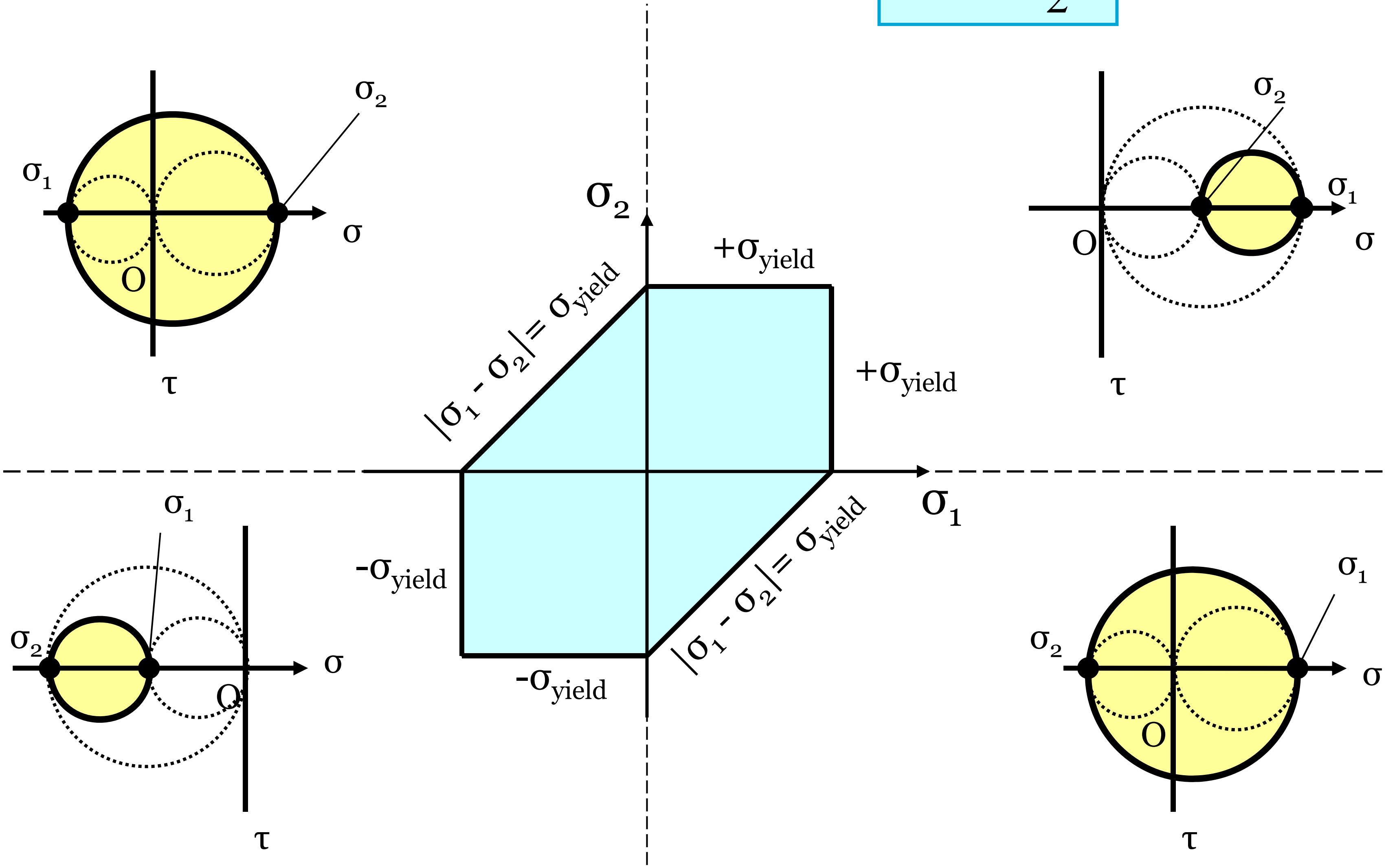
Brittle failure

- Dictated by maximum normal stresses

TU Delft

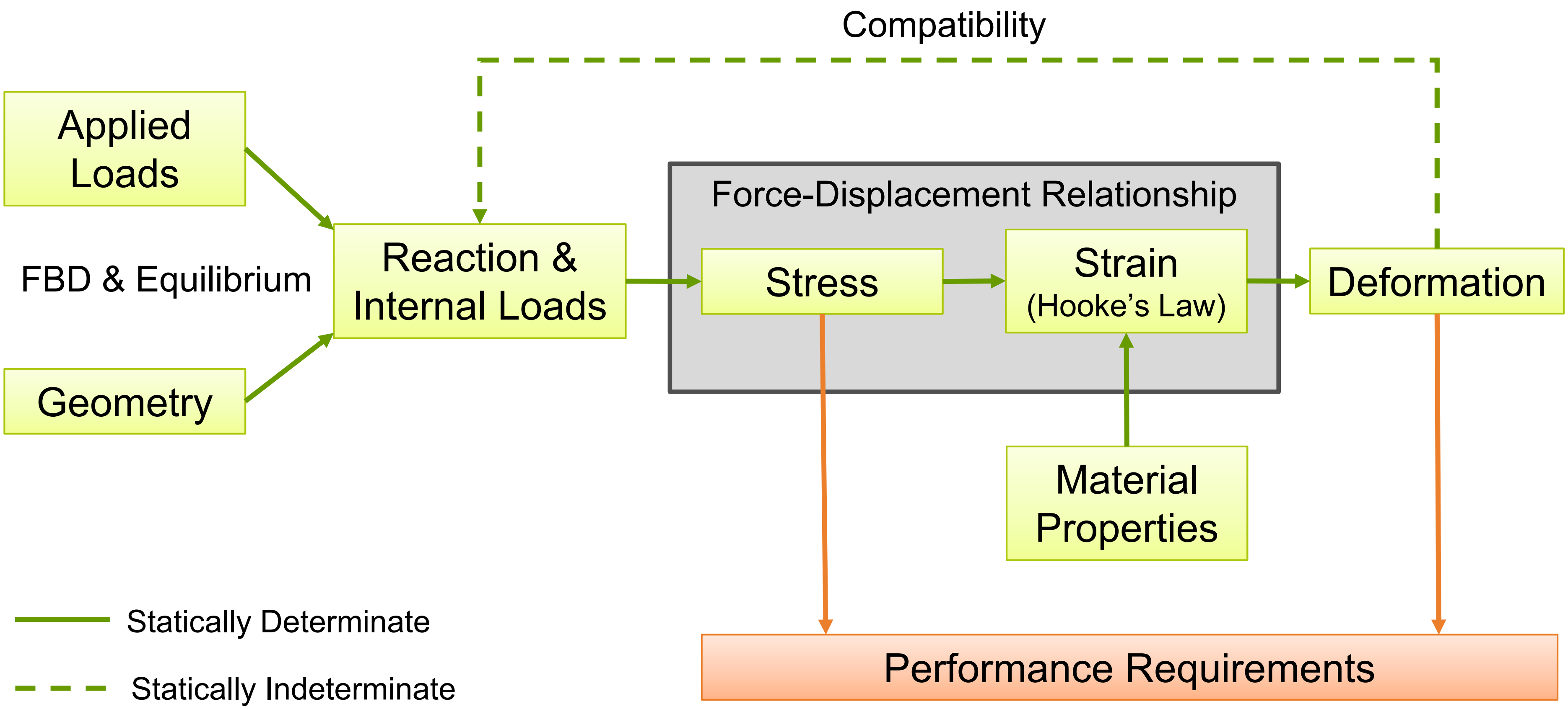
Tresca Yield Criterion

$$\tau_{max} = \frac{\sigma_{yield}}{2}$$



TUDelft

Anatomy of a Solid Mechanics Problem



TU Delft

Solving Solid Mechanics Problems

General Procedure

1. Draw FBD!
   - Establish sign convention
2. Equilibrium Equations
   - Determine if statically determinate or indeterminate
3. Compatibility Conditions
4. Force-Displacement Relations
   - If force-displacement relation is unknown, use Hooke's Law to relate stress and strain
5. Solve
   - Desired reaction forces
   - Desired internal stresses
   - Desired displacement

Solving Solid Mechanics Problems

General Procedure

- Problem may have a design element
  - Maximum load structure can carry
  - Minimum span, height, or other geometrical parameter
- Design element centres around structural requirement
  - Maximum allowable stress
  - Maximum allowable deflection
  - Principle stresses from Mohr Circle
  - Failure criteria

Solving Solid Mechanics Problems

Recommendations

- Describe your understanding of the problem in words first
    - Is the problem statically determinate or indeterminate?
    - What is the compatibility condition?
    - Is there any trick to the problem?
    - How does the design constraint affect the problem?
- **Always** look at your final numerical answer and reiterate the meaning of the sign
    - Elongation or contraction?
    - Tension or compression?
    - Clockwise or counter clockwise rotation?
- Does the answer make sense, and if not, describe where you think you went wrong

Exam Hints

- There is always a Mohr Circle question!
- Difficult questions have a twist on a concept or condition you have analysed before: identify it and describe it

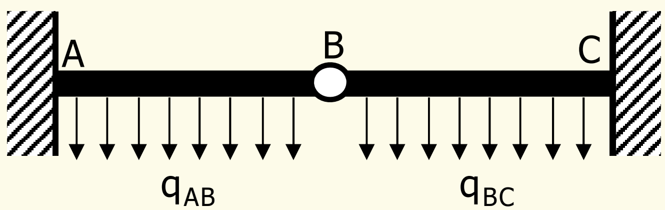
TU Delft

Past Exam Questions

exam question (1)

Two identical thin walled beams (AB & BC) are connected at B via a hinge. At A and at C the beams are clamped. Beam AB is loaded in bending by the load q_{AB} and beam BC is loaded in bending by the load q_{BC}.

Assume:
$(EI)_{AB}=(EI)_{BC}=EI$
$L_{AB}=L_{BC}=L$



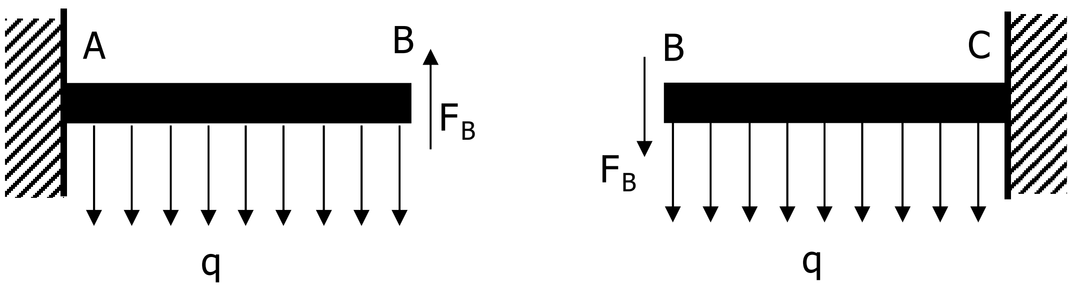
a) What is the deflection of point B when $q_{AB}=q_{BC}=q$?
b) What is the deflection of point B when $q_{AB}=2q_{BC}=2q$?

What is the twist?

Hinge mid-span. Hinges can carry shear, but not moments, therefore the moment is zero midspan.
Problem is both statically determinate and indeterminate

a) What is the deflection of point B when $q_{AB}=q_{BC}=q$?

(I) FBD: (Reactions at fixed boundary conditions not shown)



Problem is symmetric!

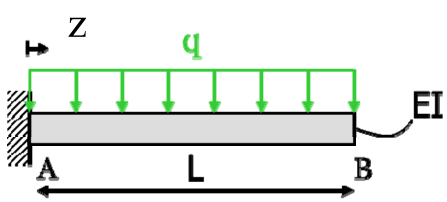
$$\therefore F_B = 0$$

(since FBD must be symmetric, and F_B must be equal and opposite on opposing faces)

Since we only want displacement, we can recognize the standard case and skip directly to solving for displacement:

(IV) Force-Displacement:

Standard Solution:



$$v_B = \frac{qL^4}{8EI}$$

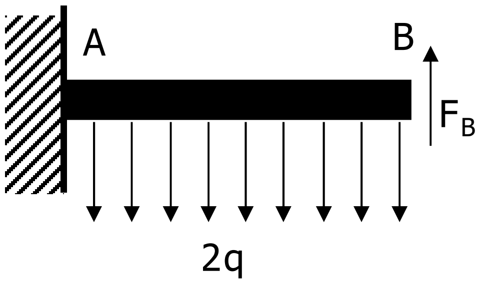
$$\theta_B = \frac{qL^3}{6EI}$$

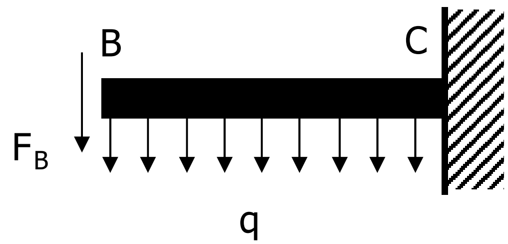
$$\therefore v_B^{\downarrow +} = \frac{qL^4}{8EI}$$

Deflection is downwards, which is logical for a distributed load acting downwards

b) What is the deflection of point B when $q_{AB}=2q_{BC}=2q$?

(I) FBD: (Reactions a fixed boundary conditions not shown)





Problem is not symmetric!

$$\therefore F_B \neq 0$$

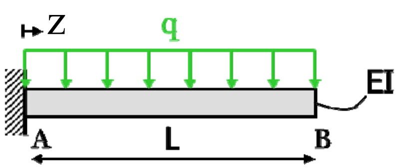
Problem is actually statically indeterminate!

(III) Compatibility:

Beam deflection at B must be the same for both beams $\therefore (v_B)_{AB} = (v_B)_{BC}$

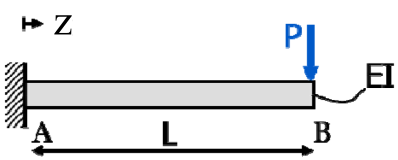
(IV) Force-Displacement:

Standard Solutions:



$$v_B = \frac{qL^4}{8EI}$$

$$\theta_B = \frac{qL^3}{6EI}$$



$$v_B = \frac{PL^3}{3EI}$$

$$\theta_B = \frac{PL^2}{2EI}$$

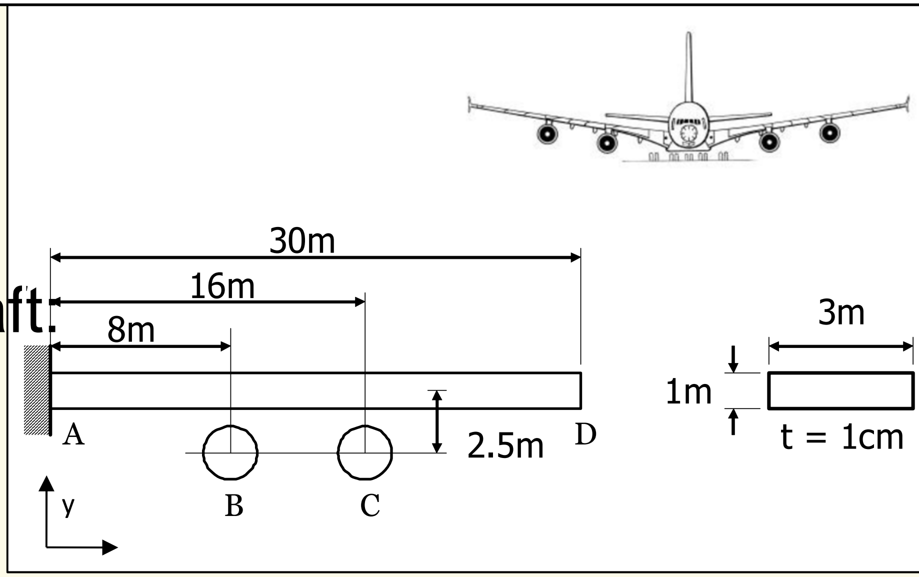
$$\therefore \left(\frac{2qL^4}{8EI} - \frac{F_B L^3}{3EI}\right) = \left(\frac{qL^4}{8EI} + \frac{F_B L^3}{3EI}\right)$$

$$\Rightarrow F_B = \frac{3}{16} qL$$

$$\left(v_B^{\downarrow +}\right)_{AB} = \frac{2qL^4}{8EI} - \frac{F_B L^3}{3EI} = \frac{2qL^4}{8EI} - \frac{\frac{3}{16}qL^4}{3EI} = \frac{3qL^4}{16EI}$$

Deflection is downwards, which is logical for a distributed load acting downwards

exam question(2)



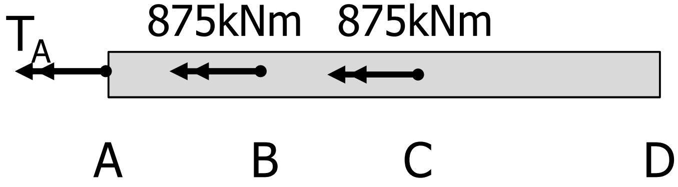
Consider the Airbus A380 aircraft:

- The engines of the A380 are suspended below the wing, having a distance of 2.5 m between the centre of the wing box and the engine thrust line. This 2.5 m distance results in the engines imparting a torsional moment into the wingbox. Each of the A380 engines can generate approximately 350kN of thrust. The inboard engine is located 8m away from the wing root and the outboard engine is located 16m from the wing root.
- The wingbox is approximated as a constant cross section (figure above) with a fixed support condition at the wing-to-fuselage connection.
  a. Determine the maximum shear stress in the wingbox at the wing root (A) and the angle of twist relative to point A at each engine location in the wing (B and C) DUE TO THE TORQUE OF THE ENGINES ONLY. (G = 26 GPa)
  b. Part b is later (too much text for here!)
  c. Part c is later (too much text for here!)

What is the twist? Twist is in part b, will get to it later

a. Determine the maximum shear stress in the wingbox at the wing root (A) and the angle of twist relative to point A at each engine location in the wing (B and C) DUE TO THE TORQUE OF THE ENGINES ONLY. (G = 26 GPa)

(I) FBD:



Engine torque = 350kN x 2.5m = 875kNm

(II) Equilibrium: $\sum T \Rightarrow T_A = -875 - 875 = -1750 kNm$ Statically determinate!

(IV) Solve: Shear Stress

Thin walled torque box: $\therefore \tau = \dfrac{T}{2tA_m}$

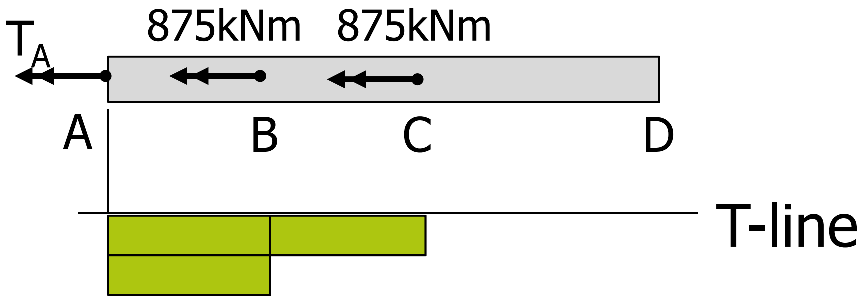
@A:

$$A_m \approx 3m \times 1m = 3m^2$$
$$t = 1cm = 0.01m$$
$$T = -1750 kNm$$

$$\therefore \tau_A = \frac{-1750 kNm}{2(0.01m)(3m^2)} = -29.2 MPa$$

a. Determine the maximum shear stress in the wingbox at the wing root (A) and the angle of twist relative to point A at each engine location in the wing (B and C) DUE TO THE TORQUE OF THE ENGINES ONLY. (G = 26 GPa)

(I) FBD:



$$T_{CD} = 0\,kNm$$

$$T_{BC} = -875\,kNm$$

$$T_{AB} = -1750\,kNm$$

(IV) Solve: Angle of Twist

Thin walled torque box: $$\theta = \frac{TL}{4GA_m^2}\oint \frac{ds}{t}$$

$$A_m = 3\,m^2$$

$$\oint \frac{ds}{t} = 2\frac{1}{0.01} + 2\frac{3}{0.01} = 800\,(m/m)$$

$$\phi_B = \frac{-1750 \cdot 10^3 \cdot 8}{4 \cdot 26 \cdot 10^9 \cdot 9}800 = -0.01197\,rad \approx -0.6856\,degrees$$

$$\phi_C = \phi_B + \frac{-875 \cdot 10^3 \cdot 8}{4 \cdot 26 \cdot 10^9 \cdot 9}800 = -0.01795\,rad \approx -1.0283\,degrees$$

b. The expression for the deflection of the wing due to distributed lift over the wing and engine weight is given below. Based on that expression (where x is distance from the wing root in meters, and force units in the expression are given in kN, and positive v is upwards in the direction of lift), plot the shear force diagram for the wing and the distributed load diagram (do not plot the bending moment diagram!)

$$\text{for } 0 \le x \le 8: \quad EIv = \frac{55}{12}x^4 - \frac{1205}{3}x^3 + 85660x^2$$

$$\text{for } 8 \le x \le 16: \quad EIv = \frac{55}{12}x^4 - \frac{1235}{3}x^3 + 87100x^2 - 1920x + 5120$$

$$\text{for } 16 \le x \le 30: \quad EIv = \frac{110}{1680}(30-x)^5 + 222445x - \frac{3868444}{3}$$

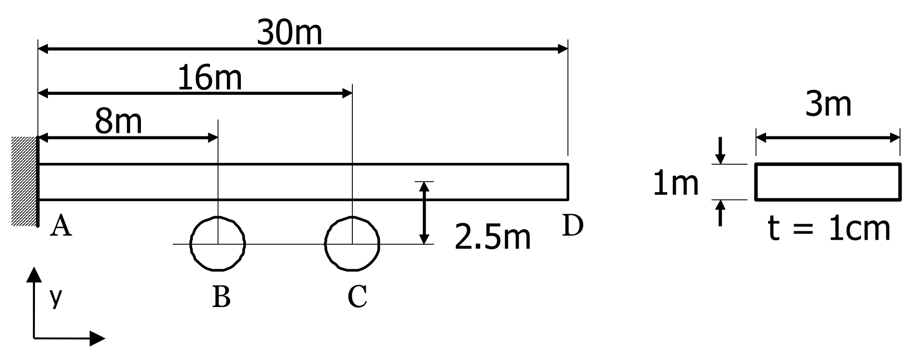
What is the twist? You can differentiate deflection to get loading

for $v^{\downarrow +}$

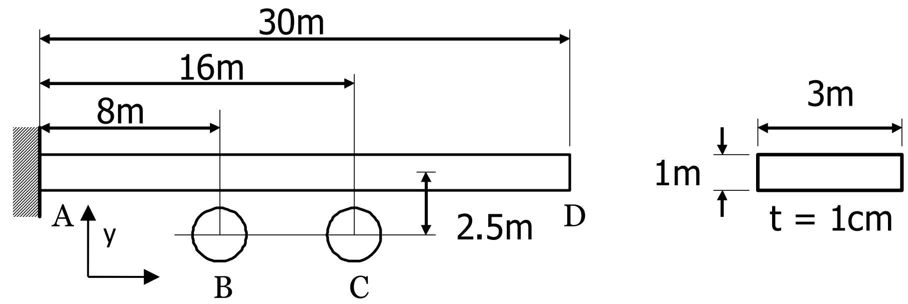
$$EIv'''' = w(x)$$
$$EIv''' = -V(x)$$
$$EIv'' = -M(x)$$

for $v^{\uparrow +}$

$$EIv'''' = -w(x)$$
$$EIv''' = V(x)$$
$$EIv'' = M(x)$$



b. Plot the shear force diagram for the wing and the distributed load diagram



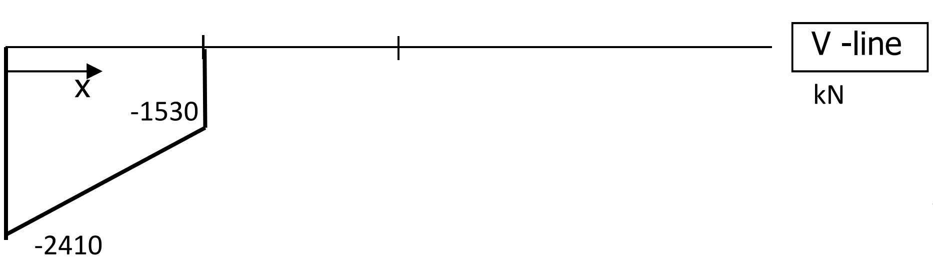
for $0 \le x \le 8$: $$EIv = \frac{55}{12}x^4 - \frac{1205}{3}x^3 + 85660x^2$$

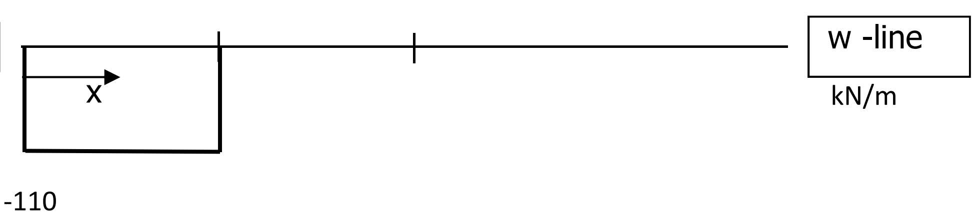
$$\Rightarrow EIv' = \frac{55}{3}x^3 - 1205x^2 + 171320x$$

$$\Rightarrow EIv'' = 55x^2 - 2410x + 171320$$

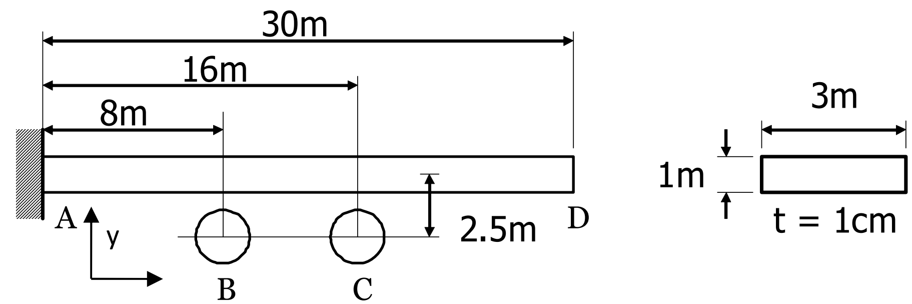
$$\Rightarrow EIv''' = 110x - 2410 = V(x)$$

$$\Rightarrow EIv'''' = 110 = -w(x)$$





b. Plot the shear force diagram for the wing and the distributed load diagram



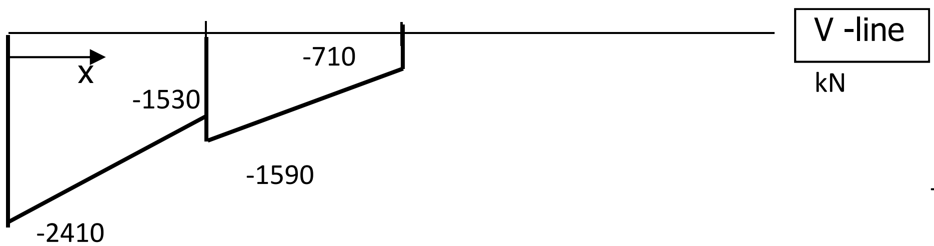
$$\text{for } 8 \le x \le 16: \quad EIv = \frac{55}{12}x^4 - \frac{1235}{3}x^3 + 87100x^2 - 1920x + 5120$$

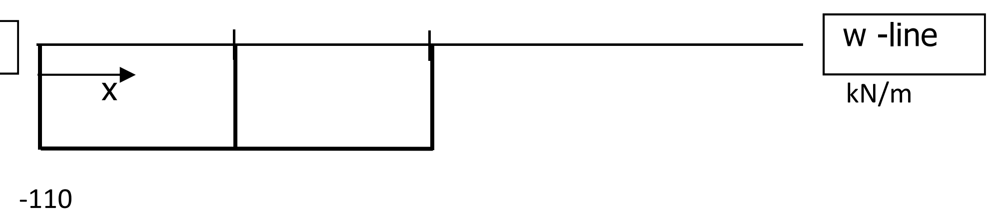
$$\Rightarrow EIv' = \frac{55}{3}x^3 - 1235x^2 + 174200x - 1920$$

$$\Rightarrow EIv'' = 55x^2 - 2470x + 174200$$

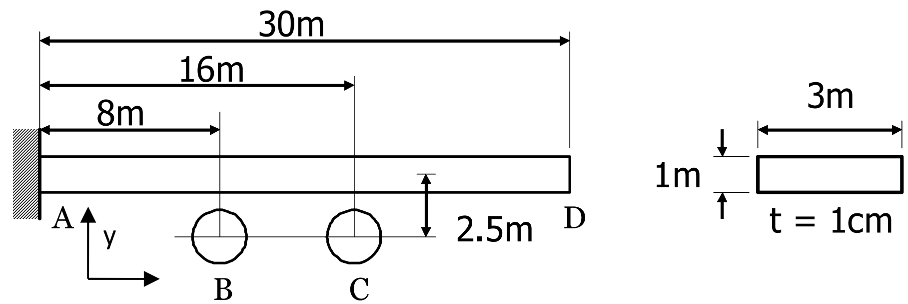
$$\Rightarrow EIv''' = 110x - 2470 = V(x)$$

$$\Rightarrow EIv'''' = 110 = -w(x)$$





b. Plot the shear force diagram for the wing and the distributed load diagram



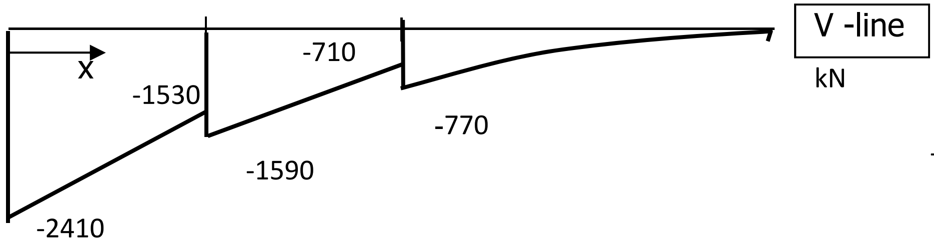
$$\text{for } 16 \le x \le 30: \; EIv = \frac{110}{1680}(30-x)^5 + 222445x - \frac{3868444}{3}$$

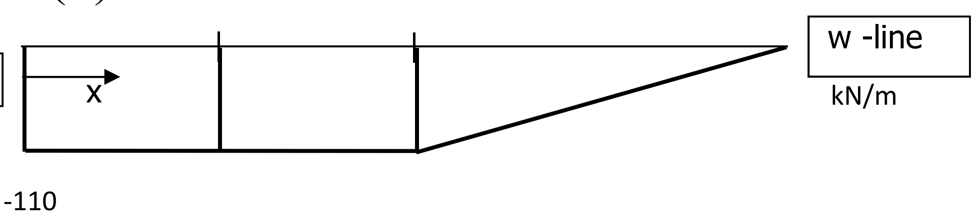
$$\Rightarrow EIv' = -1 \cdot \frac{110}{336}(30-x)^4 + 222445$$

$$\Rightarrow EIv'' = \frac{110}{84}(30-x)^3$$

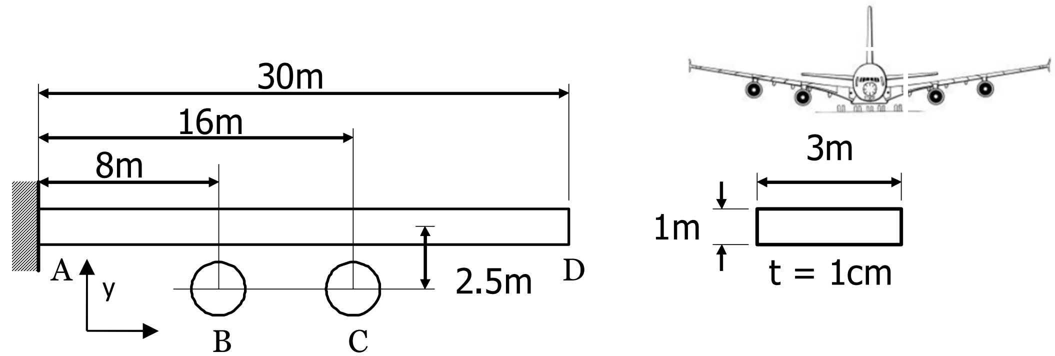
$$\Rightarrow EIv''' = -1 \cdot \frac{55}{14}(30-x)^2 = V(x)$$

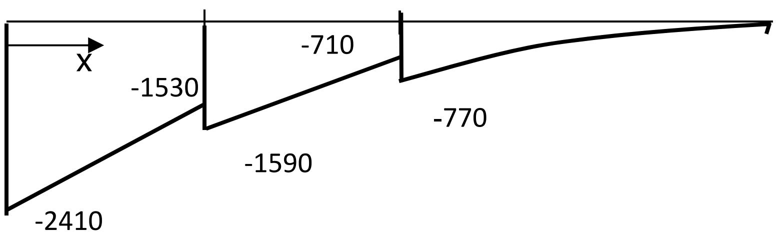
$$\Rightarrow EIv'''' = \frac{55}{7}(30-x) = -w(x)$$

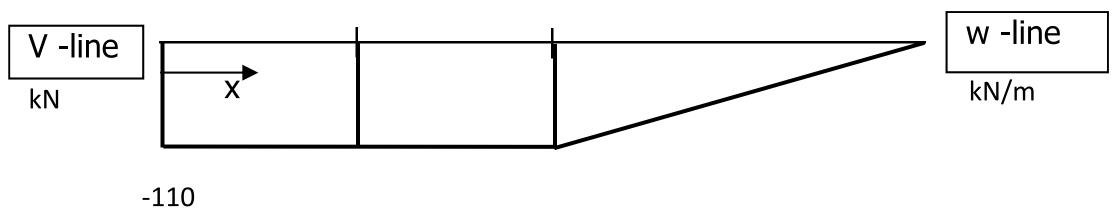




c. Based on your previous analysis, determine what the weight of each engine is (Point loads at B and C in kN) and determine the total lift produced by the wing (resultant of distributed loads)







Engine weight : -1530 - (-1590) = 60kN (positive is downwards, makes sense!)

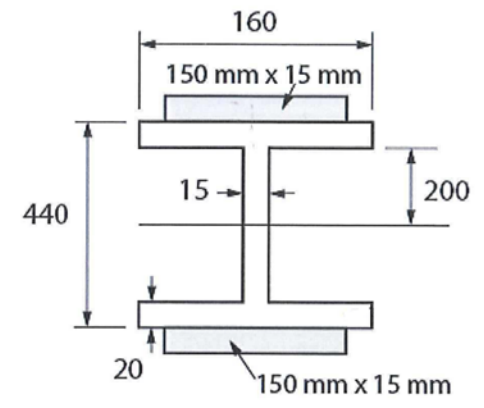
Lift : (-110)(8) + (-110)(8) + (0.5)(-110)(14) = -2530 kN

(negative is upwards, makes sense!)

(total thrust for one wing is (2)(350kN) = 700 kN, thus T/W < 1, which makes sense!)

exam question(3)

$$\sigma_{steel_{\max}} = 150MPa$$
$$\tau_{steel_{\max}} = 80MPa$$
$$\tau_{adhesive_{\max}} = 5MPa$$



A steel I-beam is to be reinforced by bonding plates of the same material to the top and bottom flanges as shown in the figure to the right. Given the above allowables:

a) Determine the maximum bending moment that can be carried by the unreinforced beam.

b) Determine the maximum shear force that can be carried by the unreinforced beam.

c) Determine the maximum bending moment that can be carried by the reinforced beam.

d) Determine the maximum shear force that can be carried by the reinforced beam.

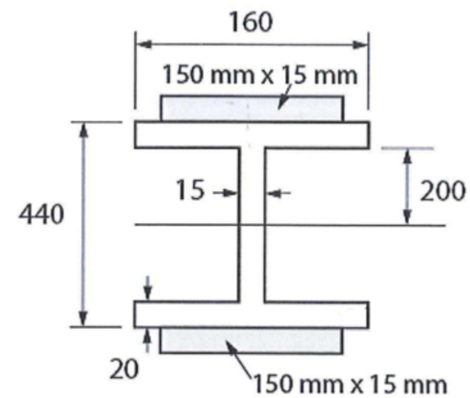
What is the twist?

For reinforced beam, two conditions need to be checked for max shear force (part d): Transverse shear in steel I-beam and shear along adhesive bondline

$$\sigma_{steel_{\max}} = 150MPa$$

$$\tau_{steel_{\max}} = 80MPa$$

$$\tau_{adhesive_{\max}} = 5MPa$$



A steel I-beam is to be reinforced by bonding plates of the same material to the top and bottom flanges as shown in the figure.

a) Determine the maximum <u>bending moment</u> that can be carried by the <u>unreinforced</u> beam.

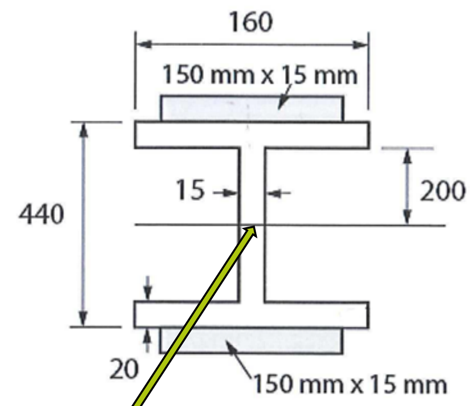
$$I = 2\cdot\left[\frac{1}{12}(160mm)(20mm)^3 + 20mm(160mm)(210mm)^2\right] + \frac{1}{12}(15mm)(400mm)^3 = 3.625\times10^{-4}m^4$$

$$\sigma = \frac{My}{I}, so\, M = \frac{\sigma I}{y} = \frac{(150MPa)\left(3.625\times10^{-4}m^4\right)}{0.220m} = 247.2kNm$$

$$\sigma_{steel_{\max}} = 150MPa$$

$$\tau_{steel_{\max}} = 80MPa$$

$$\tau_{adhesive_{\max}} = 5MPa$$



A steel I-beam is to be reinforced by bonding plates of the same material to the top and bottom flanges as shown in the figure.

b) Determine the maximum shear force that can be carried by the unreinforced beam.

$$\tau = \frac{VQ}{Ib} \; so V = \frac{\tau Ib}{Q}$$

Maximal shear is at the middle.

$$I = 3.625 \times 10^{-4} m^4$$

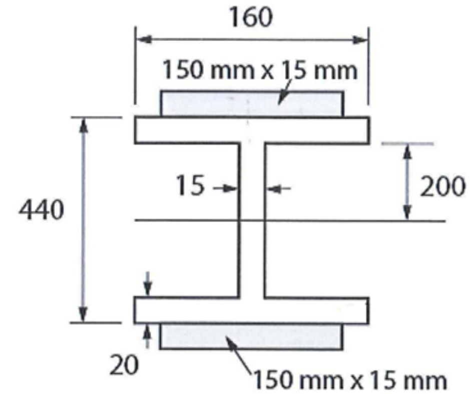
$$Q = (200mm + 10mm) \cdot (160mm \times 20mm) + (100mm) \cdot (200mm \times 15mm) = 9.72 \times 10^{-4} m^3$$

$$V_{max} = \frac{80MPa \cdot 3.625 \times 10^{-4} m^4 \cdot 0.015m}{9.72 \times 10^{-4} m^3} = 447.5kN$$

$$\sigma_{steel_{\max}} = 150MPa$$

$$\tau_{steel_{\max}} = 80MPa$$

$$\tau_{adhesive_{\max}} = 5MPa$$



A steel I-beam is to be reinforced by bonding plates of the same material to the top and bottom flanges as shown in the figure.

c) Determine the maximum <u>bending moment</u> that can be carried by the <u>reinforced</u> beam.

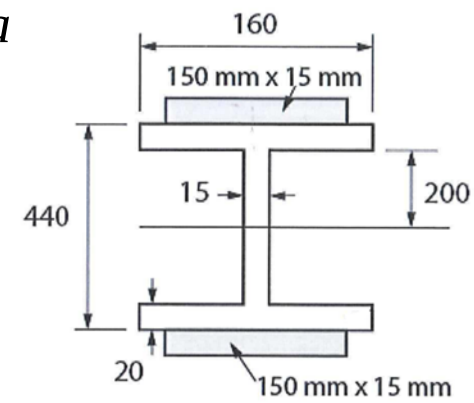
$$I_{reinf} = 3.625\times10^{-4} + 2\left[\frac{1}{12}150mm\cdot(15mm)^3 + (150mm)(15mm)\left(220mm + \frac{15mm}{2}\right)^2\right] = 5.96\times10^{-4}\,m^4$$

$$M_{reinf_{max}} = \frac{150MPa\cdot5.96\times10^{-4}\,m^4}{0.235m} = 380.4kNm$$

$$\sigma_{steel_{\max}} = 150MPa$$

$$\tau_{steel_{\max}} = 80MPa$$

$$\tau_{adhesive_{\max}} = 5MPa$$



A steel I-beam is to be reinforced by bonding plates of the same material to the top and bottom flanges as shown in the figure.

d) Determine the maximum shear force that can be carried by the reinforced beam.

$$V = \frac{\tau I b}{Q}$$

$$I_{reinf} = 5.96 \times 10^{-4} m^4$$

$$Q_{reinf_{max}} = 9.72 \times 10^{-4} + (0.220 + \frac{0.015}{2}) \times 0.150 \times 0.015 = 1.484 \times 10^{-3} m^3$$

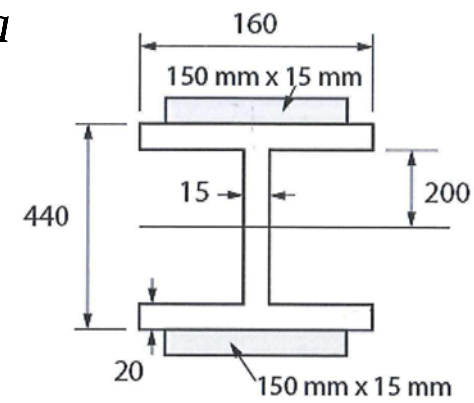
$$V_{reinf_{max}} = \frac{80MPa \cdot 5.96 \times 10^{-4} m^4 \cdot 0.015m}{1.484 \times 10^{-3} m^3} = 481.9kN$$

AND THE BONDLINE?

$$\sigma_{steel_{\max}} = 150MPa$$

$$\tau_{steel_{\max}} = 80MPa$$

$$\tau_{adhesive_{\max}} = 5MPa$$



A steel I-beam is to be reinforced by bonding plates of the same material to the top and bottom flanges as shown in the figure.

d) Determine the maximum <u>shear force</u> that can be carried by the <u>reinforced</u> beam.

BONDLINE!

$$V = \frac{\tau I b}{Q} \qquad I_{reinf} = 5.96 \times 10^{-4} m^4$$

$$Q_{adh} = \left[\left(220mm + \frac{15mm}{2} \right) \cdot (150mm \times 15mm) \right] = 5.12 \times 10^{-4} m^3$$

$$V_{adh_{max}} = \frac{5MPa \cdot 5.96 \times 10^{-4} m^4 \cdot 0.15m}{5.12 \times 10^{-4} m^4} = 872kN \qquad V_{reinf_{max}} = 481.9kN$$