Structured Electronic Design

Building the nullor: Noise
What we did so far

Known:
- Topology
- Best nullor implementation
- Voltage and current swing
- Power consumption
- Noise level

Nullor design

\[
A_t = A_{t\infty} \frac{-L}{1-L}
\]
The nullor becomes a circuit

\[ C = B^2 \log \frac{S + N}{N} \]

bandwidth - noise - distortion
noise - bandwidth - distortion
noise - distortion - bandwidth
distortion - noise - bandwidth
distortion - bandwidth - noise
Trans-impedance amplifier

Source
- Current
- Amplitude 1mA
- $R_S = 4.7\,M\Omega$

Load
- Voltage
- $R_L = 1\,M\,\Omega$

Transfer
- $\frac{v_{out}}{i_s} = 1M\Omega$
- ........

Noise measured at the output:

\[ S_{out} = \frac{4kT}{4.7M} 10^{12} \Rightarrow 60 \frac{nV}{\sqrt{Hz}} \]
Trans-impedance amplifier

\[ R_f = 1 \text{M} \Omega \]

\[ i_s \uparrow \quad R_S = 4.7 \text{M} \Omega \quad \text{Nullor} \quad + \quad - \quad R_L = 1 \text{M} \Omega \quad \downarrow v_{out} \]

Noise measured at the output:

- Noise: 144 nV/√Hz
- Infinite bandwidth
- No distortion

\[ S_{out} = \frac{4kT}{4.7M} \cdot 10^{12} \Rightarrow 60 \frac{nV}{\sqrt{\text{Hz}}} \]

\[ S_{out} = \frac{4kT}{4.7M} \cdot 10^{12} + \frac{4kT}{1M} \cdot 10^{12} \Rightarrow 144 \frac{nV}{\sqrt{\text{Hz}}} \]
bandwidth – noise – distortion
bandwidth – distortion - noise
noise - bandwidth – distortion
noise – distortion - bandwidth
distortion - noise – bandwidth
distortion – bandwidth - noise

• Noise: 144 nV/√Hz
• Infinite bandwidth
• No distortion

Noise behavior can be analyzed in the presence of a nullor
Modeling the noise of the active circuit

Equivalent sources at the input
Modeling the noise of the active circuit

Equivalent sources at the output
Equivalent sources at the output

Calculated at
• input
  • easily compared to source
• output
  • can be measured
Noise optimization and Orthogonality
Orthogonality by design

**IF** the first stage has a large gain,
**then** the noise contribution of the following stages is negligible.
IF the first stage has a large gain...
When optimizing non-linear behavior (distortion):
- No dynamic (bandwidth) effects
- No noise

When optimizing bandwidth:
- No non-linearity
- No noise

When optimizing noise behavior:
- ☺ No non-linearity
- ☺ No bandwidth details
- Noise behavior can be analyzed in the presence of a nullor

When optimizing non-linear behavior (distortion):
- No dynamic (bandwidth) effects
- No noise
Why is it sufficient to consider the design of the first stage only during noise optimization?

a) Because at the input the noise is the largest
b) Because at the input the signal is the smallest
(c) Because the gain of the first stage is large
d) Because the equivalent noise source is at the input
Orthogonality by design: distortion

IF the last stage has a large gain,
then the clipping hazard for the preceding stages is negligible
Noise optimization is possible with nullor in the loop.
Bandwidth...

- All stages contribute to frequency behaviour
- Frequency behaviour can be improved anywhere
  - Not necessarily at input stage
  - Not necessarily at output stage
  - Bandwidth optimization is done last
Most orthogonal sequence: NDB

bandwidth – noise – distortion

noise – bandwidth – distortion

distortion – bandwidth – noise

distortion – noise – bandwidth

Specs

Verification
The first stage.

Trans-impedance amplifier

Noise measured at the output:

\[ S_{\text{in}} = \frac{4kT}{4.7M} \approx 10^{-10} \text{ V}^2/\text{Hz} \]

\[ S_{\text{out}} = \frac{4kT}{4.7M} \cdot 10^{10} + \frac{4kT}{1N} \cdot 10^{10} \approx 144 \text{ V}^2/\text{Hz} \]

\[ R_f = 1M\Omega \]

\[ R_s = 4.7M\Omega \]

\[ R_L = 1M\Omega \]

\[ v_{\text{out}} \]
The first stage...

Exercise 1

(a) Carefully review the performance of each of the practical implementations below.

Answer questions like:

- What is the (detailed) expected behavior of the ideal blocks?
- How well does the behavior of the practical implementations agree with this?
- What is the gain?
- How well defined are the input and output impedance?
- How reliable are the input and output impedance?
- What is the sensitivity to the source impedance and the load impedance?

The input impedance of a CB stage is the lowest.
The first stage..

Trans-impedance amplifier

\[ R_f = 1\, \text{M} \Omega \]

\[ R_S = 4.7 \, \text{M} \Omega \]

\[ i_s \]

\[ S_{in} = \frac{4kT}{4.7\, \text{M}} \times 10^{12} \Rightarrow 60 \frac{\mu \text{V}}{\sqrt{\text{Hz}}} \]

\[ S_{vis} = \frac{4kT}{4.7\, \text{M}} \times 10^{12} + \frac{4kT}{1\, \text{M}} \times 10^{12} \Rightarrow 144 \frac{\mu \text{V}}{\sqrt{\text{Hz}}} \]
Noise at the output via PSPICE

\[ R_f = 1\text{MΩ} \]

\begin{align*}
R_s & : \ 60 \ \text{nV/√Hz} \\
R_f & : \ 128 \ \text{nV/√Hz} \\
R_{bQ1} & : \ 0.4 \ \text{nV/√Hz} \\
I_{cQ1} & : \ 14 \ \text{nV/√Hz} \\
I_{bQ1} & : \ 130 \ \text{nV/√Hz} \\
\end{align*}

Gyrator: \[ \frac{60 \ nV}{\sqrt{\text{Hz}}} \]

Resistor feedback: \[ \frac{144 \ nV}{\sqrt{\text{Hz}}} \]

Input transistor: \[ \frac{193 \ nV}{\sqrt{\text{Hz}}} \]
### Influence second stage

![Circuit Diagram]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_s$</td>
<td>60</td>
<td>nV/√Hz</td>
</tr>
<tr>
<td>$R_f$</td>
<td>128</td>
<td>nV/√Hz</td>
</tr>
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<td>$R_{bQ1}$</td>
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</tr>
<tr>
<td>$I_{bQ1}$</td>
<td>130</td>
<td>nV/√Hz</td>
</tr>
<tr>
<td>$R_{bQ2}$</td>
<td>0.0024</td>
<td>nV/√Hz</td>
</tr>
<tr>
<td>$I_{cQ2}$</td>
<td>95</td>
<td>nV/√Hz</td>
</tr>
<tr>
<td>$I_{bQ2}$</td>
<td><strong>1300.0</strong></td>
<td>nV/√Hz</td>
</tr>
</tbody>
</table>

**Gyrator:**

\[
60 \frac{nV}{\sqrt{Hz}}
\]

**Resistor feedback:**

\[
144 \frac{nV}{\sqrt{Hz}}
\]

**Input transistor:**

\[
193 \frac{nV}{\sqrt{Hz}}
\]

1314 nV/√Hz
Reduction of the noise contribution of $Q_2$.

$2qI_b$ lower

Biasing $Q_2$ lower $\Rightarrow$ noise lower
Noise

Distortion

Orthogonality!!
The real cause of the problem?

\[ R_f = 1\text{M}\Omega \]

\[ R_S = 4.7\text{M}\Omega \]

\[ R_L = 1\text{M}\Omega \]

\[ D_{Q1} = 1 \]
Noise with CE input stage

\[ R_s : 54 \text{ nV/}\sqrt{\text{Hz}} \]
\[ R_f : 120 \text{ nV/}\sqrt{\text{Hz}} \]
\[ R_L : 0.7 \text{ nV/}\sqrt{\text{Hz}} \]
\[ R_{bQ1} : 0.4 \text{ nV/}\sqrt{\text{Hz}} \]
\[ I_{cQ1} : 14 \text{ nV/}\sqrt{\text{Hz}} \]
\[ I_{bQ1} : 130 \text{ nV/}\sqrt{\text{Hz}} \]
\[ R_{bQ2} : 0.0024 \text{ nV/}\sqrt{\text{Hz}} \]
\[ I_{cQ2} : 95 \text{ nV/}\sqrt{\text{Hz}} \]
\[ I_{bQ2} : 13.0 \text{ nV/}\sqrt{\text{Hz}} \]

\[ 208 \text{ nV/}\sqrt{\text{Hz}} \]

Gyrator: \[ 60 \frac{nV}{\sqrt{\text{Hz}}} \]

Resistor feedback: \[ 144 \frac{nV}{\sqrt{\text{Hz}}} \]

Input transistor: \[ 193 \frac{nV}{\sqrt{\text{Hz}}} \]
The first stage...

Gain is most important!
There is no right or wrong, just best, second best, .....
Design rule for noise

1st stage large gain

Noise behavior dominated by 1st stage

Orthogonality
Design rule for distortion

Last stage large gain

Clipping only in last stage

Orthogonality
Design rule for bandwidth

- Effects at any place in the loop
- Not necessarily at input
- Not necessarily at output

After steps noise and distortion
The most orthogonal sequence

- bandwidth - noise - distortion
- bandwidth - distortion - noise
- noise - bandwidth - distortion
- noise - distortion - bandwidth
- distortion - noise - bandwidth
- distortion - bandwidth - noise

Specs

1 → 2 → N → S → B → Bias → verification
We want it all…

Input and output magnitude are fixed
Demonstration
Vacuum tubes
Triode
Small-signal model
Demonstration
Negative feedback gives orthogonality

Feedback network

Total gain of active part is not limited to overall gain

All stages can have unlimited gain
Conclusions

• All stages should have maximal gain (CE CS)
• First stage: noise behavior
• Last stage: clipping
• Most orthogonal order: N, D, B
Conclusions

Negative feedback gives superior noise performance because:

1. First stage can be perfectly optimized for noise
2. First stage suppresses noise of cascading stages maximally
   - The Loop Gain can be much higher than the input-to-output gain of the amplifier
   - All stages can have maximum gain
3. Input impedance does not depend on first stage
Step 3: Noise design

\[ 4kT \beta_b \]

\[ 2qI_c \]

\[ 2qI_b \]

Nullor
What do we know about a noise source?

E.g.: \( S(v_n) = 4kTR \)

\[
P_n = \int_{B_{\inf}} S(v_n) \, df = \bar{v}_n^2
\]
Representation

• Voltage source
  • Use value \( v_n \) although only \( P_n \) is known
  • Transform them as voltages
  • Shift them as voltages

• Calculate the equivalent power
  • Add correlated voltages
  • Add power uncorrelated voltages

Similar representation for current sources
• Current source
  • Use value $i_n$ although only $P_n$ is known
  • Transform them as currents
  • Shift them as currents

• Calculate the equivalent power
  • Add correlated currents
  • Add power uncorrelated currents
The equivalent noise source

\[ V_{source} \]

\[ R_{source} \]

\[ V_{neq} \]

\[ i_{neq} \]

\[ \text{Noise free} \]

\[ v_{neq} + i_{neq} R_{source} \]

\[ R_{source} \]

\[ \text{Noise free} \]
The equivalent noise source

\[ v_{neq} \]

\[ i_{neq} \]

\[ R_{source} \]

Noise free

\[ I_{source} \]

\[ i_{neq} + \frac{v_{neq}}{R_{source}} \]

Noise free
Signal to noise ratio

\[ C = B \log_2 \frac{S + N}{N} \]

\[ v_{\text{in}}, v_{\text{n,eq,in}}, Z_s, Z_i, v_{\text{out}}, v_{\text{n,eq,out}}, Z_{\text{load}}, i_{\text{in}}, i_{\text{n,eq,in}}, Z_{\text{load}}, Z_{\text{out}} \]
Optimization via transformer

Choose ratio $n$ to minimize equivalent noise source

$$v_{neq} = \frac{v_n}{n} + nZ_{s}i_n$$
How to find the equivalent source?

Four methods:

- Voltage source shift
- Current source shift
- Norton-Thevenin transform
- Shift through two-ports
Voltage source shift

\[ v_{n,1} = v_{n,2} = v_n \]
Current source shift

\[ i_{n1} = i_{n2} = i_n \]
Norton-Thevenin transform

\[ v_n = Z i_n \]

\[ Z = \frac{1}{G} \]
Shift through two-ports
1. **Identification** of noise sources : \( v_n, i_n \)

2. Determine **equivalent** noise source (input/output) : \( v_{n,eq} \) or \( i_{n,eq} \)

3. Determine **power spectral density** of \( v_{n,eq} / i_{n,eq} \) : \( S_{n,eq} \)

4. Determine **power** of \( v_{n,eq} \) or \( i_{n,eq} \) : \( P_{n,eq} \)

5. **Minimize** noise power : \( P_{\text{optimal}} \)
1. Identification of noise sources

Noise due to quantized character of charge

• Thermal noise: due to collisions of carriers

\[ S_{v_{n,R}}(f) = 4kTR \]

• Shot noise: due to carriers crossing a junction

\[ S_{i_{n,d}}(f) = 2qI_d \]
Bipolar transistor

- thermal noise base resistance $r_b$
- shot noise collector-basis junction
- shot noise base-emitter junction
- $1/f$ - noise base-emitter junction

\[ S_{u_{n,r_b}}(f) = 4kT r_b \]
\[ S_{i_c}(f) = 2qI_c \]
\[ S_{i_b}(f) = 2qI_b \]
\[ S_{i_{b,f}}(f) = 2qI_b \frac{f_1}{f} \]
2. Determination of equivalent source

\[ 4kTR_s \]

\[ R_s \quad r_b \quad v_{n,b} \quad Bi_c \]

\[ s \quad + \quad - \]

\[ 4kTr_b \]

\[ B^2 2qI_c \]

\[ 2q(DI_c) \]

\[ D^2 2qI_c \]
$V_r = \frac{kT}{q}$

$B = \frac{V_T}{I_c}$

$4kTR_s$

$R_s$

$r_b$

$v_{n,b}$

$+\quad Bi_c\quad -$
$4kT R_s$

$R_s$

$r_b$

$v_{n,b}$

$B_i c$

$(R_s + r_b)i_b$

$4kT r_b$

$\frac{2qV_T^2}{I_c}$

$|R_s + r_b|^2 D2qI_c$

Power spectral densities!
\[ 4kTR_s + 4kTr_b + \frac{2qV_T^2}{I_c} + \left| R_s + r_b \right|^2 D2qI_c \]
Determine power of equivalent source

\[ B_{\text{inf}} \left( 4kT R_s + 4kT r_b + \frac{2qV_T^2}{I_c} + \left| R_s + r_b \right|^2 D2qI_c \right) \]
Minimize noise power

\[
\frac{dP_{n,eq}}{dI_C} = 0 \Rightarrow I_{C,\text{opt}}
\]

\[
B_{\text{inf}} \left( 4kTR_s + 4kTr_b + \frac{2qV_T^2}{I_c} + \left| R_s + r_b \right|^2 D2qI_c \right)
\]

\[
I_{C,\text{opt}} = \frac{V_T}{\left( R_s + r_b \right)\sqrt{D}}
\]
\[ P_{n,eq} \]

\[ B_{\text{inf}} \left( 4kT R_s + 4kT r_b + \frac{2qV_T^2}{I_c} + \left| R_s + r_b \right|^2 D2qI_c \right) \]
\[ P_{n,eq} \]

\[ \text{specification} \]

\[ I_{C,\text{opt}} \rightarrow I_C \]

\[ B_{\text{inf}} \left( 4kTR_s + 4kTR_b + \frac{2qV_T^2}{I_c} + \left|R_s + r_b\right|^2 D2qI_c \right) \]
Demonstration with capacitive source
\[ B_{\text{inf}} \left( 4kTR_s + 4kTR_b + \frac{2qV_T^2}{I_c} + \left| \frac{1}{j\omega C} + r_b \right|^2 D2qI_c \right) \]
Conclusions

- All stages should have maximal gain (CE CS)
- First stage: noise behavior
- Last stage: clipping
- Most orthogonal order: N, S, B
- Negative feedback gives orthogonality
- Bias current of first stage can be optimized for noise performance without restriction