Structured Electronic Design

Building the nullor: Distortion
Topology
Best nullor implementation
Voltage and current swing
Power consumption
**FINAL** Noise level
All stages maximal gain (CE CS)
Most orthogonal order: N, D, B

![Diagram with labels for noise, bandwidth, clipping, and loopgain]
bandwidth – noise – distortion
bandwidth – distortion - noise
noise - bandwidth – distortion
noise – **distortion** - bandwidth
distortion - noise – bandwidth
distortion – bandwidth - noise
When optimizing bandwidth:
Small-signal models (no non-linearity)
No noise

When optimizing noise behavior:
Small-signal models (no non-linearity)
No bandwidth details

When optimizing non-linear behavior:
No dynamic effects
No noise
Non-dynamic active circuit
(Not a nullor anymore)
In

Out

Weak distortion

Clipping

$In$
Clipping

$I_c$

$V_{be}$

$V_{ce}$
Weak distortion

Simplest model that suffices

\[ i_{out} = -I_s \left( \frac{v_{in}}{V_T} e^{\frac{v_{be}}{V_T}} - 1 \right) \]

\[ v_r = \frac{kT}{q} \]
Strategy

2 distortion types

Completely different approach
Clipping breaks the loop

More loop gain does not repair this
Clipping breaks the loop

More loop gain makes it worse
Strategy

Clipping breaks the loop

**local** problem

**local** solution

Weak distortion does not break the loop

can be in **any stage**

reduced by **the loop** (everywhere)
The bipolar transistor model

\[
\begin{pmatrix}
\tilde{A} & \tilde{B} \\
\tilde{C} & \tilde{D}
\end{pmatrix}
\]
Simulation of $B$
Simulation of $D$
D-type distortion is much less than B-type distortion
Location

\[ \tilde{B} \text{ or } \tilde{D} \quad \tilde{D} \quad \tilde{D} \quad \tilde{D} \]
Select a suitable tangent

\[ I_c = I_c^Q + \frac{dI_c(V_{be})}{dV_{be}} [V_{be} - V_{be}^Q] + \frac{d^2 f(V_{be})}{2dV_{be}^2} [V_{be} - V_{be}^Q]^2 + \ldots \]
Translation to the origin (Biasing)
Adding and subtracting offsets = Biasing

\[ i_c = \frac{1}{B_{V_{beQ}}} v_{in} + \frac{d^2 f(V_{be})}{2dV_{be}^2} \bigg|_{V_{be}=V_{beQ}} v_{be}^2 + \ldots \]
Bias parameters for transfer $B$

**IF:**
- Parameter 1 = $I_{cQ}$
- Parameter 2 = $V_{beQ}$

**Then:**
Suitable tangent in origin $B$ is still non-linear

\[ i_c = \frac{1}{B_{V_{beQ}}} v_{be} + \left. \frac{d^2 f(V_{be})}{2dV_{be}^2} \right|_{V_{be}=V_{beQ}} v_{be}^2 + \ldots \]
More bias parameters
4 bias parameters

**IF:**

- Parameter 1 = $I_cQ$
- Parameter 2 = $V_{beQ}$
- Parameter 3 = $V_{cQ}$
- Parameter 4 = $I_{bQ}$

**Then:**

- Suitable tangent in origin
- No clipping
4 bias sources needed for the translations

Implementation will be **done later** in the biasing step
Now biasing quantities are **just parameters**
Biasing does **not** make the circuit linear
Summary: Clipping and weak distortion

Still 1-1 relation with input

Weak distortion

Clipping

No 1-1 relation with input
Design the last stage: clipping

$V_{beQ}$ and $I_{bQ}$ not independent

$V_{cQ} \Rightarrow V_{\text{min}} < v_c < V_{\text{max}}$

$I_{cQ} \Rightarrow I_{\text{min}} < i_c < I_{\text{max}}$

Slewing
Power bandwidth $\omega_{\text{max}} > \omega_{\text{power}}$

$i_s = M \sin(\omega t)$

$I_c$ clips at $I_c = I_{bias}$

$M_{\text{full BW}} \rightarrow M_{\text{max}}$
Check all stages

\[ V_{cQ} \Rightarrow V_{\text{min}} < v_c < V_{\text{max}} \]
\[ I_{cQ} \Rightarrow I_{\text{min}} < i_c < I_{\text{max}} \]

Can be verified with small-signal models (AC analysis)
Conclusions: Clipping

The loop is broken!

**Loop gain does not help** (Disaster)
Can only be prevented locally

**Last stage large gain**

Clipping in last stage
Dominant criterion for last stage
Dominant influence on power consumption

Clipping of other stages should not be a problem
Weak distortion

Proper bias signals added (no offsets)

**No** clipping

Small distortion components $\rightarrow$ Third order is enough

\[ i_c = G_t v_{be} + G_{t2} v_{be}^2 + G_{t3} v_{be}^3 \]
Weak distortion, 4 cases
Weak distortion, Bipolar transistor

\[
\begin{pmatrix}
1 & 1 \\
\frac{g_m r_o}{g_m} & \frac{g_m}{g_m} \\
\frac{1}{\beta_F r_o} & \frac{1}{\beta_F}
\end{pmatrix}
\]

\[g_m\text{ or }B\text{ -type distortion}\]

\[\beta\text{ or }D\text{ -type distortion}\]
Weak distortion, bipolar transistor

\[ i_c = \frac{1}{B} v_{be} + \frac{d^2 f (V_{be})}{2dV_{be}^2} \bigg|_{V_{be}=V_{beQ}} \]

\[ v_{be}^2 + \frac{d^3 f (V_{be})}{6dV_{be}^3} \bigg|_{V_{be}=V_{beQ}} \]

\[ v_{be}^3 \]

\[ i_c = \frac{1}{D} i_b + \frac{d^2 f (I_b)}{2dI_b^2} \bigg|_{I_b=I_{bQ}} \]

\[ i_b^2 + \frac{d^3 f (I_b)}{6dI_b^3} \bigg|_{I_b=I_{bQ}} \]

\[ i_b^3 \]
**B-type distortion**

\[ I_c = I_s \left( \frac{V_{be}}{e^{V_T} - 1} \right) \]

\[ \frac{dI_c}{dV_{be}} = \frac{1}{V_T} I_s e^{\frac{kT}{V_{be}}} = \frac{I_c}{V_T} = \frac{1}{B} \]

\[ \frac{d^2I_c}{dV_{be}^2} = \frac{1}{V_T^2} I_s e^{\frac{kT}{V_{be}}} = \frac{I_c}{V_T^2} = \frac{1}{BV_T} \]

\[ \frac{d^3I_c}{dV_{be}^3} = \frac{1}{V_T^3} I_s e^{\frac{kT}{V_{be}}} = \frac{I_c}{V_T^3} = \frac{1}{BV_T^2} \]

\[ i_{out} = -\frac{1}{B} \left( v_{in} + \frac{v_{in}^2}{2V_T} + \frac{v_{in}^3}{6V_T^2} \right) \]
$B$ -type distortion is bias independent

$$i_{out} = -\frac{1}{B} \left( v_{in} + \frac{v_{in}^2}{2V_T} + \frac{v_{in}^3}{6V_T^2} \right)$$

$B$ -type at the **input**, when the nullator is a voltage sensor

**Reduce via loopgain** (no interference with noise optimum)
Reduction of $B$–type distortion

\[ i_{out} = -\frac{1}{B_1 D_2 D_3} \left( v_{in} + \frac{v^2_{in}}{2V_T} + \frac{v^3_{in}}{6V^2_T} \right) \]

Secondary effect: Increase bias first stage $\rightarrow B_1$ smaller $\rightarrow$ more loopgain
Without feedback, increasing bias first stage does not help
Influence of location of distortion

\[ i_{\text{ina}} = D_a \left[ D_b D_c i_{\text{out}} + \delta_2 \left( D_c i_{\text{out}} \right)^2 + \delta_3 \left( D_c i_{\text{out}} \right)^3 \right] \]

\[ i_{\text{inb}} = D_b i_{\text{outb}} + \delta_2 i_{\text{outb}}^2 + \delta_3 i_{\text{outb}}^3 \]

\( D_a \) has a linear effect on distortion

\( D_c \) has a **quadratic** and **cubic** effect on distortion
Gain after non-linear stage is best

\[ i_{ina} = D_a \left[ D_b D_c i_{out} + \delta_2 \left( D_c i_{out} \right)^2 + \delta_3 \left( D_c i_{out} \right)^3 \right] \]

Gain last stage important!
Insert extra gain after distorting stage
Maximum gain all stages
Worst distortion often at input (B -type)
Distortion caused by first transistor

\[ i_L = L i_{L1} + l_2 i_{L1}^2 + l_3 i_{L1}^3 \]

\[ L = \frac{R_F}{B_a D_b D_c} \]

\[ l_2 = -\frac{1}{2V_T} \frac{B_a^2 D_b^2 D_c^2}{R_F} = \frac{1}{2V_T} \frac{1}{L} B_a D_b D_c \]

\[ l_3 = -\frac{1}{3V_T^2} \frac{B_a^3 D_b^3 D_c^3}{R_F} = \frac{1}{3V_T^2} \frac{1}{L} B_a^2 D_b^2 D_c^2 \]

Gain of stage itself works as if it’s after its non-linearity
Weak distortion, FET

Always $g_m$ distortion
Distortion caused by last transistor

\[ i_L = L i_{L1} + l_2 i_{L1}^2 + l_3 i_{L1}^3 \]

\[ L = \frac{R_F}{A_a A_b B_c} \]

\[ l_2 = -\frac{1}{4} \frac{A_a A_b B_c}{R_F I_{dQc}} = -\frac{1}{4} \frac{1}{L I_{dQc}} \]

\[ l_3 = -\frac{1}{8} \frac{A_a A_b B_c}{R_F I_{dQc}^2} = -\frac{1}{8} \frac{1}{L I_{dQc}^2} \]

\[ A \propto \left( \text{Channel length} \right)^{-1} \]

Longer transistor \(\Rightarrow\) lower distortion
Bipolar versus FET

\[ l_2 = -\frac{1}{2V_T} \frac{B_a^2 D_b^2 D_c^2}{R_F} = \frac{1}{2V_T} \frac{1}{L} B_a D_b D_c \]  
Distortion from first stage

\[ l_3 = -\frac{1}{3V_T^2} \frac{B_a^3 D_b^3 D_c^3}{R_F} = \frac{1}{3V_T^2} \frac{1}{L} B_a^2 D_b^2 D_c^2 \]  
Reduce via bias, loopgain

\[ l_2 = -\frac{1}{4} \frac{A_a A_b B_c}{R_F I_{dQc}} = \frac{1}{4} \frac{1}{L} \frac{1}{I_{dQc}} \]  
Distortion from last stage

\[ l_3 = -\frac{1}{8} \frac{A_a A_b B_c}{R_F I_{dQc}^2} = \frac{1}{8} \frac{1}{L} \frac{1}{I_{dQc}^2} \]  
Reduce via bias, loopgain, layout
Local feedback

Reduces non-linearity of stage

Impedance in active part

Reduces the overall loopgain

Never better than overall loop

Increases distortion other stages

Especially for last stage not a good idea

Only when non-linearity makes variation of loopgain too large
Conclusions weak distortion

$B$ -type distortion (only at nullor input)
$D$-type distortion (for bipolar)

Increase the loopgain

Via biasing ($B(I_Q)$)
Via Layout ($FET$)
Extra stage (preferably after distortion source)

No local feedback

Use high gain last stage (also reduces clipping preceding stage)
Dynamic distortion

Volterra series

Dynamic eigenvalues

Loopgain helps

Still research
Conclusions
Specifications

- Harmonic distortion
- Intercept points
- Compression point
- Intermodulation
- Power bandwidth
Harmonic distortion

Compare amplitudes fundamental harmonic with \( n^{th} \) order harmonic

\[
HD_n = \frac{y_n}{y_1}
\]

\[
THD = \sqrt{\sum_{n=2}^{m} |HD_n|^2}
\]
Intercept points

output amplitude (log)

$y_1$ $y_2$ $y_3$

$IP_2$ $IP_3$

input amplitude (log)
Compression point

-3 dB

input amplitude
(log)

output amplitude
(log)

y₁  y₂  y₃
Intermodulation

\[ \omega_1 - \omega_2 \]
\[ 2\omega_1 - \omega_2 \]
\[ \omega_2 - \omega_1 \]
\[ \omega_1 + \omega_2 \]
\[ 2\omega_1 + \omega_2 \]

Magnitude vs. Frequency

IM2, IM3, fundamental IM2