Structured Electronic Design

The Rosenstark Method

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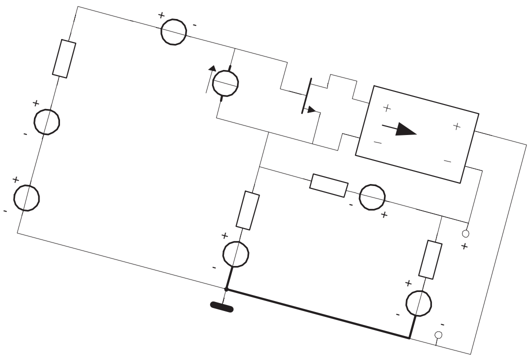
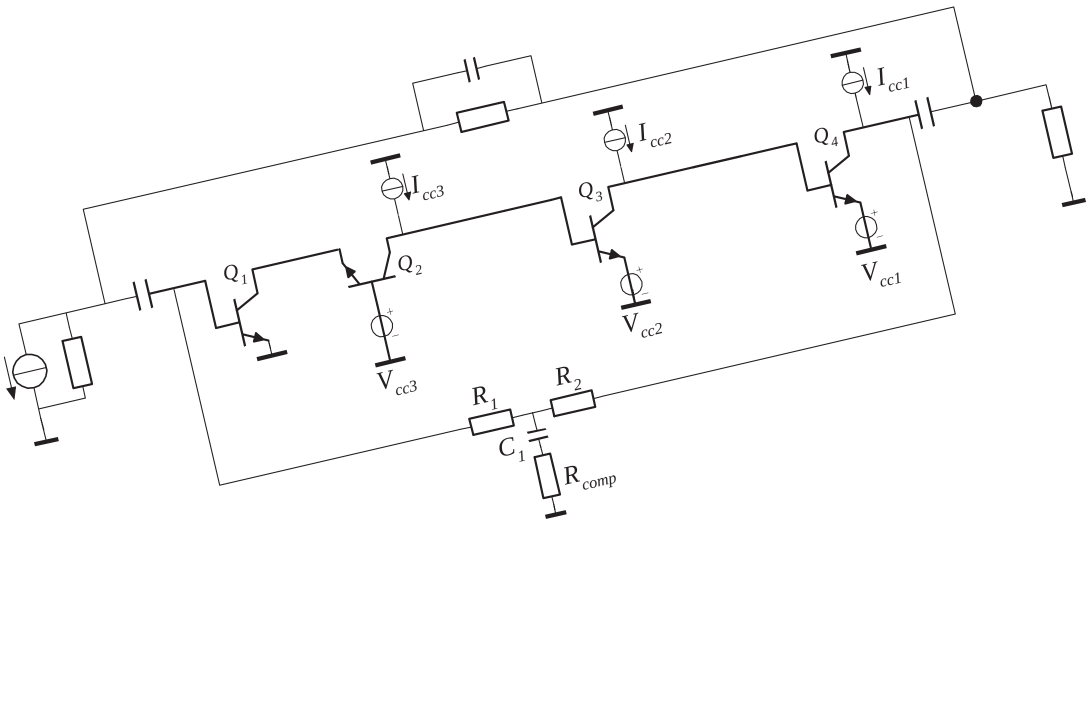
Edition 2005





1 The Rosenstark method

1.1 Introduction

This text presents the Rosenstark method to evaluate the frequency behavior of an electronic circuit. When properly understood and used, this method can give a designer a quick insight in the effectiveness a frequency compensation component can have in a circuit. This text is intended as background material for students using the book Structured Electronic Design [1] in the courses ET3400, ET8016 and ET8011. Without studying the book, some parts of the text may be more difficult to study.

2 Two methods for frequency compensation

The goal of frequency compensation is to alter the characteristic polynomial of a circuit such that the poles of the transfer function are placed at the required positions in the s-plane. A general expression for the characteristic polynomial of the system is given by:

$$CP(s) = s^n - \sum_{i=1}^{n} p_i s^{n-1} + \cdots + LP. \tag{1}$$

Frequency compensation is equivalent to changing the coefficients of the s terms without changing the LP-product, i.e. the maximum attainable bandwidth is not lowered. Two different methods can be used:

- Frequency compensation via the relation between the loop poles and the system poles. The system poles are found from the loop poles via the root-locus method. Via various procedures this root locus is influenced. The negative feedback concept plays a crucial role in this. It is well described in the book Structured Electronic Design [1] and will not be discussed here.
- . Frequency compensation without using the feedback concept or any other method to partition the amplifier. The system poles are directly manipulated. This method is generally applicable, independent of the number of loops etc., and it uses the Rosenstark [5] method for determining these system poles. Unfortunately it is difficult to relate the effect of the frequency compensation done in this way to other performance aspects like noise, distortion, accuracy etc. and the powerful concept of orthogonal design as described in [1] cannot be used here.

2.1 The Rosenstark method

The method was introduced by Rosenstark [5] and is based on Cramers rule. With this method the characteristic polynomial of a system is by inspection and relatively simple DC calculations. As it is decribed here, the method described is limited to systems without inductors.

An electronic circuit can be represented by an n-port, with n equal to the number of capacitors in the system. The n-port contains practical active devices like transistors, resistors, dependent and independent sources, but no inductive elements. This is shown in fig.1 models an electronic circuit.

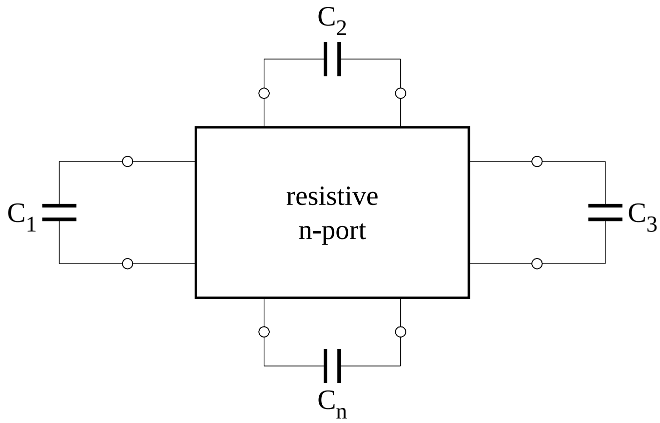


Fig. 1: A model of an electronic circuit

The coefficients of the characteristic polynomial are found from the DC impedances at the n ports by inspection. When the characteristic polynomial is given by:

$$CP(s) = 1 + sa_1 + s^2a_2 + \cdots + s^n a_n \tag{2}$$

the coefficients are found as follows. The coefficient of the first-order term is found by summing the time constants of the n ports. These time constants, τ_i, are given by the DC port impedance, R_i, times the loading capacitor C_i:

$$a_1 = \sum_{i=1}^{n} \tau_i = \sum_{i=1}^{n} R_i C_i. \tag{3}$$

For the second-order coefficient all the *interactions* between two time constants are summed. An interaction between, for instance, time constant 1 and time constant 2 is found by short circuiting port 2, calculating the time constant at port 1, τ_{12}, and multiplying this time constant with the first-order time constant of port two, τ_2. When all the possible interactions between two ports are summed, the second-order coefficient is found:

$$a_2 = \sum_{i=1}^{n} \sum_{j=i+1}^{n} \tau_{ij} = \sum_{i=1}^{n} \sum_{j=i+1}^{n} R_{ij} C_i \cdot \tau_j. \tag{4}$$

The impedance R_{ij} is the dc impedance found at port i when port j is short circuited. The meaning of these interaction terms can be visualized as follows. When the time constants do not interact a contribution of two time constants τ_1 and τ_2, to the second-order term, is just $\tau_1\tau_2$. This can easily be seen from expanding the product of n independent poles:

$$CP(s) = \prod_{i=1}^{n} (s\tau_i + 1). \tag{5}$$

However, when the two time constants interact, this is not the actual contribution. For instance, when the first-order time constant at port 2 is much lower than the first-order time constant at port 1, and the impedance at port 1 is made much lower by the short-circuit action of C_2, the actual time constant at port 1 is much lower. This is taken into account in expression (4). The time constant at port 1 is found by assuming an infinite frequency at port 2 such that a kind of $\tau(\infty)_{12}$ is found with a maximal influence of port 2. Subsequently, this time constant is multiplied by the first-order time constant of port 2, accounting for the difference between the maximal influence and the actual frequency dependent influence of port 2 on port 1.

The third-order coefficient is found by extrapolating the previous sums. The interaction between three poles is evaluated by making a short circuit at two ports and determine the effect on the impedance of the third port. This results for a_3 in:

$$a_3 = \sum_{i=1}^{n} \sum_{j=i+1}^{n} \sum_{k=j+1}^{n} C_i \tau_{ijk} = \sum_{i=1}^{n} \sum_{j=i+1}^{n} \sum_{k=j+1}^{n} R_{ijk} C_i \tau_{jk}. \tag{6}$$

To make the method more clear an example of a third-order network is given.

2.1.1 An example

The characteristic polynomial of the third-order network of fig.2 is derived. The network has

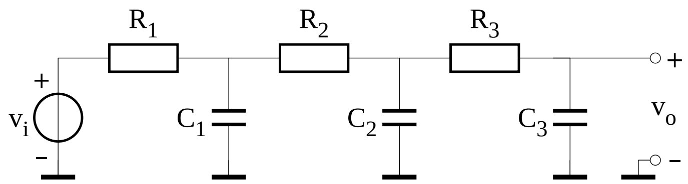


Fig. 2: A third-order network

three capacitors and thus for the Rosenstark method three ports can be distinguished. The numbers of the capacitor corresponds to the port number.

The first-order coefficient is given by the sum of the three first-order time constant:

$$\begin{aligned} \tau_1 &= R_1 C_1, \\ \tau_2 &= (R_1 + R_2) C_2, \\ \tau_3 &= (R_1 + R_2 + R_3) C_3, \\ a_1 &= \tau_1 + \tau_2 + \tau_3. \end{aligned} \tag{7}$$

The second-order coefficient is found by the summing the three second-order interaction terms:

$$\begin{aligned} \tau_{12} &= R_{12} C_1 \tau_2 = \frac{R_1 R_2}{R_1 + R_2} C_1 (R_1 + R_2) C_2 = R_1 C_1 R_2 C_2, \\ \tau_{13} &= R_1 (R_2 + R_3) C_1 C_3, \\ \tau_{23} &= R_3 (R_1 + R_2) C_2 C_3, \end{aligned}$$

$$a_2 = \tau_{12} + \tau_{13} + \tau_{23}. \tag{8}$$

The third-order term is given by the third-order interaction term(s). As for this third-order interaction terms two port are short circuited, only one interaction term exists. When ports 2 and 3 are short circuited a_3 equals:

$$a_3 = R_{123}C_1\tau_{23} = \frac{R_1R_2}{R_1+R_2} \cdot C_1 \cdot R_3(R_1+R_2)C_2C_3 = R_1R_2R_3C_1C_2C_3. \tag{9}$$

When for instance port 1 and 3 are short circuited, the following expression is found:

$$a_3 = R_{213}C_2\tau_{13} = \frac{R_2R_3}{R_2+R_3} \cdot C_2 \cdot R_1(R_2+R_3)C_1C_3 = R_1R_2R_3C_1C_2C_3. \tag{10}$$

Obviously it does not matter which ports are short circuited and which are used for evaluation to see the effect of an interaction between ports . However, a clever choice make a lot of difference in the number and the difficulty of the calculations to be done. It is most convenient to short circuit the ports for which the largest part of the circuit is "removed".

2.2 Frequency compensation

Frequency compensation with the Rosenstark method concentrates on changing time constants such that the characteristic polynomial gets the required form. When for instance a second-order system is given by:

$$CP(s) = 1 + s(\tau_1 + \tau_2) + s^2\tau_{12} \tag{11}$$

and the first-order coefficients need to be enlarged without changing the second-order coefficient, significantly, ($a_2 = 1/LP$) an additional capacitor, C_x, is placed at a third port (fig.3). Now the characteristic polynomial changes to:

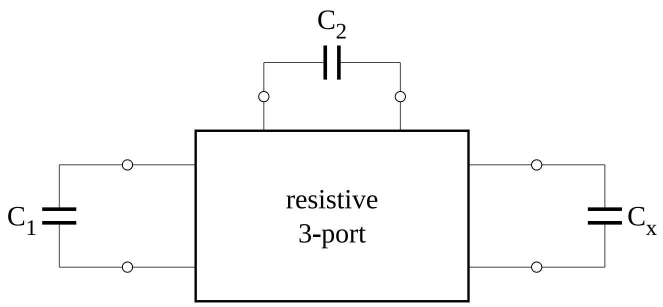


Fig. 3: Addition of an extra capacitor for frequency compensation purposes

$$CP(s) = 1 + s(\tau_1 + \tau_2 + \tau_x) + s^2(\tau_{12} + \tau_{1x} + \tau_{2x}) + s^3\tau_{12x}. \tag{12}$$

The third-order coefficient τ_{12x} needs to be small enough, such that the number of poles that has to be taken into account, is not increased, i.e. the number of the dominant poles remains two. This can be obtained by using a port of which the impedance becomes very low when port 1 and 2 are short circuited. This impedance is a part of τ_{x12} and, consequently, τ_{12x} is also low, hopefully resulting in a non-dominant pole. An example of this is the discussion on the effectiveness of a phantom zero in [1]. Introduction of a phantom zero also introduces a pole which is in the case of an effective phantom zero a non-dominant pole. The Rosenstark method can be used to quickly evaluate this effectiveness.

In the case of a compensation capacitor between a base and a collector of a transistor (to obtain, in asymptotic-gain model terms [1], pole-splitting) short circuiting the base-emitter port and base-collector port results in a zero impedance at the compensation port and thus $\tau_{x12} = 0$. Consequently, the order is not increased. When the third-order coefficient is

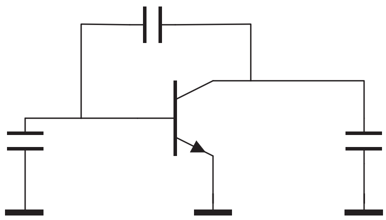


Fig. 4: A transistor with a source, a load and a frequency compensation capacitor.

negligible small, the second-order coefficient is still a measure for the bandwidth. Of course, it is not allowed to decrease this bandwidth significantly and thus τ_{1x} and τ_{2x} needs to be negligible small with respect to τ_{12}. This means that ports should be used which, when they are short circuited, cause the impedance of port 1 and 2 to become relatively low. In the case of a base-collector compensation capacitor (fig.4), short circuiting this capacitor results in a diode connected transistor. The impedances at the two ports become both approximately $1/g_m$. This is much smaller than the impedance at port 1 when for instance port 2 is short circuited, i.e. r_π and thus the two additional second-order time constants are very small. This gives a small effect on the LP-product. Shorting two capacitors to find the third other term reveals that the third order term is zero. This three-capacitor circuit is second order!

3 Conclusion

The Rosenstark method relies on finding the characteristic polynomial of the total system by means of inspection and simple dc calculations. With this method, the maximum attainable bandwidth can easily be found by only calculating the coefficient of the highest dominant order. Because this term equals $1/LP$ (where LP is the LP-product as described in [1]) it predicts the maximum attainable bandwidth. As this method concentrates directly on the characteristic polynomial, frequency compensation is done via the addition of time constants to the system to modify the characteristic polynomial. This can be done by first identifying the effective ports and after that finding again the correct value(s) for the compensation component(s). Unfortunately it is difficult to relate the effect of the frequency compensation done in this way to other performance aspects like noise, distortion, accuracy etc. and the powerful concept of orthogonal design as described in [1] cannot be used here. It can however be very useful to rank various compensation solutions found with the structured electronic design method on the basis of effectiveness.

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