

Assignment 3

Subject: Parameter estimation and optimization in time and frequency domain, parameter accuracy, predicted variance

Deadline: May 10, 2010, 08:45h

Part I: Optimization in time domain

Introduction

This assignment covers some examples of parameter estimation and optimization. In part I some basic characteristics of optimization with grid and gradient search methods are discussed. Part II will familiarize you with a genetic algorithm.

Assignment 3.1: Grid & gradient optimization

For this part you need the data in `dataset1.mat`. The dataset contains two vectors x and y , which could have been the result of some kind of measurement. Let us say we have reason to believe that the output of the measured system is related to the input as follows:

$$y_i = (t - a) \cdot (\sin(b \cdot t) + \cos(b \cdot t)).$$

- a) Estimate the unknown constants a and b using a grid search of sufficient resolution in the region $[0,8]$ for both a and b . Also, plot the error as a function of a and b in a surface plot. Hint: you will first have to write a least squares criterion function (=error function) of the form $e = f(p, t, y)$, where output e is the summed square error, p is the parameter vector (containing a and b) and t and y are the "measured" signals.
- b) Draw the measured signal and best estimate of y in one plot. Comment on the results.
- c) Use Matlab's function `fmincon()` to perform a gradient search instead of a grid search. Try 3 different starting points; do they affect the outcome? Explain why (not).
- d) Plot the measured signal y together with the best estimate of y . Compare with the results in b) and comment.
- e) Compare the total number of function evaluations used with the grid search and used with `fmincon()`. Comment on the efficiency and accuracy of the two methods.
- f) You probably have noticed that the error function has multiple local minima. Suggest and implement a combination of a grid search and gradient search which robustly finds the global minimum independent of the starting point (range $[0,8]$) in less function evaluations than just a grid search.

Assignment 3.2: Genetic optimization

This part will introduce you to genetic algorithms (GAs). First, it covers some basic properties of GAs based on the problem in Assignment 4.1. General note: when answering these questions, always repeat your optimization a couple of times. A genetic algorithm optimization involves a lot of random processes!

- g) Use a genetic algorithm (function `ga()`) to find the optimum of the problem in part I (using `dataset1.mat` again). Take a population of 20 individuals. Set the initial range to $[0,8]$ for both parameters. Each new generation needs to be created by taking two elite individuals and using a crossover fraction of 0.6. Use the provided custom plot function `customplotfcn.m` for monitoring the optimization process. All these options can be set using the `gaoptimset()` command. Comment on the optimal solution and convergence of the algorithm.
- h) Set the number of elites to zero and the cross over fraction to 0.05. Repeat the optimization. Comment on the optimal solution and convergence of the algorithm.
- i) Keep the number of elites zero and set the cross over fraction to 0.95. Repeat the optimization. Comment on the optimal solution and convergence of the algorithm.

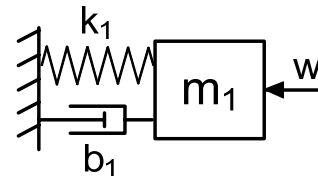
- j) Vary the following settings and describe their effect on optimization result (or accuracy) and the time required to find the optimum: population size, number of elites, crossover fraction. Draw a table which clearly illustrates the effects that you describe.

Part II: Optimization in frequency domain

In this part of the assignment you will estimate parameters in the frequency domain. For this part you need the data in `dataset2.mat`. The dataset was obtained in an experiment where a subject's wrist was perturbed with a torque $w(t)$ and the wrist angle $x(t)$ was measured. You will be asked to estimate the mechanical properties with two different model structures.

Assignment 3.3: Mass-spring-damper

Often, the human control behavior can be reasonably well represented with a simple mass-spring-damper model. Use the transfer function from perturbation torque $w(t)$ to wrist angle $x(t)$ to estimate the parameters. Use `lsqnonlin()` as the optimization function.



- k) Use the Fourier transformed signals to estimate the frequency response function (FRF)

according to: $\hat{H}_{xw} = \frac{S_{xw}}{S_{ww}}$. Make a Bode diagram (gain and phase) and a graph of the

coherence (apply frequency averaging over 8 frequency bands).

- l) Explain how the dynamic characteristics can be used to estimate parameters of a model in the frequency domain? In what frequency ranges do the mass, the spring and the damper affect the frequency response function (the most)?
- m) Derive the transfer function in the s-domain and include it in your report.
- n) Use `lsqnonlin()` to estimate the parameters with the error definition from the lecture notes, including coherence and frequency scaling. Plot the fitted transfer function on top of the Bode diagram from k). Does the fit describe the dynamics well?

Assignment 3.4: Mass-spring-damper with feedback

We know that humans use afferent feedback for motion control. Supplement your model with velocity feedback. The feedback should contain a feedback gain k_v , a time delay t_d and a first order filter with time constant 0.030 s, representing activation dynamics.

- o) Derive the transfer function in the s-domain and include it in your report.
- p) Use `lsqnonlin()` to estimate the parameters with the error definition from the lecture notes, including coherence and frequency scaling. Plot the fitted transfer function on top of the Bode diagram from k). Does the fit describe the dynamics well?

Assignment 3.5: Frequency range selection

At high frequencies the noise dominates the FRF and may affect the parameter fit. Redo the fit from Assignment 4.4, but now only select the frequencies up to 20 Hz.

- q) Use `lsqnonlin()` to estimate the parameters with the error definition from the lecture notes, including coherence and frequency scaling. Plot the fitted transfer function on top of the Bode diagram from k). Does the fit describe the dynamics well?

Part III: Comparison optimization in frequency and time domain

So far, you have been estimating parameters without having the means of quantitatively evaluating the accuracy of the parameters and the goodness of the fit. In this part, you will be using two such measures: the standard error of the mean (SEM) and the variance accounted for (VAF). Both SEM and VAF will be compared between the time and frequency domain.

Assignment 3.6: mass-spring-damper

In assignment 4.3 the simple model structure described the human control behavior reasonably well. In this part the model will be fitted to the same dataset as used in Part II (`dataset2.mat`) but now in the time domain. Note that you will have to build a Simulink model of the provided schematic model to simulate the response of the system. Use the boundaries for the optimization as provided in the Appendix.

- r) Give the optimal parameter m_1 , b_1 and k_1 and plot the model output together with the measured output. Is your fit good?
- s) Now, determine the VAF of your fit. Explain how VAF expresses the goodness of the fit.
- t) Determine the SEM of the parameters. Plot the parameter values together with the SEM in an error bar graph (see Assignment Guide). Interpret and discuss the results.

Assignment 3.7: mass-spring damper with feedback

We know that humans use afferent feedback for motion control.

- u) Could this explain the relatively low VAF you have found in Assignment 4.6?

Supplement your Simulink model with velocity feedback that is multiplied by a gain k_v , delayed with a time delay t_d , and filtered with first order activation dynamics with time constant 0.030 s. Note that there are no non-linearities in the model.

- v) Fit your model and give the optimal parameters m_1 , b_1 , k_1 , t_d and k_v . Plot the model output and measured output.
- w) Determine the VAF. Compare to Assignment 4.6. Explain that even if we would have had the exact correct model structure, the VAF would never be exactly 1.
- x) Plot an error bar graph of the parameter values and the SEM. Comment on the results.

Assignment 3.8: Comparison time and frequency domain approach

- y) Make a table of the parameters with the SEMs and the VAFs of both the time domain and the frequency domain approaches (Part II) and discuss similarities and differences.
- z) Is there a difference in calculation time to estimate the parameters between the two approaches? Explain.
- aa) In the frequency domain approach you needed to fit your model only to the frequencies up to 20 Hz to obtain an accurate fit. In the time domain approach we did not filter the signal to get rid of the (high frequent) noise, but we still obtained a good fit. Explain why.

Appendix: recommended parameter boundaries

Parameter	Unit	Lower	Upper
m_1	[Nm s ² /rad]	0.001	0.008
b_1	[Nm s/rad]	0.05	0.25
k_1	[Nm/rad]	1	10
k_v	[Nm/rad]	0.01	0.3
t_d	[s]	0.010	0.035
m_2	[Nm s ² /rad]	5	100
b_2	[Nm s/rad]	0.01	10
k_2	[Nm/rad]	5	100